EFFECTIVE AND NEUTRAL STRESSES IN SOILS USING BOUNDARY ELEMENT METHODS

E. Alarcón, C. García-Suárez, A. Reverter

Escuela Técnica Superior de Ingenieros Industriales
Polytechnical University of Madrid. Spain

The evaluation of neutral pressures in soil mechanics problems is a fundamental step to evaluate deformations in soils. In this paper we present some some results obtained by using the boundary element method for plane problems, describing the undrained situation as well as the consolidation problem.

UNDRAINED CASE

Since long ago (Scott,1965), it has been recognized that, for loads normal to the boundary, the undrained case can be reduced to a potential problem. This has produced several analytical solutions, (P.N.Sundaram, 1980;J.Bielak, 1982.),but only recently (Alarcón et al.1983),it has been clear where are the limitations of that approach.
In the hypothesis of working in a linear media, the Beltrami equations for zero body forces produce

$$\nabla^2 I=0 \qquad \text{or} \qquad \nabla^2 \theta=0 \qquad \ldots(1)$$

where I is the first stress invariant and θ the volumetric deformation.Taking into account the principle of effective stresses it is possible to write

$$3\,\nabla^2 u + \nabla^2 I'=0 \qquad \ldots(2)$$

or

$$\nabla^2 u=-(\nabla^2 I')/3 \qquad \ldots(3)$$

where u is the pore pressure and I' is the first invariant of effective stresses.
For continuty it is necessary now to stablish the

compatibility of deformations between the soil skeleton and the water filling completely the voids in the case of a fully saturated soil.The volumetric soil skeleton change is

$$\Delta V=V(I'/K') \qquad \text{...(4)}$$

where V is the volume under study and K'the effective bulk modulus of the soil skeleton.
In the water case

$$\Delta V_w=(nV/K_w)u \qquad \text{...(5)}$$

where n is the porosity and K_w the bulk modulus of the water.As

$$I'=I - 3\,u \qquad \text{...(6)}$$

equating eqs.(4)&(5),and inserting eq.(6) it is possible to obtain the following expression for u:

$$u = \frac{I}{K'((3/K')+(n/K_w))} \qquad \text{...(7)}$$

We have now in Equation (7) the fundamental relation we were looking for.It expresses the neutral pressure as a function of soil properties as well as the total pressures.Inserting eq.(1) into eq.(7) it is possible to state the proposed problem as the pair of the following equations:

$$\begin{array}{ll} \nabla^2 u=0 & \text{in } \Omega \\ u=\text{eq.}(7) & \text{in } \delta\Omega \end{array} \qquad \text{...(8)}$$

where Ω is the domain and $\delta\Omega$ its boundary.
In the special case in which it is possible to assume $K_w=\infty$,that is,full incompressibility

$$u=I/3 \qquad \text{in } \delta\Omega \quad \text{...(9)}$$

or,when there are only normal pressures acting on the boundary

$$u=p \qquad \text{in } \delta\Omega \quad \text{..(10)}$$

which is Scott's equation.
From what has been said we see that the Laplace's equation approach is only useful in the fully incompressible case,or when it is possible to define the first total stress invariant in the boundary.In the general case it is better to work directly with eq.

(7) after having solved an elasticity problem. It is possible then, to use the enormous amount of analytical experience stored in books like that from Poulos (1974).

As an example we show in fig.1 the total effective and neutral pressures obtained, by using known analytical expressions in the classical elasticity theory, under the corner of a shallow foundation when there is a uniform distribution of tangential stresses acting at the interface area. Fig.1a. represents an aspect ratio of 0.5, while fig.1.b is for a square foundation.

