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TRANSITION FROM ISENTROPIC TO ISOTHERMAL EXPANSION IN LASER PRODUCED PLASMAS

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Abstract—The transition that the expansion flow of laser-produced plasmas experiences when one moves from long, low intensity pulses (temperature vanishing at the isentropic plasma–vacuum front, lying at finite distance) to short, intense ones (non-zero, uniform temperature at the plasma–vacuum front, lying at infinity) is studied. For planar geometry and large ion number Z_i the transition occurs for $d\phi/dt \simeq 0.14(27/8)k^{7/2}Z_i^{3/2}n_c^{\,2}/m_i^{3/2}\bar{K}$; ϕ, n_c, m_i, and $\bar{K}$ are laser intensity, critical density, ion mass, and Spitzer's heat conduction coefficient. This result remains valid for finite Z_i, though the numerical factor in $d\phi/dt$ is different. Shorter wavelength lasers and higher Z_i plasmas allow faster rising pulses below transition.

1. INTRODUCTION

THE EXPANSION flow of plasmas produced by irradiating solid targets with laser light, changes non-trivially as one moves from long, low intensity pulses (MULSER, 1970; COOPER, 1973) to short, intense ones (CLARKE *et al.*, 1973; MASON and MORSE, 1975). In the first limit the neighbourhood of the plasma–vacuum boundary, which lies at a finite distance at any given time, behaves isentropically, and the electron temperature T_e vanishes there (SANMARTIN and BARRERO, 1978a); in the opposite limit, and assuming a short enough mean-free-path (and quasineutrality), the flow extends to infinity at any time, and T_e is non-zero and uniform in the rarefied plasma (SANMARTIN and BARRERO, 1978b). This transition has important consequences: in an isothermal expansion the mean-free-path (and the Debye length) grows indefinitely, leading to a breakdown of the assumption just mentioned; phenomena undesirable for laser fusion, such as significant ion acceleration or a non-thermal electron distribution function (hot and cold populations, truncated Maxwellian) (MORSE and NIELSON, 1973; CROW *et al.*, 1975; PEARLMAN and MORSE, 1978; DECOSTE, 1978), follow from that breakdown.

In this paper we find that the transition occurs, in a sense, discontinuously. In a rising pulse, the rate of entropy generation in the absorption process increases with the laser intensity $\phi(t)$. If the increase is slow enough, the plasma is able to convey away all the entropy produced in the region of absorption, outside which conduction is negligible. The convection is less efficient for a faster increase. There is a finite value of $d\phi/dt$, for a given plasma and a given laser frequency, above which conduction is important throughout the expansion, and the rarefied plasma is isothermal.

We present the mathematical problem in Section 2. To carry out the analysis we use simplifications such as considering large ion charge number, planar geometry, and a linear pulse. In Sections 3 and 4 we study the isentropic and isothermal limits, and the general case, respectively. In Section 5 we discuss the results obtained and their validity for more general situations.

2. BASIC EQUATIONS

The equations describing the expansion flow of a plasma produced by irradiating a solid target with a laser-light pulse are

$$\frac{Dn}{Dt} = -n\frac{\partial v}{\partial x}, \qquad \left(\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right), \tag{1}$$

$$\frac{m_i n}{Z_i}\frac{Dv}{Dt} = -\frac{\partial}{\partial x}(nkT_e), \tag{2}$$

$$nT_e\frac{D}{Dt}\left(k\ln\frac{T_e^{3/2}}{n}\right) = \frac{\partial}{\partial x}\left(\bar{K}T_e^{5/2}\frac{\partial T_e}{\partial x}\right) + \phi(t)\delta(x - x_c), \tag{3}$$

where n and T_e are electron density and temperature, v is ion or electron velocity, and m_i and Z_i are ion mass and charge number; k, $\bar{K}$, δ, and x_c are Boltzmann's constant, Spitzer's heat conduction coefficient, Dirac's function, and critical plane position respectively. We have assumed planar geometry, absorption at the critical density $[n_c \equiv n(x_c)]$, and a quasi-neutral, collision dominated plasma. Also, we took Z_i to be large in order to simplify the equations neglecting ion pressure and internal energy ($T_e \gg T_i/Z_i$); then the ion temperature is decoupled from system (1)–(3), and is given by

$$\frac{n}{Z_i}T_i\frac{D}{Dt}\left(k\ln\frac{T_i^{3/2}}{n}\right) = \tfrac{3}{2}kn^2\frac{T_e - T_i}{\bar{t}_{ei}T_e^{3/2}},$$

where $\bar{t}_{ei}T_e^{3/2}/n$ is the ion-electron energy relaxation time (SPITZER, 1962). We shall comment on these assumptions in the last section. The light pulse, starting at $t = 0$, is incident from $x = -\infty$ on the solid half-space $x > 0$.

