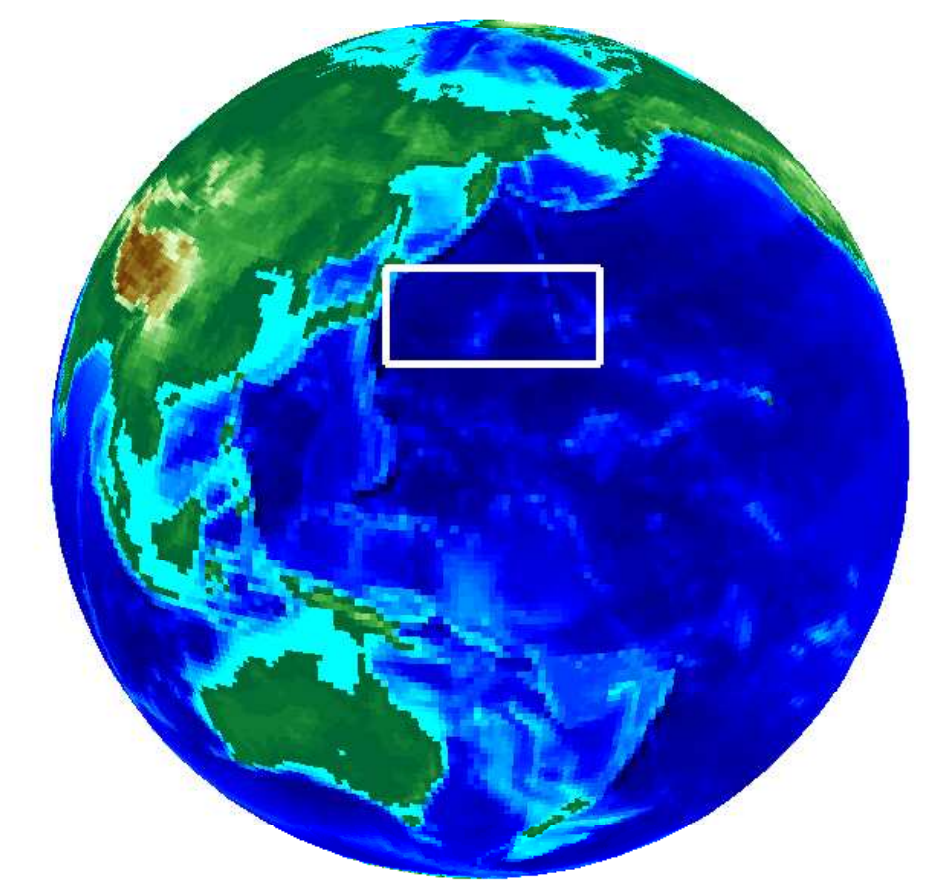# A Lagrangian descriptor applied to the Kuroshio current

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The understanding of the circulation of ocean currents, the exchange of CO_2 between atmosphere and oceans, and the influence of the oceans on the distribution of heat on a global scale is key to our ability to predict and assess the future evolution of climate [1]. Global climate change is acting on sea breathing through mechanisms not yet understood [2]. The ocean is important in the regulation of heat and moisture fluxes, and oceanic physical and bio-geochemical processes are major regulators of natural greenhouse gases. Understanding how oceans mix their waters is necessary to provide sound forecasts on climate [1]. Global change also acts on marine biodiversity and threatens the survival of ecosystems and exploitable resources. To predict not only the efects of global change on the oceans, but also the response time of climate feedback requires to improve detection systems and to open new lines of research.

