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PROBABILISTIC ANALYSIS OF TUNNEL LINERS

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Abstract

The use of probabilistic methods to analyse reliabiliy of structures is being applied to a variety of engineering problems due to the possibility of establishing the failure probability on rational grounds. In this paper we present the application of classical reliability theory to analyse the safety of underground tunnels.

1 General

The continuous development of transportation networks with stringent geometric conditions has induced an increasing pace of tunnel construction. Generally the techniques used to build them are based on a careful control of the structural response of the lining and this implies the use of sophisticated numerical techniques of analysis to predict and interpret the experimental results. In particular it would be interesting to have a measure of the liner reliability in relation with the different conditions that could be expected to limit the tunnel performance.

In that sense the use of the Limit State philosophy, widely admitted in other areas of structural analysis, seems to be very promising. The same can be said in relation with the so-called Level II methods of structural reliability that allows the implementation of a general procedure as well as the possibility of calibration of Codes of Practice based on the factored load and resistance properties.

In this paper we are going to show the way in which we have tried to establish a procedure to quantify the reliability of road tunnel liners.

The plan of the paper is as follows: first of all we shall describe some details of the physical process, then a series of Limit States will be defined. After that, a simple mathematical model will be defined according to the very well known New Austrian Tunneling Method (NATM) procedure. After summarizing some results obtained with Level II methods, more general models will be presented that can be applied to a variety of situations including classical Terzaghi approach or the Panet procedure to simulate 3D conditions using 2D procedures.

2 The physical model

The problem that arises in tunnel piercing can be explained using figure 1. In it four zones have been established in a sequence through which all sections of the tunnel have to pass.

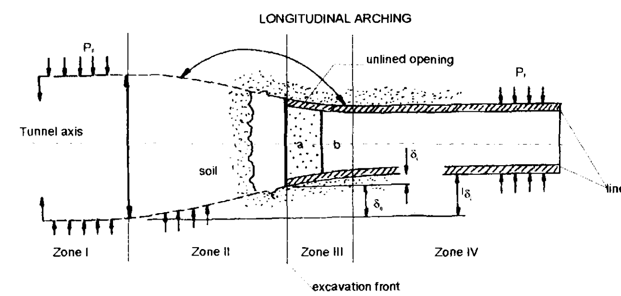

Figure 1

Zones I and IV are stabilized situations. In the first, the soil is unaltered, because the effect of the perforation has not attained it, while in zone IV all the interaction effects between the soil and the liner have reached an equilibrium so the lined opening can be considered stable.

In zone II the so-called convergence (that is, displacements directed towards the tunnel axis) of the soil not yet excavated, starts. The length of the front influence varies depending on the soil properties between one and two tunnel radii. The significance of this zone is that before any opening has been materialized the soil has already suffered a decompression that alters the initial "at rest" stress-state.

In zone III the excavation has been finished and two subzones can be identified according to whether or not the linning has been built. In zone (III a) the soil or rock is unsustained so a rapid convergence happens.

Generally it is very difficult to measure those convergences due to the complicated operations that have to be done in order to mantain the workers safety, but some of them can be obtained. As could be expected the length of the subzone is very dependent on the soil strength and is part of some empirical classifications (see for instance ref 2)

In zone (III b) the liner has been input and the process of soil-structure interaction starts until the equilibrium of zone IV reached. It is generally in this area where most measurements are done: convergence is one of them, of course, but also total stresses in cells included in the liner or differential displacements measured with extensometers anchored to the liner. From those measurements it is possible to establish a data bank to be used in the calibration of numerical models that can be utilized to compute the failure probability of the system as indicated in figure 2.