For a linear pulse

$$\phi(t) = \phi_0 t/\tau = t\,\mathrm{d}\phi/\mathrm{d}t,$$

we may introduce self-similar variables

$$\eta = x[(3\tau/4)(t/\tau)^{4/3}(Z_i kT_r/m_i)^{1/2}]^{-1}, \qquad \nu = n/n_r,$$
$$y = v[(t/\tau)^{1/3}(Z_i kT_r/m_i)^{1/2}]^{-1}, \qquad z = T[(t/\tau)^{2/3}T_r]^{-1},$$

to transform equations (1)–(3) into the system

$$\frac{\mathrm{d}\nu}{\mathrm{d}\eta} = \frac{\nu}{\eta - y}\frac{\mathrm{d}y}{\mathrm{d}\eta}, \tag{4}$$

$$y - 4(\eta - y)\frac{\mathrm{d}y}{\mathrm{d}\eta} = -\frac{4}{\nu}\frac{\mathrm{d}(\nu z_e)}{\mathrm{d}\eta}, \tag{5}$$

$$\nu\left[z_e\left(1 + \frac{4}{3}\frac{\mathrm{d}y}{\mathrm{d}\eta}\right) - 2(\eta - y)\frac{\mathrm{d}z_e}{\mathrm{d}\eta}\right] = \frac{\mathrm{d}}{\mathrm{d}\eta}z_e^{5/2}\frac{\mathrm{d}z_e}{\mathrm{d}\eta} + \frac{8\nu_c^2}{(\alpha_c Z_i)^{3/2}}\delta(\eta - \eta_c); \tag{6}$$

to simplify the equations, we chose a convenient reference temperature

$$T_r = \left(\frac{9Z_i k^2 \tau n_r}{16 m_i \bar{K}}\right)^{2/3},$$

leaving n_r arbitrary momentarily, and used the parameter defined by SANMARTIN and BARRERO (1978a, b), $\alpha_c \equiv (9k/4m_i)(k^2 n_c^2 \tau/\bar{K}\phi_0)^{2/3}$.

For n_c/n_0 small (n_0 = solid density), and $d\phi/dt$ not so large as to generate a thermal wave (SHEARER and BARNES, 1971), there exists a well defined ablation surface separating the rarefied expansion flow from the high-density, compressed region on the right. The motion of that surface is slow when compared with velocities in the expansion flow, so that, to analyze the expansion, the surface may be set at $\eta = 0$, where, therefore, the density goes to infinity and the velocity vanishes, while the pressure takes a finite value; then for some appropriate n_r, we may write

$$y = z = 0, \qquad \nu z = 1 \quad \text{at} \quad \eta = 0. \tag{7}$$

In addition, at the plasma–vacuum interface, which may lie at either finite or infinite distance, we have zero density and heat flux:

$$\nu \to 0, \qquad z_e^{5/2}\, dz_e/d\eta \to 0 \quad \text{as} \quad y \to \eta. \tag{8}$$

Finally, for convenience, we drop the δ-term in equation (6) using instead a jump condition across the critical plane,

$$z_e^{5/2}\left(\frac{dz_e}{d\eta}\bigg|_{\eta_c^-} - \frac{dz_e}{d\eta}\bigg|_{\eta_c^+}\right) = \frac{8\nu_c^2}{(\alpha_c Z_i)^{3/2}}. \tag{9}$$

Neither η_c nor ν_c are known *a priori* (because n_r is unknown).

Any solution to equations (4)–(6) and (7) behaves, in the neighbourhood of $\eta = 0$, as

$$z \simeq A(-\eta)^{2/5} \qquad \nu \simeq A^{-1}(-\eta)^{-2/5}, \qquad y \simeq -\frac{3}{25}A^{7/2}(-\eta)^{2/5}, \tag{10}$$

A being an arbitrary constant. We expect (and find) that such a solution satisfies conditions (8), for any given A within some positive range, if a discontinuity in $dz/d\eta$ is permitted at some appropriate η (which will be the critical plane). Once the solution has been obtained condition (9) yields the value of $\alpha_c Z_i$ corresponding to the chosen A.