We use a novel Lagrangian descriptor (function M, introduced in [3, 4]. It is based on the measure of the arclength of particle trajectories on the ocean surface at a given time. In [5, 6, 7] this technique has been proved successfully to characterize the Kuroshio current. We employ this tool on velocity data sets on the Kuroshio current from SURCOUF project in the oceanic currents. In particular, invariant manifolds, hyperbolic and non-hyperbolic flow regions are detected.
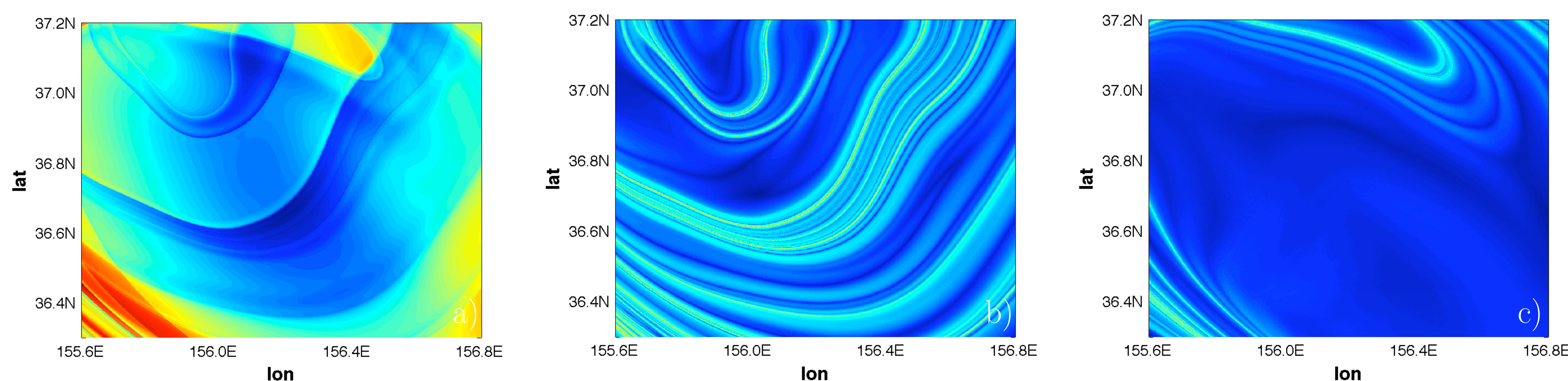

Fig. 1. Comparison of diverse Lagrangian techniques for $\tau = 50$ at the same area; a) The function M; b) the forward FTLE field; c) the backward FTLE field.

The capability of function M is illustrated in figure 1, where is compared with traditional Lyapunov methods at the Kuroshio current. Function M is able to reproduce a richer structure than forward and backward FLTE methods with i) lower computational time (about one third), ii) simpler numerical algorithm which helps in the reliability of the results, iii) half of the runs required than in combined forward/backward FTLE to obtain finer structures.

Lagrangian tools provide a skeleton for the characterization of fluid flows. Underlying their description is Poincaré's idea of seeking geometrical structures on the ocean surface (the phase portrait) that can be used to organize particles schematically by regions corresponding to qualitatively different types of trajectories. Finding this partition of the phase portrait for aperiodic geophysical flows is still a challenge. For instance, typical oceanographic spaghetti diagrams represent paths over time of messy trajectories but these do not communicate information about regions in which particle evolutions are qualitatively different. Tools such as invariant manifolds or Lagrangian Coherent Structures (LCS) are more succesful for this purpose. Recently [3,4] there has been defined a new global Lagrangian descriptor, that for a vector field reads:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t), \ \mathbf{x} \in \mathbb{R}^2 \tag{1}$$

where $\mathbf{v}(\mathbf{x},t)$ is C^r ($r \geq 1$) in $\mathbf{x}$ and continuous in t, $\mathbf{x}(t)$ is a trajectory, and $(x_1, x_2) \in \mathbb{R}^2$. For all initial conditions $\mathbf{x}^*$ in an open set $\mathcal{B} \in \mathbb{R}^2$, at a given time t^*, we define the function $M(\mathbf{x}^*, t^*)_{\mathbf{v}, \tau} : (\mathcal{B}, t) \to \mathbb{R}$ as follows:

$$M(\mathbf{x}^*, t^*)_{\mathbf{v}, \tau} = \left(\int_{t^*-\tau}^{t^*+\tau} \sqrt{\left(\frac{dx_1(t)}{dt}\right)^2 + \left(\frac{dx_2(t)}{dt}\right)^2} \, dt \right), \tag{2}$$