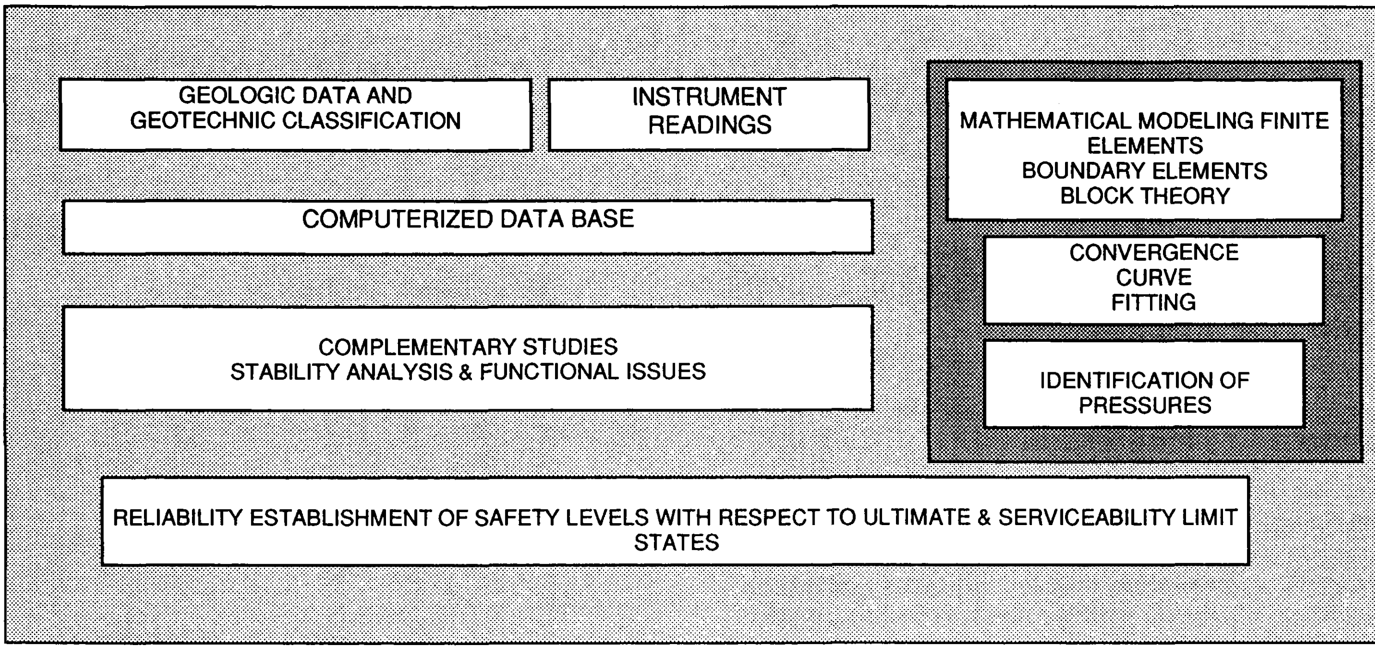

Figure 2

3 Limit States

The limit states are related, as usual, to conditions that have to be fulfilled in order to mantain the serviceability or to guarantee the resistance of the tunnel. We propose to select as a **first limit state** a value related to the convergences.

On the one hand convergences measure the degree of stabilization of the soil-liner system but on the other they also reflect the accuracy of the geometric dimensions of the structure. In case of symmetric deeply embedded structures the convergence is twice the radial component of the displacements.

The admissible value is usually taken as a proportion α, R of the liner radius, so the first condition is

$$f_1\left(K_0\,;\gamma_S\,;H\,;\text{etc}\right)\leq\alpha R$$

where f_1 (K_0, K_S, H, etc) is a function of the at rest coefficient K_0, soil density γ_S, depth of embedment H, and other parameter influencing displacements. If conditions are non symmetric the limit-state could be based on the absolute maximum of the displacements or on a weighted mean value that can take into consideration the different importance of displacements in relation with the use of space (i.e.: a ventilation duct, or a traffic space or the part outside the circulation area, etc).