3. LIMIT REGIMES

To better understand the general case, analyzed in the next section, we briefly discuss here the large and small $\alpha_c Z_i$ range, studied in detail by SANMARTIN and BARRERO (1978a, b) for arbitrary Z_i, and which are particularly simple for Z_i large.

We find that as A decreases towards a finite value 1.92, the critical plane moves to infinity and $\alpha_c Z_i$ goes to zero. The limit solution, which takes into account neither the second condition in (8) nor equation (9) (removed from the problem) is very useful; as the density vanishes ($\eta \to -\infty$) the left-hand side of (6) approaches zero and $z_e^{5/2}\, dz_e/d\eta$ reaches a constant finite value, which will differ negligibly from the rightward heat flux $z_e^{5/2}\, dz_e/d\eta|_{\eta_c^+}$ for any $\alpha_c Z_i$ small enough, and may be used to determine the corresponding value of η_c: From equation (9) we get

$$\nu_c^2 = -\frac{(\alpha_c Z_i)^{3/2}}{8}\left[z_e^{5/2}\, dz_e/d\eta|_{\eta\to-\infty, \alpha_c Z_i = 0}\right]; \tag{11}$$

the critical plane lies where the density in the $\alpha_c Z_i = 0$ solution takes the value given in (11). We neglected $z_e^{5/2}\,dz_e/d\eta|_{\eta_c^-}$ in (9) because, for ν_c small, the (nearly) constant value of the heat flux to the left of the critical plane can only be zero (uniform temperature). It may be easily verified that the flow speed in a frame moving with the local density $(y-\eta)$ is everywhere less than the isothermal sound speed.

For the opposite limit, we find that as A becomes large, the critical plane approaches the origin and $\alpha_c Z_i$ increases indefinitely. In fact, equations (5), (6), (7) and (9) show that, for $\alpha_c Z_i$ large, conduction is restricted to a thin (deflagration) layer ($\eta \ll 1$, $\eta \ll y$) connected to a much broader isentropic region where the first condition in (8) may be satisfied. In the thin layer equations (4)–(6) may be integrated once (quasi-steady flow), using (7) to get

$$\nu y = -\tfrac{3}{25}A^{5/2}, \tag{12}$$

$$\tfrac{3}{25}A^{5/2}(y^2+z_e)+y=0, \tag{13}$$

$$\tfrac{2}{25}A^{5/2}(y^2+5z_e)+z_e^{5/2}\,dz_e/d\eta=\frac{8\nu_c^2}{(\alpha_c Z_i)^{3/2}}(1-\sigma); \tag{14}$$

σ is zero (unity) for $\eta<\eta_c(\eta>\eta_c)$. Since z_e must have a maximum at the critical plane, equations (13) and (12) lead to

$$y_c=-25/(6A^{5/2}), \qquad z_c=y_c^2, \qquad \nu_c=18A^5/625; \tag{15}$$

then, evaluating equations (13) and (14) just behind the deflagration, where the Chapman–Jouguet condition is satisfied for arbitrary Z_i (SANMARTIN and BARRERO 1978a) we obtain the value of $\alpha_c Z_i$ corresponding to A,

$$\alpha_c Z_i=(6\times 2^{1/3}/25)^4 A^{25/3}.$$

In the isentropic region, the density vanishes at a finite value η_v, and near it we have

$$\frac{z}{\eta_v^2}\simeq\frac{7}{40}\left(1-\frac{\eta}{\eta_v}\right), \qquad \frac{y}{\eta_v}\simeq 1-\frac{3}{10}\left(1-\frac{\eta}{\eta_v}\right), \qquad \nu\sim(1-\eta/\eta_v)^{3/7}. \tag{16}$$

Clearly, the speed $(y-\eta)$ is subsonic near both the origin and η_v, and supersonic somewhere in between, so that the flow presents two isothermal sonic points [one point lies at the critical plane (LIÑAN, 1979) as shown in (15), and the other in the isentropic región].

