

## MODELIZATION OF LOW CYCLE FATIGUE DAMAGE IN FRAMES

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### SUMMARY

Damage models based on the Continuum Damage Mechanics (CDM) include explicitly the coupling between damage and mechanical behavior and, therefore, are consistent with the definition of damage as a phenomenon with mechanical consequences. However, this kind of models is characterized by their complexity. Using the concept of lumped models, possible simplifications of the coupled models have been proposed in the literature to adapt them to the study of beams and frames.

On the other hand, in most of these coupled models damage is associated only with the damage energy release rate which is shown to be the elastic strain energy. According to this, damage is a function of the maximum amplitude of cyclic deformation but does not depend on the number of cycles. Therefore, low cycle effects are not taking into account.

From the simplified model proposed by Flórez-López, it is the purpose of this paper to present a formulation that allows to take into account the degradation produced not only by the peak values but also by the cumulative effects such as the low cycle fatigue. For it, the classical damage dissipative potential based on the concept of damage energy release rate is modified using a fatigue function in order to include cumulative effects. The fatigue function is determined through parameters such as the cumulative rotation and the total rotation and the number of cycles to failure. Those parameters can be measured or identified physically through the characteristics of the RC. So the main advantage of the proposed model is the possibility of simulating the low cycle fatigue behavior without introducing parameters with no suitable physical meaning. The good performance of the proposed model is shown through a comparison between numerical and test results under cycling loading

### INTRODUCTION

During strong earthquakes, structures are expected to be subjected to large lateral load reversals. Consequently, relatively large inelastic cyclic rotations can be expected. These rotations have an important influence on the overall behavior of the frames and on their dynamic response since they involve energy dissipation.

Fatigue damage increases with applied cycles in a cumulative manner which may lead to fracture. Palmgren [1924] suggested the concept of linear damage accumulation rule which was first expressed in a mathematical form by Miner in 1945. Since then, the treatment of cumulative fatigue damage has received increasingly more attention. As a consequence of it, numerous papers have been published with different fatigue damage models [Fatemi and Yang, 1998; Socie and Morrow, 1976].

The problem of the estimation of cumulative damage of a component could be relatively easily solved under harmonic or block loading using the hypothesis of fatigue cumulative damage proposed by Palmgren and Miner. Under random loadings such as seismic events or wind, cycles are not well-defined and then developed cumulative hypothesis cannot provide satisfactory results due to a considerable variance of the estimation range.

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In these cases, the most important aspect is to count closed hysteresis cycles in the load history which involves converting a random loading history into an equivalent sum of cycles by a cycle counting method.

Several methods have been developed for cycle counting. The rainflow technique developed by Matsuishi and Endo [1968] is one of the most commonly used cycle counting methods. This technique allows the conversion of an irregular loading history of random nature into a set of blocks of equivalent harmonic amplitudes. In the same way, some variations of the classical rainflow method have been developed [Cacko, 1992].

However, in the last years the fatigue study has been reorientated through its incorporation in the Continuum Damage Mechanics (CDM) [Lemaitre and Chaboche, 1985; Lemaitre, 1993]. The same concepts used in CDM to model ductile failure can be extended to the low cycle fatigue damage processes, where plasticity is the key mechanism for crack initiation. Damage Mechanics deals with damage as a continuum variable and, because of it, CDM models including plasticity and damage can predict ductile crack initiation. An extension of themselves including the number of cycles could be suitable to simulate the low cycle fatigue damage. According to it, Chaboche [1985] developed a formulation for damaged materials where the fatigue phenomenon was incorporated in the CDM. However, only harmonic loads were considered being the hypotheses of fatigue cumulative damage suitable.

In the present work, a simplified model for evaluation of low cycle fatigue damage in frames is proposed. The proposed formulation is based in a generalization of the lumped plasticity models including damage effects according to the lines of the CDM such as it was developed in [Florez-Lopez, 1995; Cipollina et al, 1995]. It can be considered as a simplified damage mechanics incorporating concepts of the CDM. A reformulation of the model is developed in order to include the cumulative effects produced in a low cycle fatigue process. The main advantage of this model is the ability of representing the cumulative fatigue damage in the classical way used in the CDM without necessity of incorporating new rules. On the other hand, the nonlinear cumulative damage is obtained directly in a simple way avoiding cycle counting techniques.