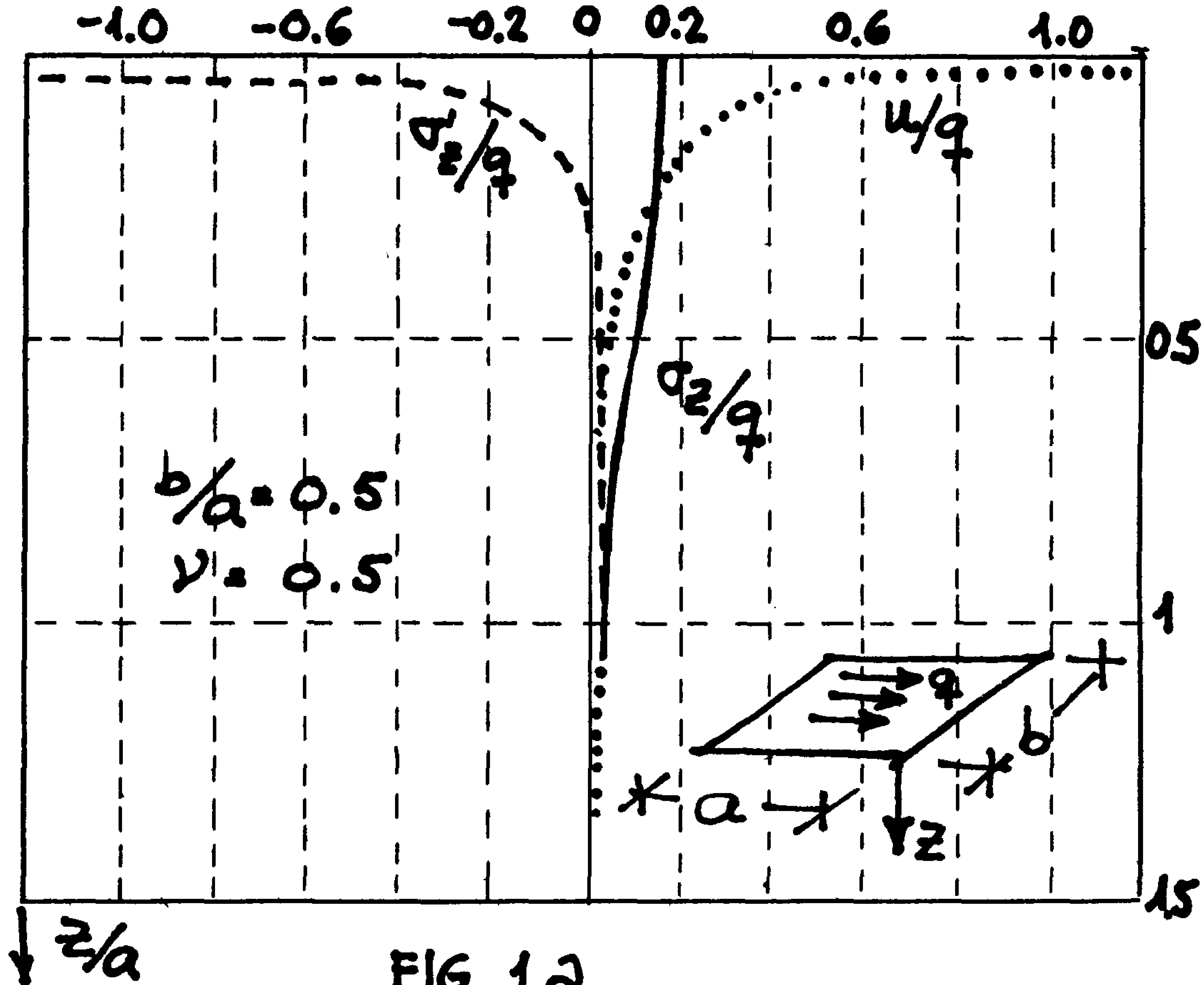
On the contrary, if as is usually the case, one assumes full incompressibility and forces acting orthogonally to the boundary, the pore pressure distribution can be obtained throuhg the solution of a potential problem . In this case the Boundary Element Method is the cheapest solution. In what follows we present some results obtained for an embedded strip foundation. The computer program was prepared for an IBM-PC, using several features to improve the efectiveness and speed of the BEM procedure as described elsewhere (Gª-Suárez et al 1983). Fig.2a. shows the discretization used at the boundary, as well as the cell's mesh utilized in the domain to interpolate the isolines. Those are shown in fig.2b. for a regular spacing of neutral pressures.

In order to produce results with practical applications we ran several cases which have been summarized in figure 3. There, the evolution of the neutral pressures under the center of the footing is collected versus the depth ratio z/H , where H refers to an impervious boundary at the bottom of a permeable stratum. Fig. 3a. represents the influence of different degrees of embeddment, while Fig.3b. shows the dependence on the stratum's depth.