4. ISENTROPIC–ISOTHERMAL TRANSITION

For arbitrary $\alpha_c Z_i$, it proves convenient to define the phase–space variables

$$Y=\frac{y}{\eta}, \qquad N=\frac{\nu}{-\eta^3}, \qquad \theta=\frac{z_e}{\eta^2}, \qquad F=\frac{z_e^{5/2}\,dz_e/d\eta}{\eta^6},$$

and write system (4)–(6) in the form

$$\frac{dN}{dY}=\frac{N}{Y-1}\left(\frac{4Y-3}{Y}\frac{\Delta_1}{\Delta_2}-1\right), \tag{17}$$

$$\frac{d\theta}{dY}=\frac{2\theta+F/\theta^{5/2}}{Y}\frac{\Delta_1}{\Delta_2}, \tag{18}$$

$$\frac{dF}{dY} = \frac{6_{\perp}}{} \quad \frac{4N[(1+4Y/3)\theta/4 - F(Y-1)/2\theta^{5/2}]}{Y} \frac{\Delta_1}{\Delta_2} - \frac{4}{3} N\theta, \tag{19}$$

$$\frac{dY}{d \ln \eta} = -Y \frac{\Delta_2}{\Delta_1}, \tag{20}$$

where

$$\Delta_1 = \theta - (Y-1)^2, \tag{21}$$

$$\Delta_2 = \theta - (Y-1)\left(Y - \frac{3}{4} - \frac{F}{Y\theta^{5/2}}\right). \tag{22}$$

The behaviour of the solution for Y large may be directly obtained from equation (10) for η small:

$$N \simeq (25Y/3)^{17/3} A^{-125/6}, \tag{23}$$

$$\theta \simeq (25Y/3)^{8/3} A^{-25/3}, \tag{24}$$

$$F \simeq -(2/5)(25Y/3)^{28/3} A^{-175/6}. \tag{25}$$

Condition (9) and the boundary conditions (8) become

$$F^{-} - F^{+} = 8N_c^2/(\alpha_c Z_i)^{3/2}, \tag{26}$$

$$N = F = 0 \quad \text{at} \quad Y = 1, \tag{27}$$

respectively.

It is possible to show that θ must vanish at $Y=1$ in the form

$$\theta \simeq (Y-1)/4. \tag{28}$$

On the other hand N and F may behave in a variety of ways in the neighbourhood of that point. We find

$$N \simeq B(Y-1)^{3/7}, \tag{29a}$$

$$F \simeq 7(Y-1)^{5/2}/1280, \tag{29b}$$

$$\eta - \eta_v \simeq -10\eta_v(Y-1)/7, \tag{29c}$$

and

$$N \simeq C(Y-1)^4 \exp\left[\frac{-1}{2(Y-1)}\right], \tag{30a}$$

$$F \simeq \tfrac{7}{12} C(Y-1)^6 \exp\left[\frac{-1}{2(Y-1)}\right], \tag{30b}$$

$$\eta \sim (Y-1)^{-1/2}, \tag{30c}$$

where B and C are arbitrary constants. One may verify that the isentropic and isothermal behaviours discussed in Section 3 correspond to equations (29) and (30), respectively. We find, however, that there exists a *third* possible type of solution near $Y=1$,

$$N \simeq 5(Y-1)^{1/2}/16, \tag{31a}$$

$$F \simeq (Y-1)^{5/2}/192, \tag{31b}$$

$$\eta - \eta_v \simeq -3\eta_v(Y-1)/2. \tag{31c}$$

Notice that (31a), contrary to equations (29a) and (30a) contains no arbitrary constant and therefore should correspond to a specific A value, marking the transition between the isentropic and isothermal behaviours.

Indeed we find that as A decreases from large values B decreases too. Consider B small. As one moves away from the plasma–vacuum interface toward growing densities, the approximation (29) breaks down for $(Y-1)=0(B^{14})\ll 1$. In that region equations (17)–(19) yield an equation involving only $F/(Y-1)^{5/2}$ and $N/(Y-1)^{1/2}$,

$$\frac{d[F(Y-1)^{-5/2}]}{d[N(Y-1)^{-1/2}]}=\frac{7}{12}\frac{1-(1280/7)F(Y-1)^{-5/2}[1+3F(Y-1)^{-5/2}/N(Y-1)^{-1/2}]}{1-192F(Y-1)^{-5/2}}. \tag{32}$$

The solution to (32), starting with the isentropic values for $(Y-1)/B^{14}\to 0$,

$$\frac{N}{(Y-1)^{1/2}}\simeq\left(\frac{B^{14}}{Y-1}\right)^{1/14}\to\infty,\qquad \frac{F}{(Y-1)^{5/2}}\to\frac{7}{1280},$$

ends at the nodal point of (32),

$$\frac{N}{(Y-1)^{1/2}}=\frac{5}{16},\qquad \frac{F}{(Y-1)^{5/2}}=\frac{1}{192}, \tag{33}$$

(see Fig. 1). Equations (33) are the *same* as (31).