## ELASTOPLASTIC DAMAGE MODEL

### Constitutive Equations

Damage in Continuum Damage Mechanics takes into account the degradation of materials resulting in a stiffness reduction.

According to the Strain Equivalence Principle proposed by Lemaitre (1971) and using the Kachanov's definition of effective stress, the stiffness of a damaged material can be obtained as  $E(1-d)$  being  $E$  the initial Young's modulus and  $d$  a scalar representing the isotropic damage. Assuming a damaged elastic material, the strain due to damage can be obtained as [Ortiz, 1985; Ju, 1989]:

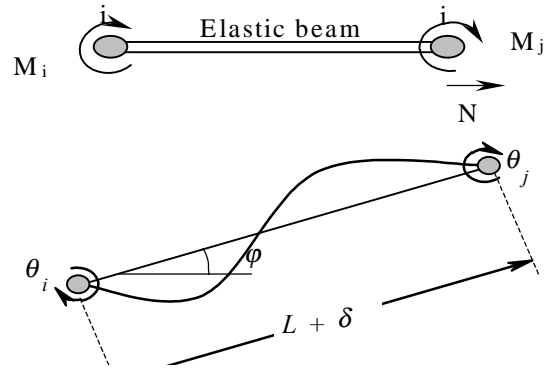
$$\varepsilon^d = \frac{\sigma d}{E(1-d)} \quad (1)$$

which is consistent with the response of reinforced concrete under uniaxial monotonic loading.

Equation (1) can be applied to a member of constant area  $A$  subjected to an axial load:

$$\delta^d = \frac{NL}{EA} \frac{d_a}{1-d_a} \quad (2)$$

where  $N$  is the axial force and  $\delta^d$  the elongation due to the axial damage  $d_a$ .



**Figure 1: Generalized stresses and strains for the model**

Equation (2) can be generalized in order to take into account the flexural damage effects in a frame member. For it, we consider an element where the stress distribution is described by a three component vector,  $q=[M_i, M_j, N]^T$ , collecting the bending moments at the two ends and the axial force (Figure 1), which is associated to the corresponding kinematic variables  $u=[\theta_i, \theta_j, \delta]^T$ . The constitutive equations expressing the relations between the flexural moments and the corresponding rotations due to damage,  $u^d = [\theta_i^d, \theta_j^d, \delta^d]^T$ , are obtained as:

$$\begin{aligned}\theta_i^d &= \frac{d_i}{1-d_i} \frac{L}{4EI} M_i \\ \theta_j^d &= \frac{d_j}{1-d_j} \frac{L}{4EI} M_j\end{aligned}\quad (3)$$

being  $d_i$  and  $d_j$  the damage variables due to flexural effects at both ends of the member. Therefore, the damage vector for each member will be defined as  $D^t = (d_i \quad d_j \quad d_a)$ .

More details about the formulation of the constitutive equations for this model can be found in [Florez-Lopez, 1995; Perera *et al*, 1998].

### Dissipative Potentials

In order to specify the complete set of equations for a damaged material accounting the CDM, it is necessary to define two dissipation potentials, one for plasticity and the other for damage; no coupling between both potentials is assumed. Then the total dissipation potential is given as:

$$F = f(q, R, X; D) + g(Y; D) \quad (4)$$

where  $f$  is the dissipative potential associated to plasticity function of the actual stress tensor,  $q$ , and  $R$  and  $X$  are the isotropic and kinematic hardening variables, respectively.;  $g$  is the dissipative potential associated to the damage process being  $Y$  the internal variable associated to damage (damage energy release rate) [Lemaitre and Chaboche, 1995].

### Evolution Laws

The Principle of Maximum Plastic Dissipation implies the normality of the flow rules in generalized stress space for plastic deformations and in  $Y$ -space for damage variables:

$$du^p = d\lambda^p \frac{\partial f}{\partial q} \quad dD = d\lambda^d \frac{\partial g}{\partial Y} \quad (5)$$

where  $d\lambda^p$  and  $d\lambda^d$  are plastic and damage consistency parameters, respectively.

The expression for these two functions,  $f$  and  $g$ , is obtained from experimental results and their consideration will be treated in the next section.