CONSOLIDATION PROBLEMS

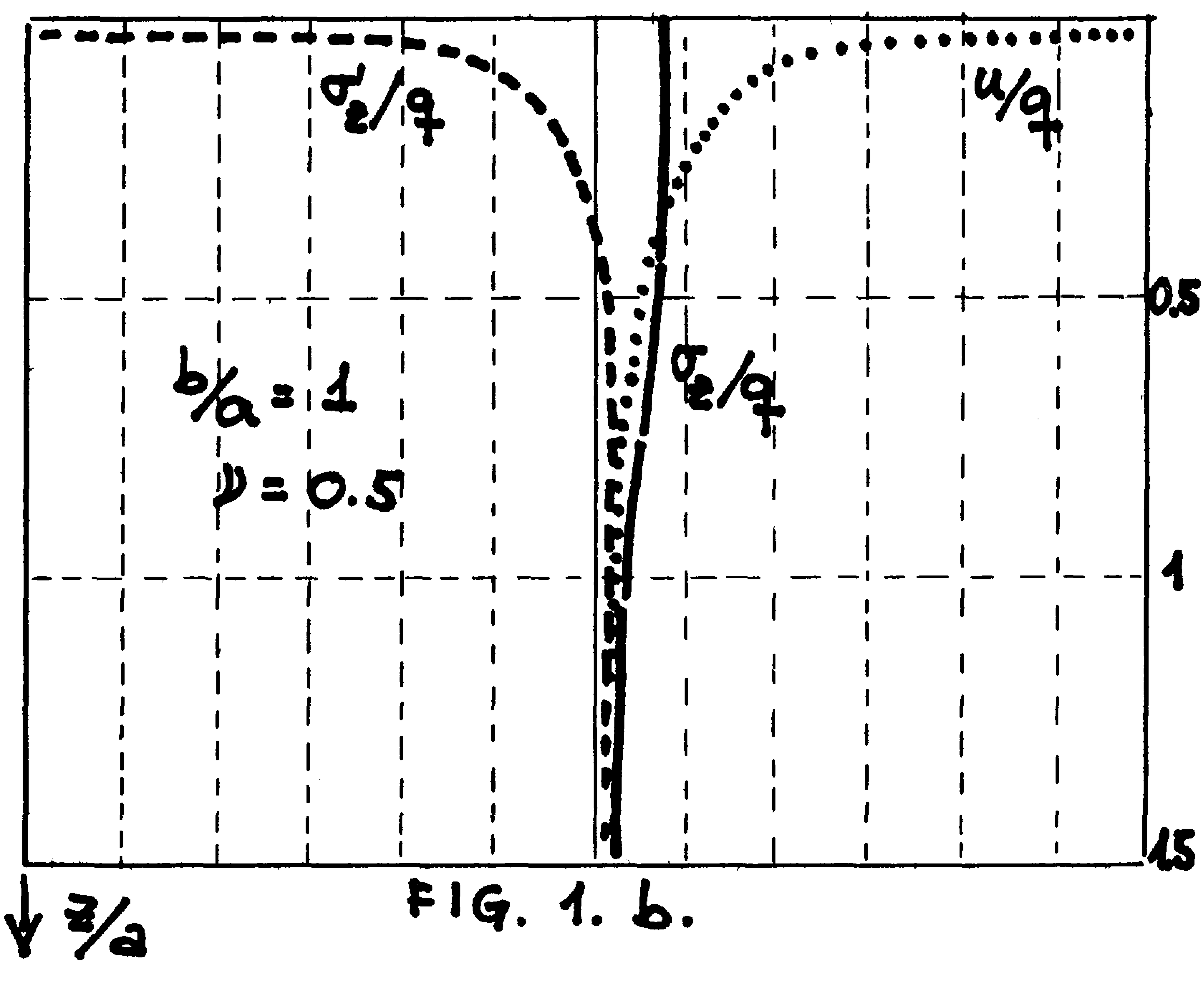
When there is the possibility of continuos drainage in the soil the problem is called Consolidation The excess pore-pressure is dissipated slowly and a continuous transfer of load is produced between the fluid and the soil skeleton. The phenomenon can be described as a coupled problem, in which the neutral pressures are mainly governed by a diffussion equation while the soil-skeleton stresses can be described by an elasticity equation. The problem is a classical one in soil mechanics, and it is well documented elsewhere. (See Verruijt 1977, for a finite element approach of it).

Two approaches were presented to that problem, when



-1.0
-0.6
-0.2
0
0.2
0.6
1.0
σ'z/q
u/q
σz/q
0.5
1
1.5
b/a = 0.5
ν = 0.5
q
a
b
z
z/a

FIG. 1.a.