We find similarly that as A increases from its lowest value, 1.92, C increases. Consider C large enough. We find that the approximation (30) breaks down when $C(Y-1)^{7/2}\exp[-1/2(Y-1)]=0(1)$, $(Y-1\ll 1)$. In that region equations (17)–(19) yield again (32). Its solution, starting with the isothermal values,

$$\frac{N}{(Y-1)^{1/2}}=C(Y-1)^{7/2}\exp[-1/2(Y-1)]\to 0,$$

$$\frac{F}{(Y-1)^{5/2}}=\left(\frac{7}{12}\right)C(Y-1)^{7/2}\exp[-1/2(Y-1)]\to\frac{7}{12}\frac{N}{(Y-1)^{1/2}},$$

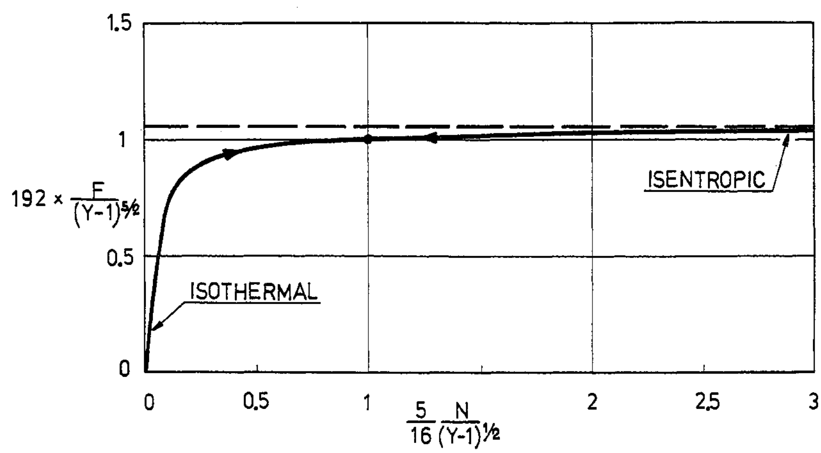

FIG 1.—Numerical solution to equation (32).

ends again at the nodal point (33) (see Fig. 1). It is clear that the solutions for small B and large C differ from each other only at low densities ($Y \simeq 1$) when the isentropic and isothermal behaviours are attained respectively, and must correspond to close A values. That density range collapses to zero as A approaches the value 2.19, from either above or below ($B \rightarrow 0$ or $C \rightarrow \infty$), and in that limit the transition behaviour (31) is valid all the way down to zero density. For $A = 2.19$ we find $\alpha_c Z_i = 3.75$ or, equivalently,

$$\mathrm{d}\phi/\mathrm{d}t = 0.14(27/8)k^{7/2}Z_i^{3/2}n_c^{\,2}/m_i^{3/2}\bar{K} \equiv (\mathrm{d}\phi/\mathrm{d}t)^*. \quad (34)$$

For $\mathrm{d}\phi/\mathrm{d}t > (\mathrm{d}\phi/\mathrm{d}t)^*$ the plasma extends to infinity (density decaying exponentially with distance, and temperature being practically uniform), though both n and T_e are exponentially small in $[(\mathrm{d}\phi/\mathrm{d}t)/(\mathrm{d}\phi/\mathrm{d}t)^* - 1]^{-1}$ (see Figs. 2 and 3).