## LOW CYCLE FATIGUE MODELLING

Very usually, the Griffith criterion is used as a damage dissipation potential. It is well known, nevertheless, that the Griffith criterion cannot describe crack propagation under repeated loads since the maximum energy released load remains constant in that case. However, the application of cyclic loading activates the dislocation motion that will lead to the formation of a fatigue crack. Therefore, models based on the simple Griffith criterion are not able to simulate the strength degradation due to fatigue effects.

Different alternatives have been proposed in the literature to perform the fatigue modelling [Chaboche, 1995; Marigo, 1985; Suaris *et al*, 1990]. In this paper, it is proposed a generalization of the Griffith criterion in order to include the low cycle fatigue effects. For it, a new function affecting the crack resistance is introduced. This function depends on the number of cycles and, so, an implicit evolution of the internal variables of the plastic and damage processes is obtained.

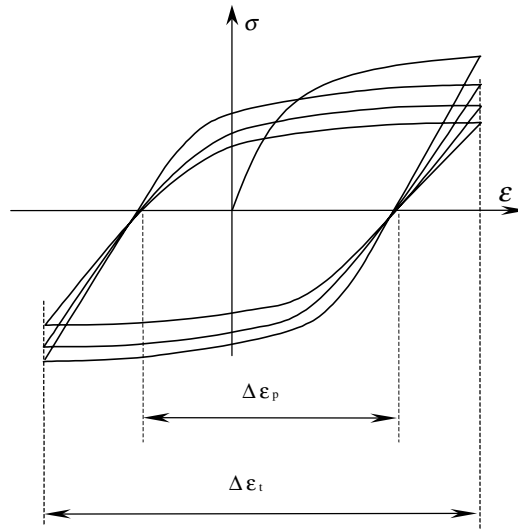
### Miner's Rule

Under the conditions of cyclic loading, the analysis can be generally based on the principle of linear summation of damage. This principle, as applied to fatigue fracture was formulated by Palmgren [1924] and Miner [1945]. Damage fractions due to each individual cycle are summed until fracture occurs. Failure is assumed to occur when these damage functions sum up to or exceed unity:

$$D_{mi} = \sum \frac{n_i}{N_f} \geq 1 \quad (6)$$

where  $n_i$  is the number of cycles for the current amplitude and  $N_f$  is the number of cycles to failure for this amplitude

The application of the linear cumulative damage model consists of converting random cycles into an equivalent number of constant amplitude cycles. Techniques like rainflow [Matsuishi and Endo, 1967] or range pair [Dowling, 1972] allow to perform this conversion.



**Figure 2: Total and plastic strain amplitude**

The quantification of the number of cycles to failure  $N_f$  is performed usually through the Manson-Coffin relationship [1953]:

$$N_f = C(\Delta\epsilon^p)^K \quad (7)$$

where  $\Delta\epsilon^p$  is the plastic strain amplitude of the hysteretic cycles (Figure 2) and  $C$  and  $K$  are parameters depending on the materials. Some authors [Kunnath *et al*, 1997; Koh and stephen, 1991] suggested the total strain amplitude could be used instead of plastic strain.

## Formulation

From the Griffith criterion, such as it was defined in Florez-Lopez [1995], it is proposed here the following dissipation potential for low cycle fatigue damage:

$$g = Y - [Y_{cr} + Z(D) \cdot \xi(\omega)] \quad (8)$$

where  $\omega$  is a cumulative parameter and  $\xi(\omega)$  is a function which allows to include the fatigue effects in the damage evolution and which has to satisfy the following conditions:

$$\begin{aligned} \xi(\omega) &= 1 \Leftrightarrow \omega \leq \omega_{\min} \\ \xi(\omega) &= 0 \Leftrightarrow \omega = \omega_{\max} \end{aligned} \quad (9)$$

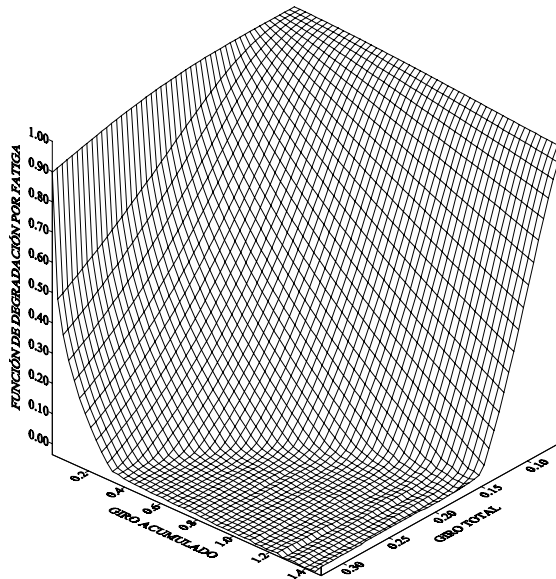
In the same way, it is defined a plastic dissipation potential including the new function as follows:

$$f = |M - X| - (M_y + R\sqrt{\xi(\omega)}) \quad (10)$$

The keypoint in the simulation of the fatigue phenomena is the choice of the fatigue function  $\xi(\omega)$ . From the Miner's rule and through different numerical evaluations a good correlation has been obtained with a function such as:

$$\xi(\tilde{\theta}, \theta_t) = 1 - \left( \frac{\tilde{\theta}}{N_f(\theta_t) \cdot \theta_t} \right)^\mu \quad (11)$$

where  $\tilde{\theta}$  and  $\theta_t$  are the total cumulative rotation and the total rotation, respectively, and  $\mu$  is the ductility. This function is represented in the Figure 3. In this expression, it can be observed that the cocient between brackets can be considered as a Palmgrem-Miner like relationship.



**Figure 3: Fatigue function**

The evaluation of the number of cycles to failure  $N_f$  in equation (11) has been performed taking the expression performed by Koh and Stephen [1991] which is based on the total strain:

$$\varepsilon_t = \frac{\Delta \varepsilon_t}{2} = 0.08(2N_f)^{-\frac{1}{3}} \quad (12)$$

## Actualization of the number of cycles

When we work with cycles of non constant amplitude, some inconsistencies due to a quick loss of strength may appear in the model. The main reason of this is the strong decrease of the value of the fatigue function when the cycle amplitude is increased. This phenomenon is not consistent with the experimental tests.

To overcome this problem, the total cumulative rotation must be recalculated when the maximum response is increased or decreased. For it, the following continuity condition has to be satisfied:

$$\xi(\tilde{\theta}^{\text{old}}, \theta_t^{\text{old}}) = \xi(\tilde{\theta}^{\text{new}}, \theta_t^{\text{new}}) \quad (13)$$

from which the new value is obtained

$$\tilde{\theta}^{\text{new}} = N_f^{\text{new}} \theta_t^{\text{new}} \left( \frac{\tilde{\theta}^{\text{old}}}{N_f^{\text{old}} \theta_t^{\text{old}}} \right)^{\frac{\mu^{\text{new}}}{\mu^{\text{old}}}} \quad (14)$$

## RESULTS

The model presented above is checked through some examples.

Figures 4a and b represent experimental and numerical results using the proposed dissipative functions. Results from Fig. 10 are referred to a circular cross section reinforced concrete column [Kunnath *et al.*, 1997] which is subjected to a constant axial load of 806 kN and lateral displacement is controlled. The numerical simulation has been done with the following parameters:  $EI/L = 2.51E+7$  Nm,  $M_{cr}^+ = M_{cr}^- = 27.420$  kNm,  $M_p^+ = M_p^- = 87.808$  kNm,  $M_u^+ = M_u^- = 98.784$  kNm,  $\theta_{pu}^+ = \theta_{pu}^- = 0.029$ ,  $\alpha^+ = \alpha^- = 1$ .

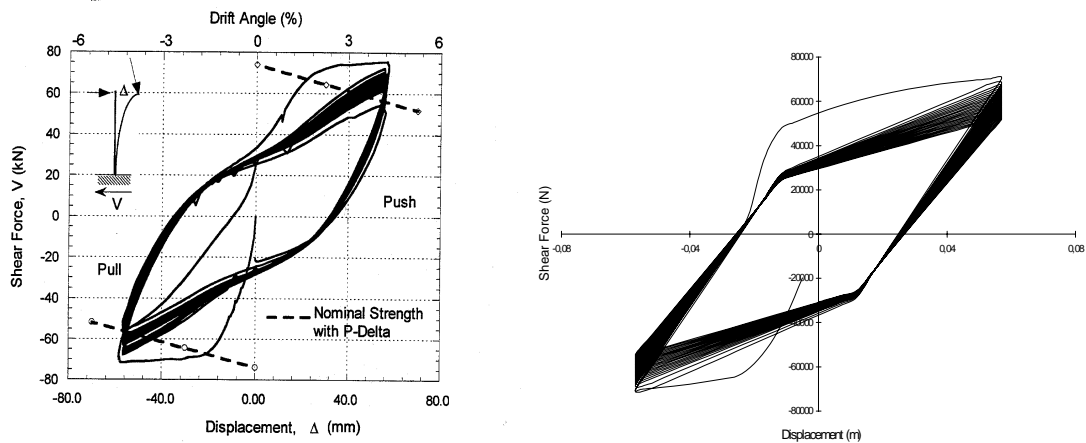


Figure 4: Experimental test (left) by Kunnath *et al.* (1997) and numerical simulation (right)