σ'z/q
u/q
σz/q
0.5
1
1.5
b/a = 1
ν = 0.5
z/a

FIG. 1. b.

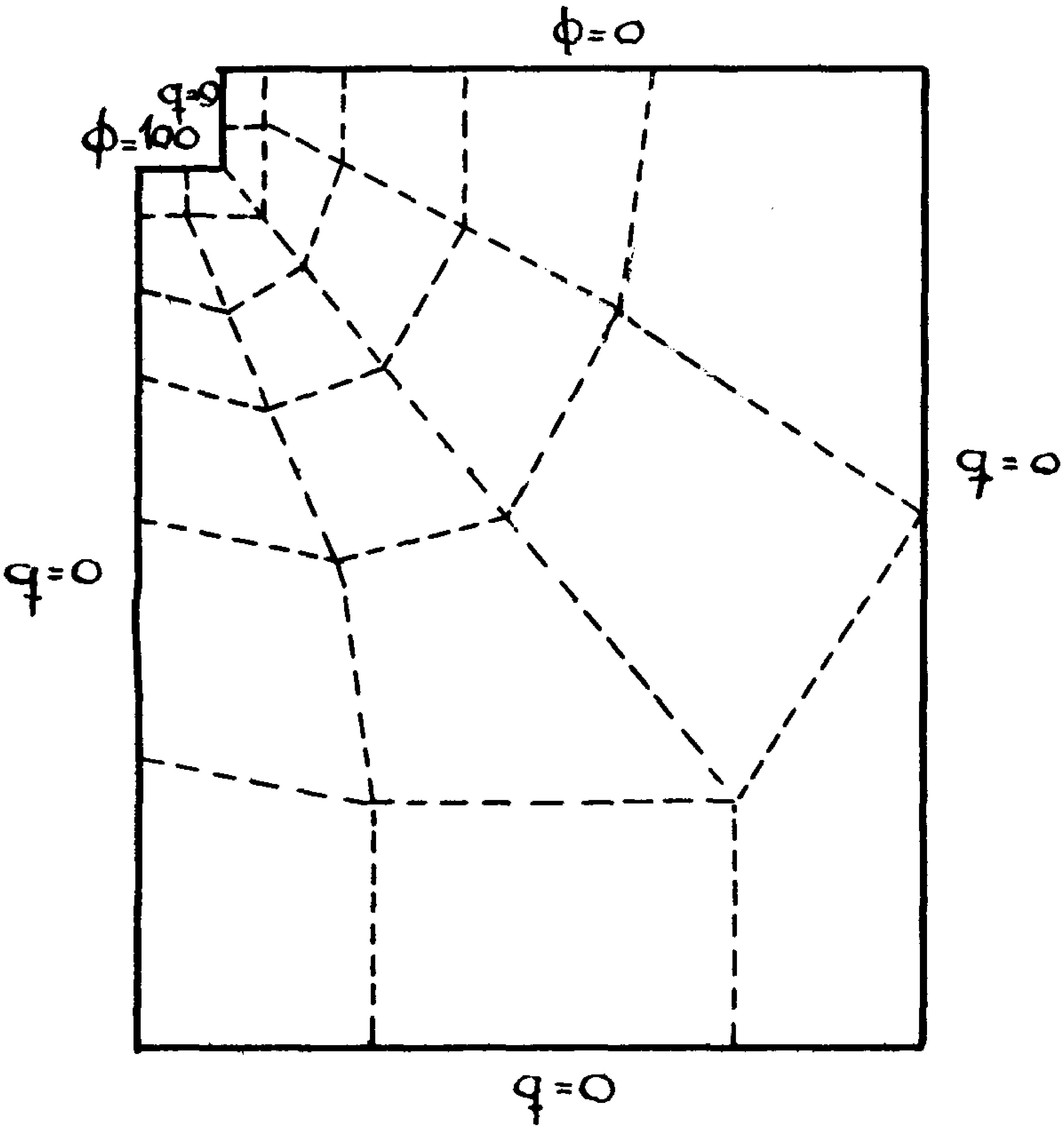