For a given initial condition, the function M measures the length of the curve outlined by a trajectory on the latitude/longitude plane. The trajectory is integrated forwards and backwards in time for an appropriate time τ.
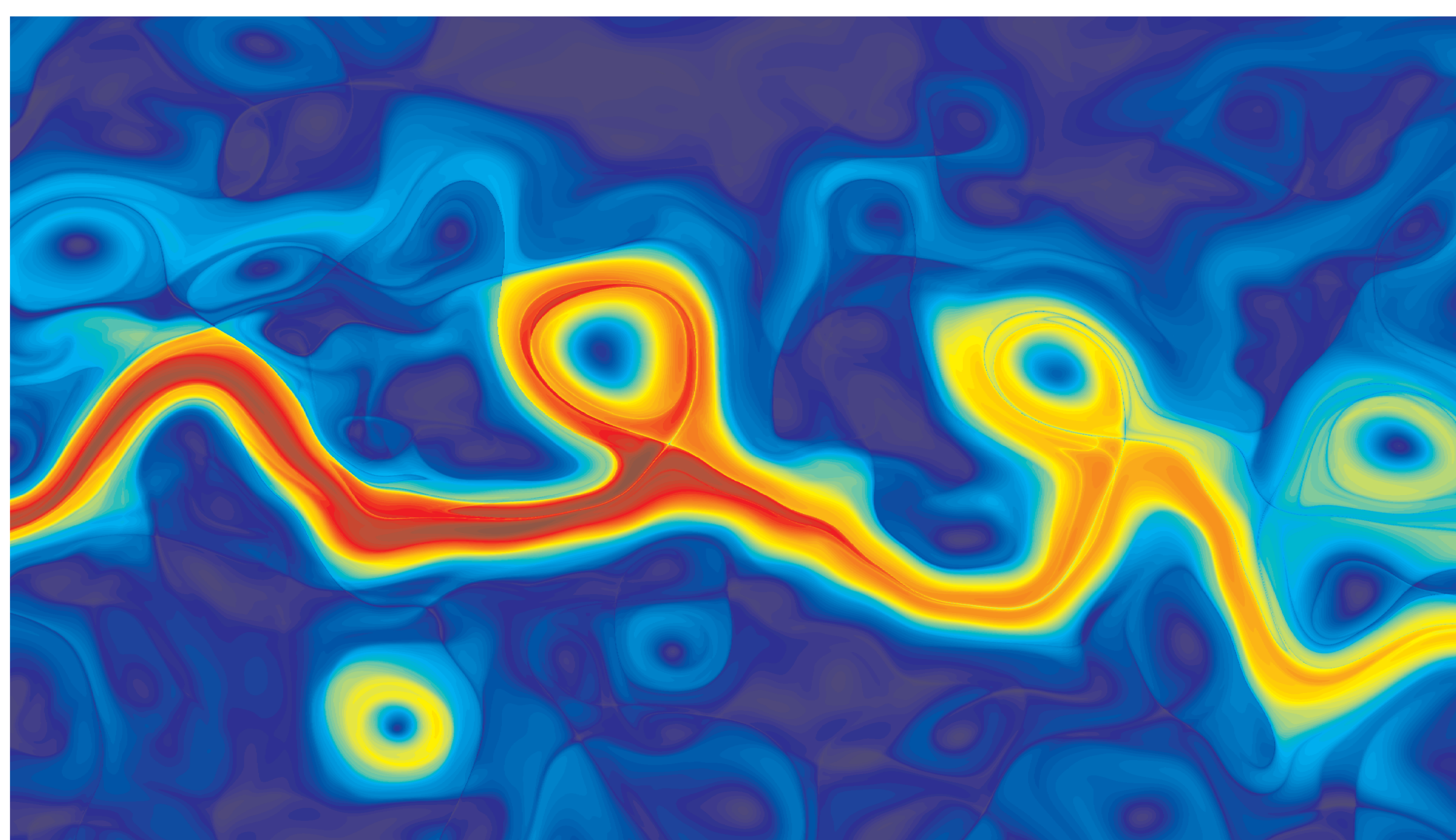
Singular lines are identified as manifolds since they are advected by the flow and are asymptotically obtained from small segments aligned with the stable and unstable subspaces of the DHT (Distinguished hyperbolic trajectory). Figure 3 shows the overlapping of M with the stable