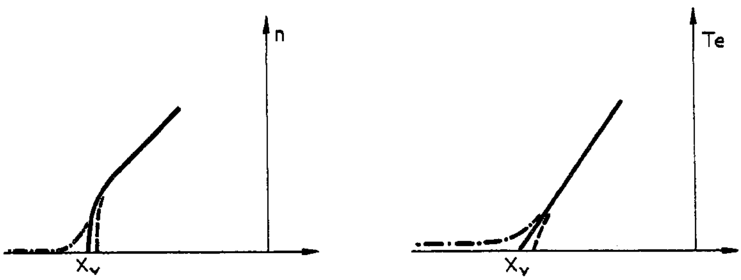

FIG. 2.—Schematics of n and T_e for $\mathrm{d}\phi/\mathrm{d}t$ just below (----), at (——), and just above (–·–·–) transition.

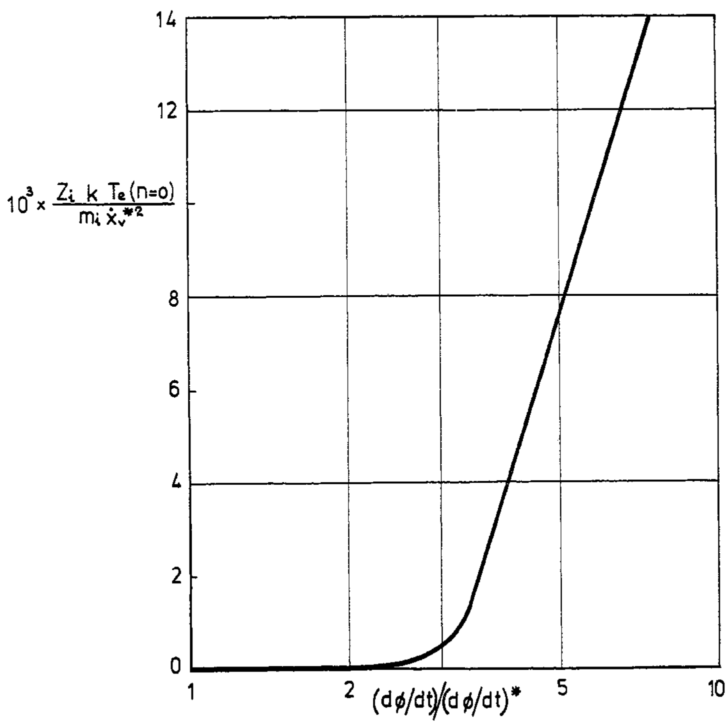

FIG. 3.—T_e at vanishing density above transition; $\dot{x}_v^* = \dot{x}_v$ at transition ($\dot{x}_v \equiv \mathrm{d}x_v(t)/\mathrm{d}t$).

For the numerical integration, we start with large A (and correspondingly large B) values, for which we know that there must be two isothermal sonic points ($\Delta_1 = 0$). Let those two points occur at Y_{s1} and Y_{s2} ($Y_{s1} > Y_{s2}$), and let the critical plane lie at Y_c. Since F must be negative to the right of the critical plane, we cannot have $Y_c < Y_{s1}$, because otherwise we would arrive at either a multivalued solution ($\Delta_2 \neq 0 \rightarrow \mathrm{d}\ln\eta/\mathrm{d}Y = 0$ at Y_{s1}) or a positive $F(\Delta_2 = 0 \rightarrow F = Y\theta^{5/2}/4$ at Y_{s1}). An analysis of the points where both Δ_1 and Δ_2 vanish shows that Y_c cannot be larger than Y_{s1} either; thus $Y_c = Y_{s1}$ as for $A \rightarrow \infty$. Starting at large Y with (23)–(25) we integrate until the solution meets the sonic curve (Y_{s1}); since F^+