FIG. 2 a. Cell's mesh & boundary conditions

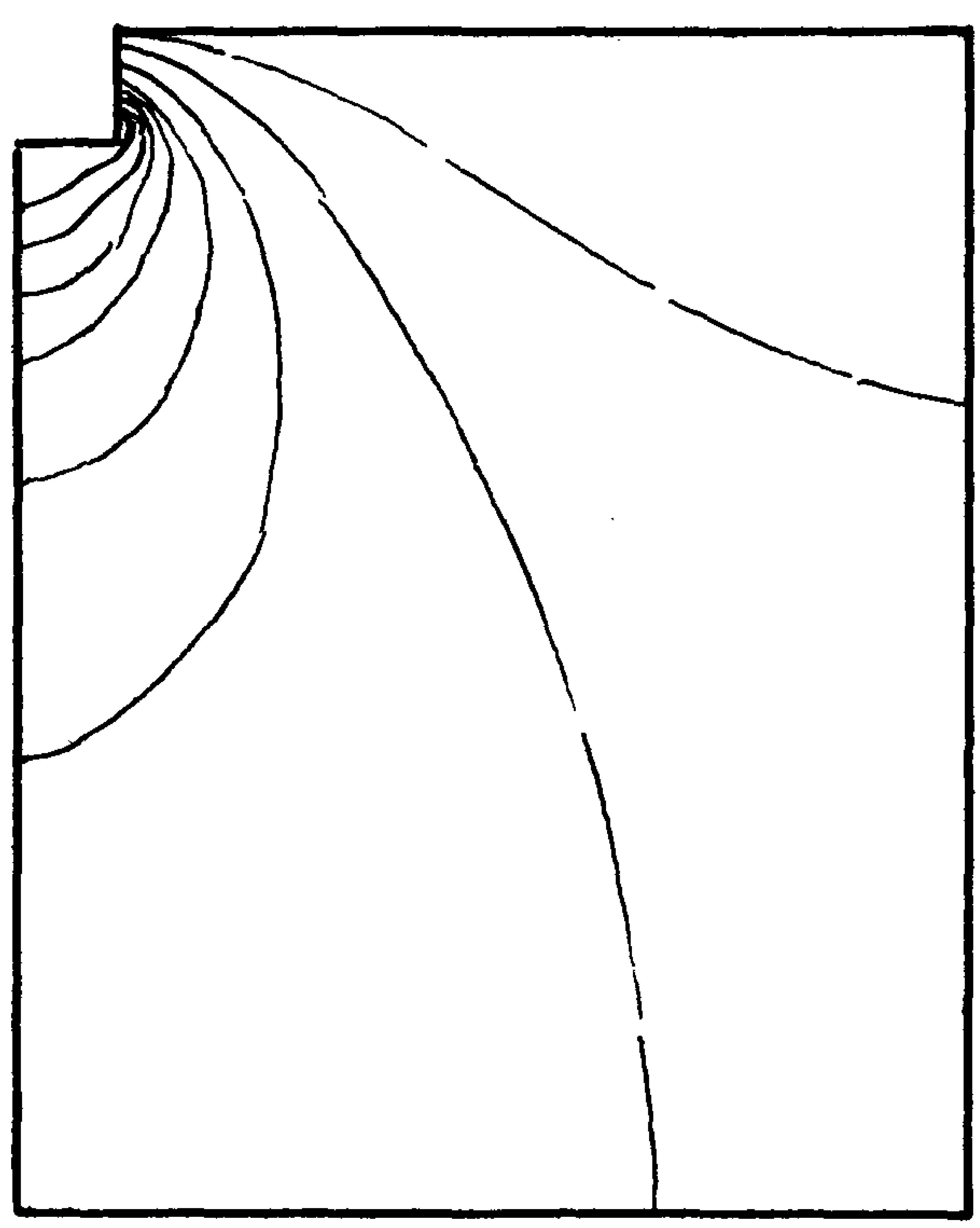


FIG 2 b. Equipotential (u= ctnt) lines.