The **second and third limit states** are related to the liner resistance and soil resistance respectively. The liner resistance can be expressed in terms of hoop stresses for the case of symmetric scenarios as a function

$$f_2\left(t, R, f'_{ck}\ \text{etc}\right)\leq\alpha_2\, f'_{ck}$$

where t, R, f'_{ck}, etc are respectively the liner thickness, liner radius, characteristic value of concrete strength, and any other parameter influencing the resistance.

In less simple cases it would be better to work with actions effects and to establish the admissibility on the basis of an interaction diagram of bending moments, and axial and shear forces.

The soil resistance is another factor to control because the key idea in the New Austrian Tunneling Method (N.A.T.M.) (3,4) it just to create a mixed structure in which the liner supplies radial confinement to the soil in the degree neccesary to obtain a ductile behavior of the rock mass. In that way a soil arch or plastified zone is created around the tunnel. This arching effect is fundamental to obtain the stability and the economy, so the **third limit state** is related to the soil resistance. A way to control the soil arch in a symmetric situation is to compare the radius of the plastified zone with the liner radius, i.e.

$$f_3\left(K_0\,,\gamma_S\,,H\,,t, R, \varphi, c, \text{etc}\right)\leq\alpha_3\, R$$

where the parameters are those of the previous states plus some others like the internal soil friction φ, cohesion c, etc.

In more complicated cases the plastified zone is not concentric with the liner and then it is better to establish a maximum distance or a weighted mean distance to the liner as a criterium.

Other limit states can be envisaged similary depending on the use of the tunnel, the soil conditions, the embedment depth, etc. For instance in shallow depths it is possible that the formation of a key block due to the rock fissuration could be more critical because the confinement pressure could be not high enough to induce ductile behavior. In those cases a different behavioral model would be used as well as a different criterium. In this paper only the above mentioned criterium will be used in order to clarify the application of the reliability philosophy.

4 Mathematical model

The classical interpretation of the N.A.T.M. can be explained using figure 3 that can be compared with figure 1.

The idea is to plot for a symmetric situation the pressures that should be applied to the boundary of a circular hole (figure 3.a) to obtain a fixed radial displacement δ. The curve AB (figure 3.b) shows after an elastic part AA' a curved branch A'B, reflecting the plastic behaviour of the continuous media.

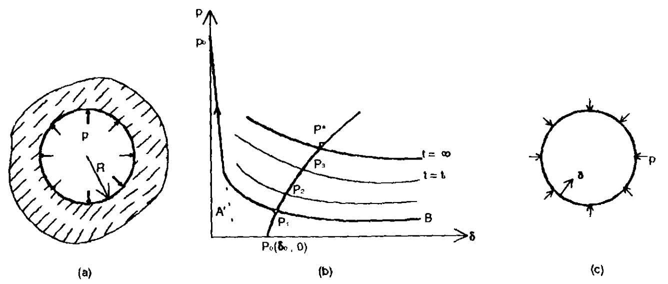

Figure 3

Similarly it is possible to plot the same relation for a ring representing the liner (figure 3.c). in this case the curve is P_0 P^* and starts at a point $(\delta_0,0)$ because before the liner is built (zones I, II and III a of figure 1) the soil has contracted. The equilibrium is reached at point P_1 except if the soil is viscous in which case a family of AB curves will develop for different time intervals t_i. In a viscoplastic soil the final curve will be reached for $t = \infty$ where all the plastic deformations are developed Point P^* will be then the design point or check-point because there all the three above mentioned limit states are in the worst conditions.