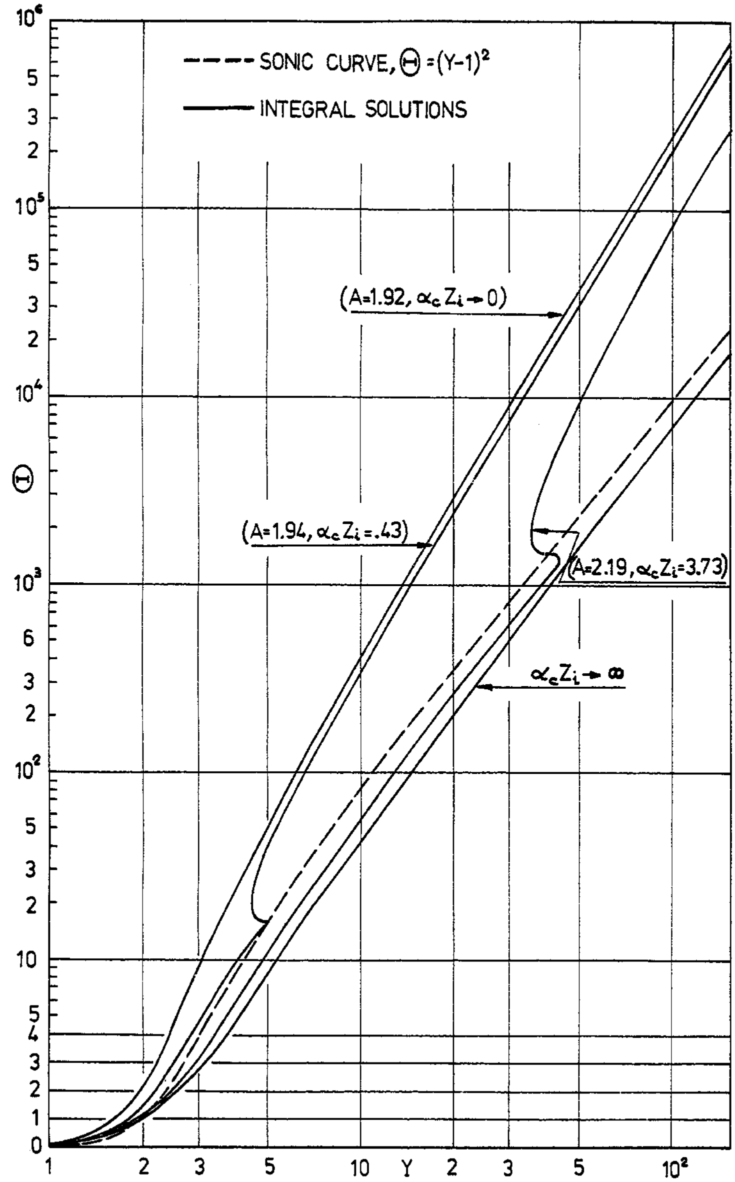

FIG. 4.—Numerical results in the space phase (θ, Y) for different values of $\alpha_c Z_i$. Notice the scale change at $\theta = 4$.

remains negative we have $\Delta_2 \neq 0$ at Y_{s1}, but the solution is not multivalued because of the jump in $F(Y_c = Y_{s1})$. To continue the solution to lower densities, beyond the critical plane, we sweep through F^- (which is unknown); for each value within a certain positive range the integral curve $\theta(Y)$ is found to meet the sonic curve ($\Delta_1 = 0$) at a second point and Δ_2 is found to vanish there (F is *now* positive, and the point has a nodal character). Starting finally at $Y = 1$, using (29), the integral curve $\theta(Y)$ for each B meets the sonic curve at a point, and again we have $\Delta_2 = 0$ there. For a certain B and a certain F^- the sonic points have the same Y and N (for given Y, conditions $\Delta_1 = 0$, $\Delta_2 = 0$ determine uniquely only θ and F); one F^- and B are obtained the solution is completed. The method remains valid as A is decreased below the transition value, 2.19, though then one must use (30) to start integration from $Y = 1$. The two sonic points are found to approach each other, and they meet for $A \simeq 1.94$. Below 1.94 the solution is everywhere subsonic. Figure 4 shows numerical results for $\theta(Y)$, for $A = 1.92$ ($\alpha_c Z_i \to 0$), $A = 1.94$ (when the sonic points coalesce), just below 2.19 (beyond transition), and ($\alpha_c Z_i \to \infty$).

5. DISCUSSION

We have studied the transition that the expansion flow of laser-produced plasmas experiences when one moves from long, low intensity pulses to short, intense ones. For planar geometry, large ion number Z_i, absorption at the critical density n_c, and a pulse reasonably linear in time ($\phi \simeq t\, \mathrm{d}\phi/\mathrm{d}t$), we find that for

$$\mathrm{d}\phi/\mathrm{d}t < (\mathrm{d}\phi/\mathrm{d}t)^* \simeq 0.14(27/8)k^{7/2}Z_i^{3/2}n_c^{\,2}/m_i^{3/2}\bar{K},$$

the plasma behaves isentropically near the plasma–vacuum front ($x_v \sim t^{4/3}$),