Figures 5a and b show experimental and numerical results of a rectangular cross section reinforced concrete column with moderate confinement tested by Wehbe *et al.* [1996].

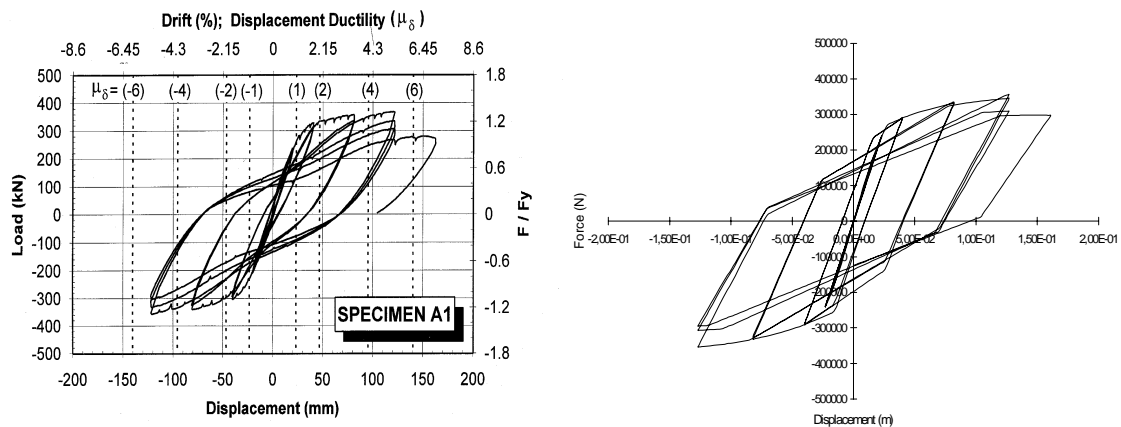


Figure 5. Experimental test (left) by Wehbe *et al.* (1996) and numerical simulation (right)

As in the previous cases the column is subjected to a constant axial load of 641 kN and lateral displacement is controlled. The numerical simulation has been done using the parameters:  $EI/L = 2.21E+7$  Nm,  $M_{cr}^+ = M_{cr}^- = 210$  kNm,  $M_p^+ = M_p^- = 643$  kNm,  $M_u^+ = M_u^- = 850$  kNm,  $\theta_{pu}^+ = \theta_{pu}^- = 0.05$ ,  $\alpha^+ = \alpha^- = 1$ . The damage index evolution in the numerical simulation is represented in Figure 6.

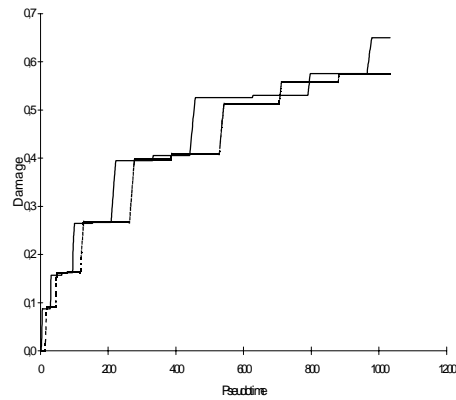


Fig.6. Damage evolution for example 3

## CONCLUSIONS

1. The strength degradation due to low cycle fatigue has been formulated through a suitable choice of the dissipative potentials
2. Good correlation between experimental and numerical results under cyclic loading has been obtained
3. Damage index is associated with the cracking level of the concrete and plastic rotations are related to plastic deformations in the reinforcement
4. Strength degradation due to fatigue effects is associated with the fatigue in the longitudinal reinforcement
5. This model could be taken into account as a framework for seismic retrofitting decision making of structures.
6. The approach presented is amenable of further generalizations

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