P. Fritz (ref 5) using a Mohr Coulomb criterium has developed the formula to quantify the soil model. If a linear behaviour of the liner is assumed it is possible to compose Table I

SOIL	LINER
$\frac{\rho}{R} = \left[\left(\sigma_p + \frac{\sigma_d}{m-1}\right)\frac{1}{\frac{\sigma_d}{m-1} + p_i}\right]^{1/m-1}$	$\varepsilon^l = \frac{\delta_l - \delta_0}{R}$
$m = \frac{1+\operatorname{sen}\varphi}{1-\operatorname{sen}\varphi} \quad \sigma_d = \frac{2c\cos\varphi}{1-\operatorname{sen}\varphi}$ $\sigma_p = \frac{2p_0 - \sigma_s}{1+m}$	$\delta_l - \delta_0 = \frac{p_i R^2}{E_l\, t}$
$\delta_i = R\frac{1+\nu_s}{E_s}\left[k_2\left(\frac{\rho}{R}\right)^{m+1} + k_1\left(\frac{R}{\rho}\right)^{m-1} + k_3\right]$	
$k_1 = \left(\sigma_p + \frac{\sigma_d}{m-1}\right)\left[(1-\nu_s)\frac{1+m^2}{2m} - \nu_s\right]$	$p^* = \frac{\sigma_{l\,lim}\cdot t}{R}$
$k_2 = (1-\nu_s)\frac{1+m}{2m}\left[\sigma_d + (m-1)\sigma_p\right]$	
$k_3 = p_0 - \sigma_p - k_1 - k_2$	

Table I

where symbols have been used the following

ρ: radius of plastified zone

R: liner radius

σ_p: radial pressure p at which plastification starts

m, σ_d: parameters reflecting soil properties according to Mohr-Coulomb criterium

p_i: internal pressure

φ: internal friction angle

c: soil cohesion

p_0: initial soil stress (assumed isotropic)

δ_i: radial displacement corresponding to p_i

ν_s E_s: Poisson coefficient and Young Modulus of soil

ε^l: liner hoop deformation

δ^l: liner radial displacement

δ_0: initial soil displacement before liner is introduced

t: liner thickness

E_l: liner Young modulus

$\sigma_{l\,lim}$: liner limit stress

Generally it is more realistic for reinforced concrete liners to use a combination of the material laws, as those presented in figure 4 and to limit not the stresses in the liner but the concrete deformation because the ultimate state will be concrete crushing (for instance $\varepsilon_u = 0.003$)

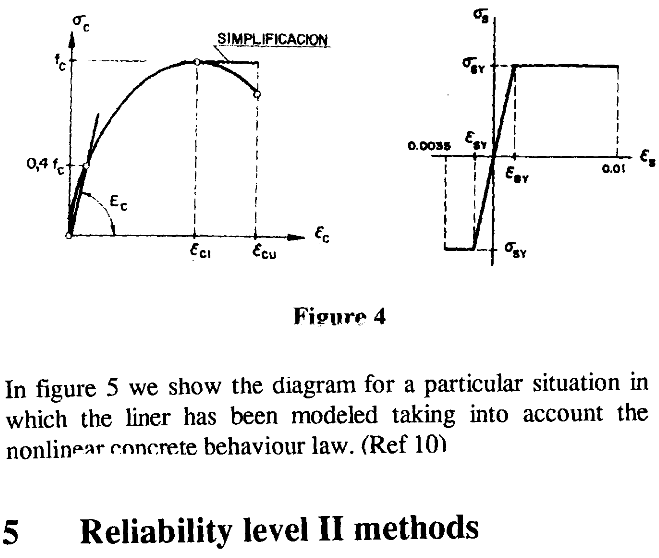

Figure 4

In figure 5 we show the diagram for a particular situation in which the liner has been modeled taking into account the nonlinear concrete behaviour law. (Ref 10)

5 Reliability level II methods

As it is well known (ref 6) in addition to limit states it is necessary to choose the basic variables which statistic properties are defined through their mean values and covariances. In the examples we have been treating, 6 variables have been chosen related to the soil (E_s, ν_s, γ_s, c, φ and H) and other 6 for the liner (δ_0, f'_{ck} t, R, ω_s, ω_c) where the meaning of each symbol has been defined in previous paragraphs, except ω_s and ω_c that are the ratios of steel reinforcement used as bars or solid profiles respectively.