$$\begin{gathered} n \simeq b(1-x/x_v)^{3/7}, \qquad v/\dot{x}_v \simeq 1-(\tfrac{3}{10})(1-x/x_v), \\ Z_i kT_e/m_i\dot{x}_v^{\,2} \simeq (\tfrac{7}{40})(1-x/x_v), \end{gathered} \tag{35}$$

where b is an arbitrary constant and $\dot{x}_v \equiv \mathrm{d}x_v(t)/\mathrm{d}t$; b decreases with increasing $\mathrm{d}\phi/\mathrm{d}t$, and vanishes at $(\mathrm{d}\phi/\mathrm{d}t)^*$. For $(\mathrm{d}\phi/\mathrm{d}t)$ slightly below $(\mathrm{d}\phi/\mathrm{d}t)^*$, i.e. b small, the approximation (35) fails very close to the front $(1-x/x_v) = 0(b^{14})$ where the flow takes smoothly the form

$$\begin{gathered} n \simeq \left(\frac{2}{3}\right)^{11/2} \frac{10\bar{K}m_i^{5/2}}{k^{7/2}Z_i^{5/2}} \left(\frac{x_v}{t^{4/3}}\right)^3 \left(1-\frac{x}{x_v}\right)^{1/2}, \\ \frac{v}{\dot{x}_v} \simeq 1-\frac{1}{3}\left(1-\frac{x}{x_v}\right), \qquad \frac{Z_i kT_e}{m_i \dot{x}_v^{\,2}} \simeq \frac{1}{6}\left(1-\frac{x}{x_v}\right). \end{gathered} \tag{36}$$

As $\mathrm{d}\phi/\mathrm{d}t$ approaches $(\mathrm{d}\phi/\mathrm{d}t)^*$ from below ($b \to 0$), the thickness of the isentropic region (35) adjoining the front collapses to zero, so that, at the value $(\mathrm{d}\phi/\mathrm{d}t)^*$ the behaviour at the front is the (non-isentropic) limit one given by (36). For $\mathrm{d}\phi/\mathrm{d}t > (\mathrm{d}\phi/\mathrm{d}t)^*$ there is no solution with finite x_v; n decays exponentially to zero at $x = -\infty$ where T_e takes a finite value shown in Fig. 3 for values close to $(\mathrm{d}\phi/\mathrm{d}t)^*$. Figure 2 shows schematically n and T_e versus x. We assumed throughout the analysis a quasineutral, collision dominated plasma; these assumptions will breakdown for $T_e(n = 0)$ large, well beyond transition, but this cannot affect the determination of $(\mathrm{d}\phi/\mathrm{d}t)^*$.

It is of interest to note that the ratio of heat flow to internal energy convection flow

$$r \equiv \frac{\bar{K}T_e^{5/2}\,\partial T_e/\partial x}{(\frac{3}{2})nkT_e(\mathrm{d}x/\mathrm{d}t|_n - v)},$$

[convection measured in a frame where the local density is constant: $\mathrm{d}x/\mathrm{d}t|_n \equiv -(\partial n/\partial t)/(\partial n/\partial x)$], which is an index of the non-isentropic character of the flow, changes discontinuously at the transition: For $\mathrm{d}\phi/\mathrm{d}t < (\mathrm{d}\phi/\mathrm{d}t)^*$ (b given) we have $r \to 0$ as $x \to x_v$ [on the other hand, $r \to \infty$ as $b \to 0$, for any given, small $(1 - x/x_v)$]. At the transition ($b = 0$), $r \to 1/30$ at the front. For $(\mathrm{d}\phi/\mathrm{d}t) > (\mathrm{d}\phi/\mathrm{d}t)^*$, we have $r \to 35/30$ as $n \to 0$.

We find the preceding results valid for finite Z_i, though the numerical value 0.14 in the expression for $(\mathrm{d}\phi/\mathrm{d}t)^*$ may change. A similar conclusion should follow from an analysis allowing absorption at densities below critical.

The results may be also used for structured pulses, for which $\mathrm{d}\phi/\mathrm{d}t$ may change dramatically in time; then, condition $\mathrm{d}\phi/\mathrm{d}t = (\mathrm{d}\phi/\mathrm{d}t)^*$ should mark the time of transition from isentropic to isothermal flow, during the pulse. Notice finally that shorter wavelength lasers and higher Z_i plasmas ($\bar{K} \sim Z_i^{-1}$) allow faster rising pulses below transition.

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