and unstable manifolds computed with the technique used in [8]. This confirms the coincidence of the lines with the manifolds.
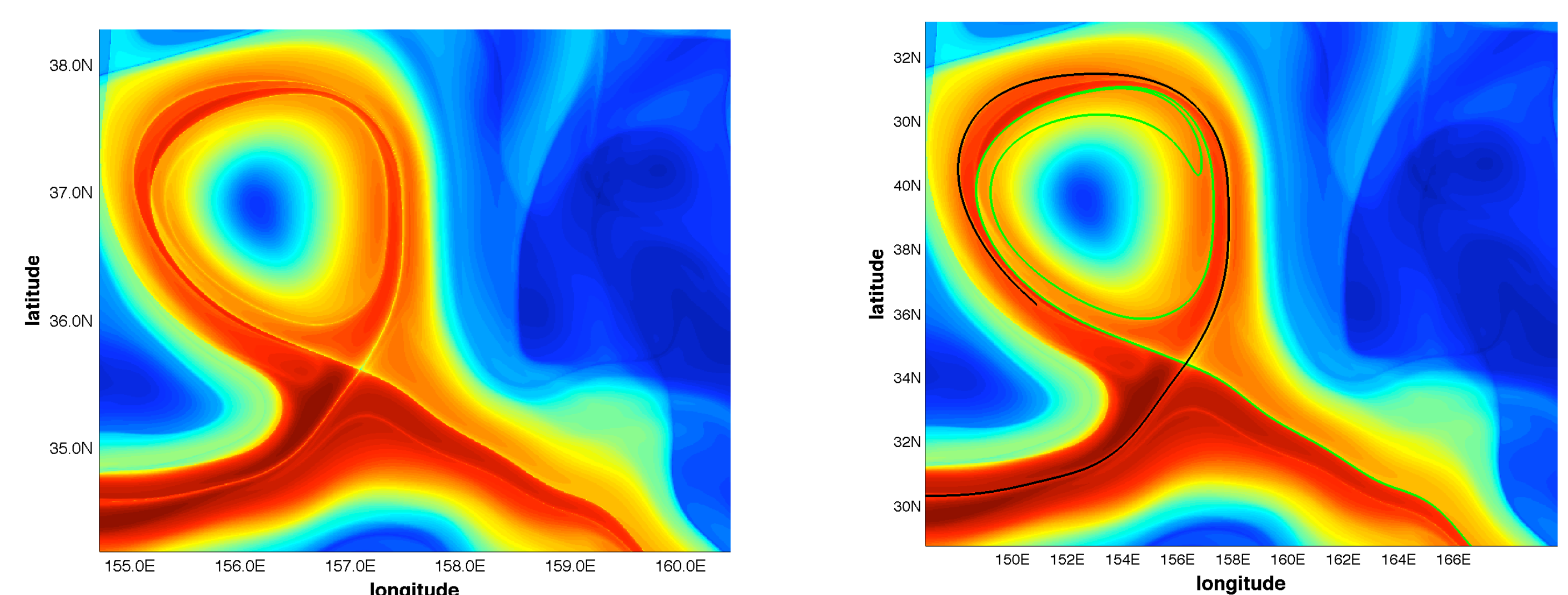

Fig. 3. The function M on May 2, 2003. $\tau = 15$; inset of Fig. 2 with a piece of stable manifold (black) and a piece of unstable manifold (green) of the DHT overlapping.

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Fig. 2. Evaluation of the function M over the Kuroshio current between longitudes 148ºE-168ºE and latitudes 30ºN-41.5ºN on May 2, 2003 take $\tau = 15$