Using the Hasofer and Lind definition (ref 7) the reliability index for every limit state is related to the minimum distance of the origin to the failure surface when the variables are standardized and make independent after the adequate transformations.

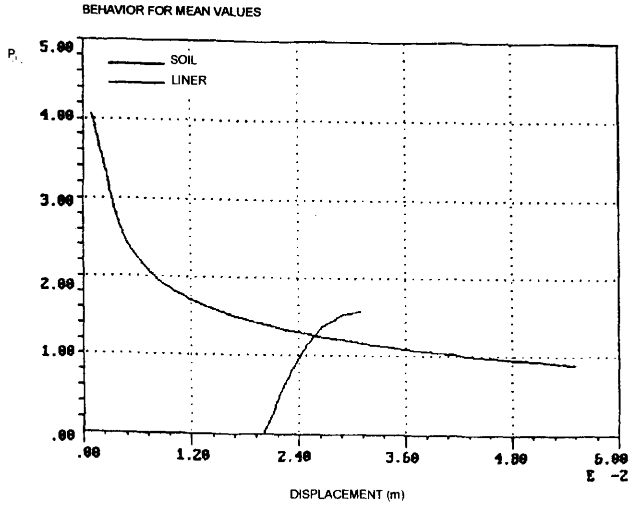

Figure 5

On the other hand, the coordinates of the point at the surface (design point) ($\beta\alpha_1$ $\beta\alpha_2$ ...... $\beta\alpha_N$) are used to obtain the sensitivities related to the different variables as well as to define weighting factors affecting to values of the stochastic variables.

To obtain the design point we have used the typical Racwitz approach but also a surface response method (ref 11) based on the point estimation (Rosenblueth, (8)).

In this method for functions of n variables it is necessary to obtain estimations at 2^n points defined as combinations of the mean plus or minus the standard derivation of every variable. Using the results of those estimates a surface can be adjusted (the simplest case being an hyperplane) and β and α_i computed to obtain the desired results.

In this case it is very easy to show that if the plane representing the failure plane is expressed as

$$W = a_o + \sum_1^N a_j z_j$$

where z_j are the typified variables (which values are ± 1) then

$$a_o = \frac{\sum_1^N W_i}{N}$$

$$a_j = \frac{\sum_1^N W_i z_j^i}{N}$$

where N is the number of point estimations.

Then the reliability index, the director cosines α_k and the design point z_k^* are given by

$$\beta = \frac{a_o}{\sqrt{\sum_j a_j^2}}$$

$$\alpha_k = \frac{a_k}{\sqrt{\sum_j a_j^2}}$$

$$z_k^* = \frac{a_k a_o}{\sum_j a_j^2}$$

Detailed values for mean values and covariance matrices ot basic variables can be found in ref 9, related to a road-tunnel in different soil conditions. As an example of the results figure 6 shows.

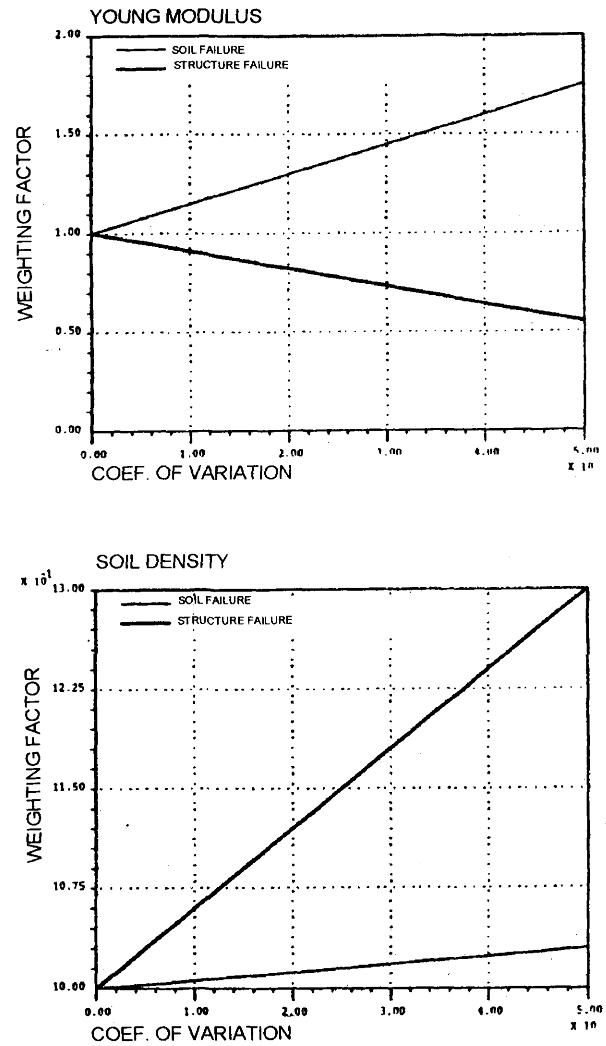

Figure 6

In the probabilistic finite element method one of tne procedures to obtain the sensitivity is the development in a Taylor series round the point under consideration by solving sequencially (reference) the systems

$$K^0 u^0 = f^0$$

$$K^0 \left.\frac{\partial u}{\partial x}\right|^0 = \left.\frac{\partial f}{\partial x}\right|^0 - \left.\frac{\partial k}{\partial x}\right|^0 u^0$$

where K is the stiffness matrix of the system, f the load vector and u the displacement vector, x is the vector of basic stochastic variables and superindex o means that all values are particularized for the basic variable set corresponding to the design point.

In this way it is possible to connect the reliability modulus with a general representation of the problem.

For instance we have used the previous approach with a Terzaghi method in which, the liner is represented by monodimensional members and the passive pressures by elastic springs (figure 7) while some active pressures are directly applied using some empirical rules (figure 8). This system is well suited for shallow tunnels or arcas near portals

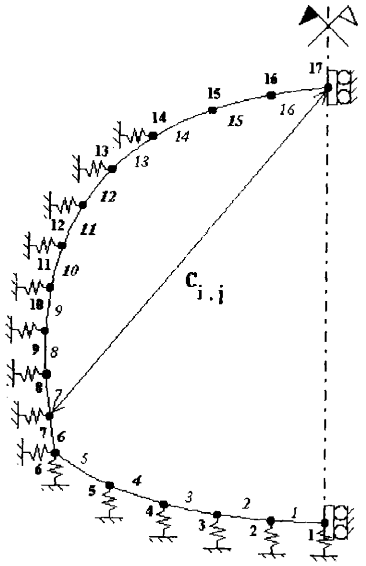

Figure 7

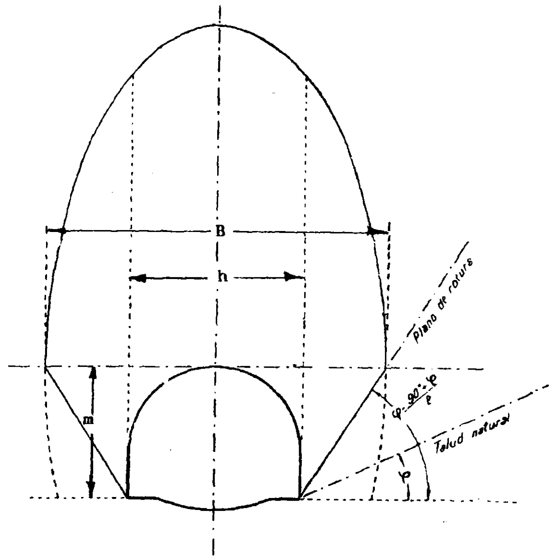

Figure 8

The next tables show the results obtained in an example using the previous definition. In it, was calculated the reliability index of the tunnel using as a failure surface the points where the vertical displacement of the liner crown was 3 cm. The stochastics variables used were the stiffness of the springs. In those tables, the point over the failure surface, the sensitivities coefficients of the springs and the reliability index are shown as well as the weighting factors of the stochastics variables.

NODAL DISPLACEMENTS

NODE	DISPL. X	DISPL. Y	ROTAT. Z
1	0	-0.03	0
2	-0.0002059	-0.0298239	0.0002515
3	-0.0003296	-0.0293388	0.0004534
4	-0.0003213	-0.0286435	0.0005919
5	-0.0001574	-0.0278221	0.0007247
6	0.0002347	-0.0268497	0.0010117
7	0.0011195	-0.0255626	0.0015959
8	0.0028333	-0.0239318	0.001977
9	0.0024892	-0.0235306	0.001977
10	0.004561	-0.0219528	0.0014093
11	0.0059476	-0.0212753	0.0005173
12	0.0060363	-0.02111	-0.0005802
13	0.0049981	-0.0209927	-0.0013695
14	0.0031523	-0.0207702	-0.0017773
15	0.0019959	-0.0174825	-0.0022707
16	0.0008091	-0.0129022	-0.002992
17	-0.0002168	-0.0074596	-0.0033018
18	-0.0003004	-0.0027106	-0.002816
19	-0.0002074	-0.0006508	-0.0017927
20	-0.0001054	0.0003652	-0.0007395
21	0	0.0006382	0

FAILURE SURFACE

Sensitivity Coeff.	
Alfa1	0.027
Alfa2	0.010
Alfa3	0.765
Alfa4	0.030
Alfa5	0.010
Alfa6	0.010
Alfa7	0.040
Alfa8	-0.035
Alfa9	0.609
Alfa10	0.111
Alfa11	0.129
Alfa12	0.095
Alfa13	0.022
Alfa14	-0.013
Alfa15	-0.001
Alfa16	0.000
Beta=	3.3453
Pf(%)=	0.0411

WEIGHTING FACTORS

Variable	phi
1	0.977
2	0.992
3	0.360
4	0.975
5	0.991
6	0.992
7	0.966
8	1.029
9	0.490
10	0.907
11	0.892
12	0.921
13	0.982
14	1.011
15	1.001
16	1.000

Table II

It is also possible to model a general shape by a 2-D finite element method by using the Panet approach as described by Leca and Clough (ref. 14), i.e.: representing the excavation by a softening ground of the central part of the mesh and the application of an initial stress state as indicated in figure 9.

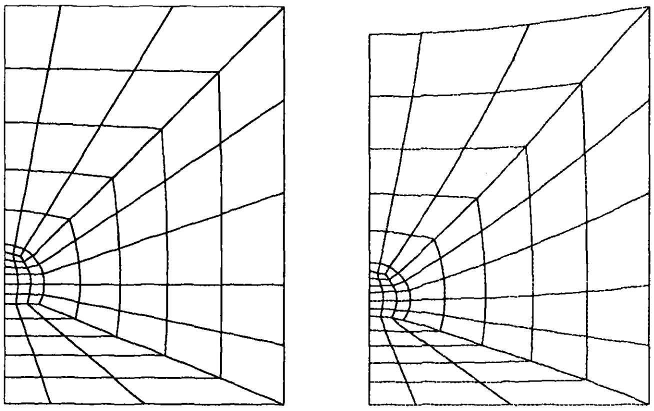

Figure 9

6 Conclusions

In the paper we have tried to show how different classical methods of structural reliability can be applied to the assesment of the safety of tunnel liners. Although more work is necessary it is hoped that this line of thought will improve the current measures of safety. In addition it is clear that the procedure can be used as an identification tool for back analysis fixing as limit state the variables to be adjusted.

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