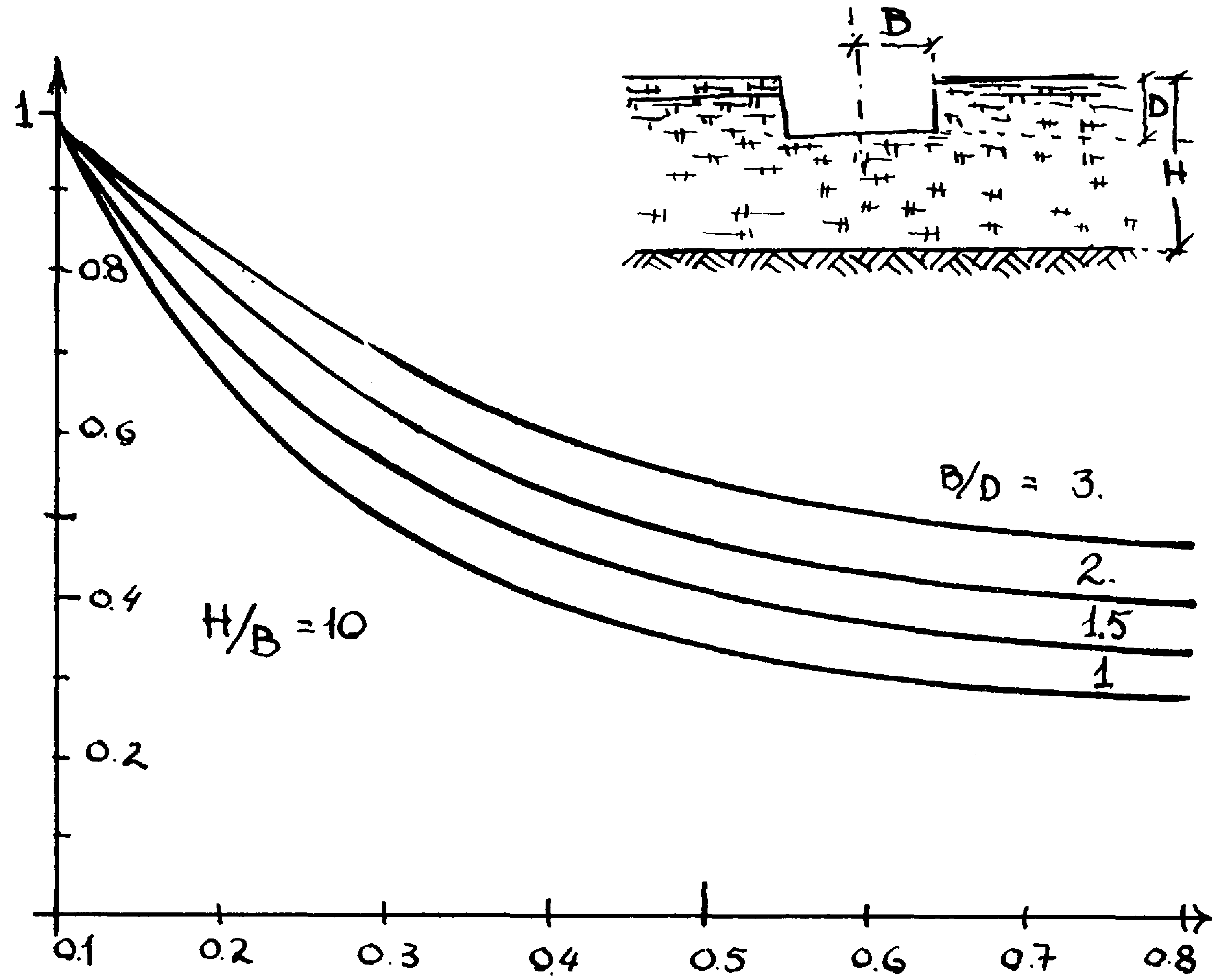


FIG. 3a. Neutral pressures for different footing's widths

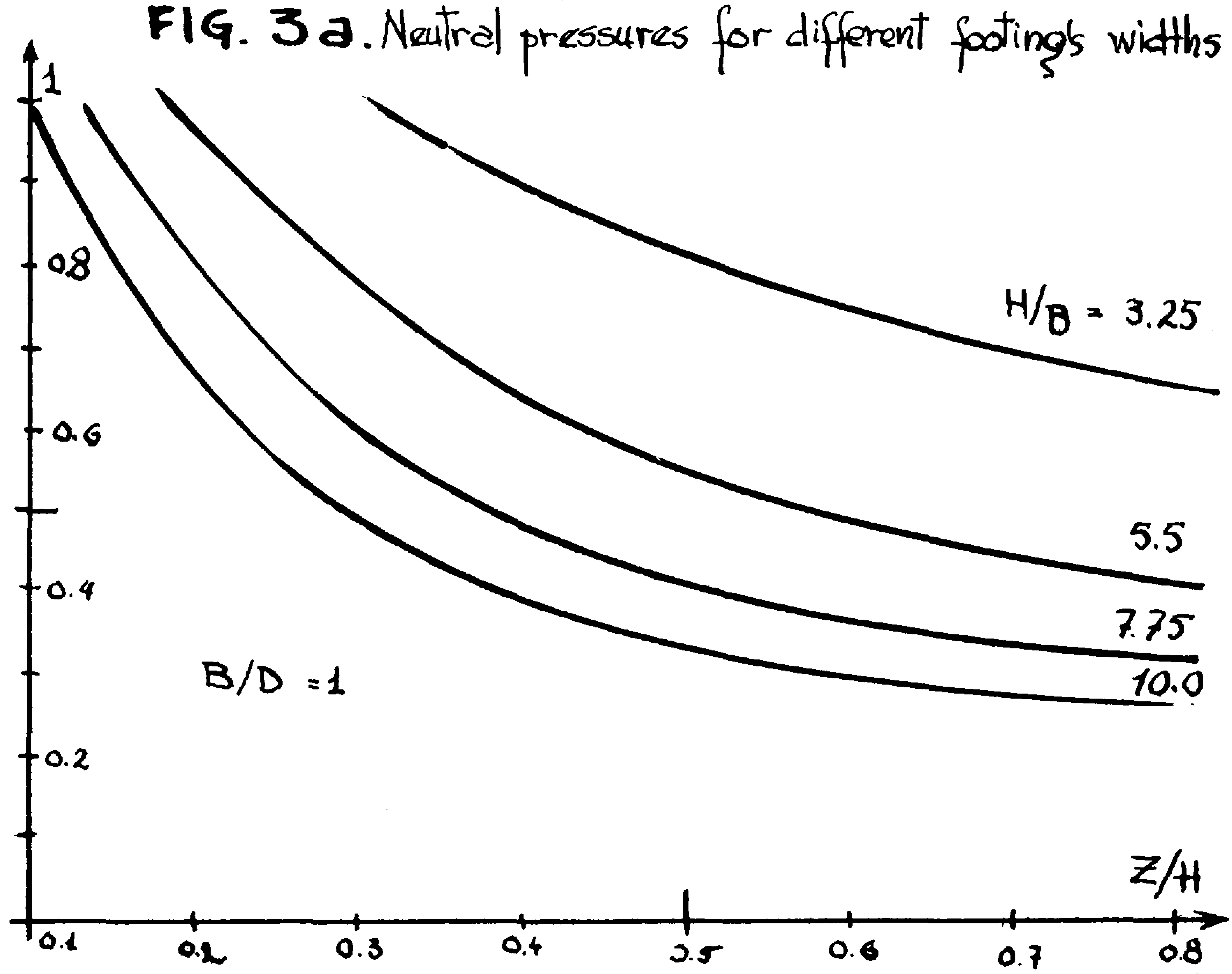


FIG. 3b. Neutral pressures for different layer's depths

solved by boundary elements. The first one (Onishi et al.) works directly with a fundamental solution that includes the time, while the second (Gª Suárez et al.) works with the fundamental solution to a Helmoltz-type equation produced after accepting a finite difference aproach in the time domain. The coupling term between equations is due to the first invariant of stresses, that is related to the neutral pressure value. After the classical transformations in a direct B.E.M. the elasticity equation can be written as

$$c.u_i+\int_{\delta\Omega}\sigma_{ij}n_j U_{ji}-\int_{\delta\Omega}\sigma^0_{ij}n_j u_i+\int_{\Omega}\emptyset U_{ji,i}+\int_{\Omega}X_i U_{ji}=0 \quad (11)$$

The effective stresses can be obtained by the input of eq. (11) into the Lamé's relationships, getting

$$\sigma'_{ij}=\int_{\delta\Omega}t_k D_{kij}-\int_{\delta\Omega}u_k S_{kji}-\frac{2G}{1-2\nu}\delta_{ij}\frac{\delta I^m}{\delta x_m}+G\left(\frac{\delta I^i}{\delta x_j}+\frac{\delta I^j}{\delta x_i}\right) \quad (12)$$

$$I^k=\int_{\Omega}\emptyset U_{ki,i} \quad \ldots(13)$$

The derivatives of eq(13) have to be done carefully

$$\frac{\delta I^k}{\delta x_m}=\int_{(\Omega-B\epsilon)}\emptyset\frac{\delta U}{\delta x_m}-\emptyset\lim_{\epsilon\to 0}\int_{\delta(\Omega-B\epsilon)}U_{ki,i}\,r_{,k} \quad \ldots(14)$$

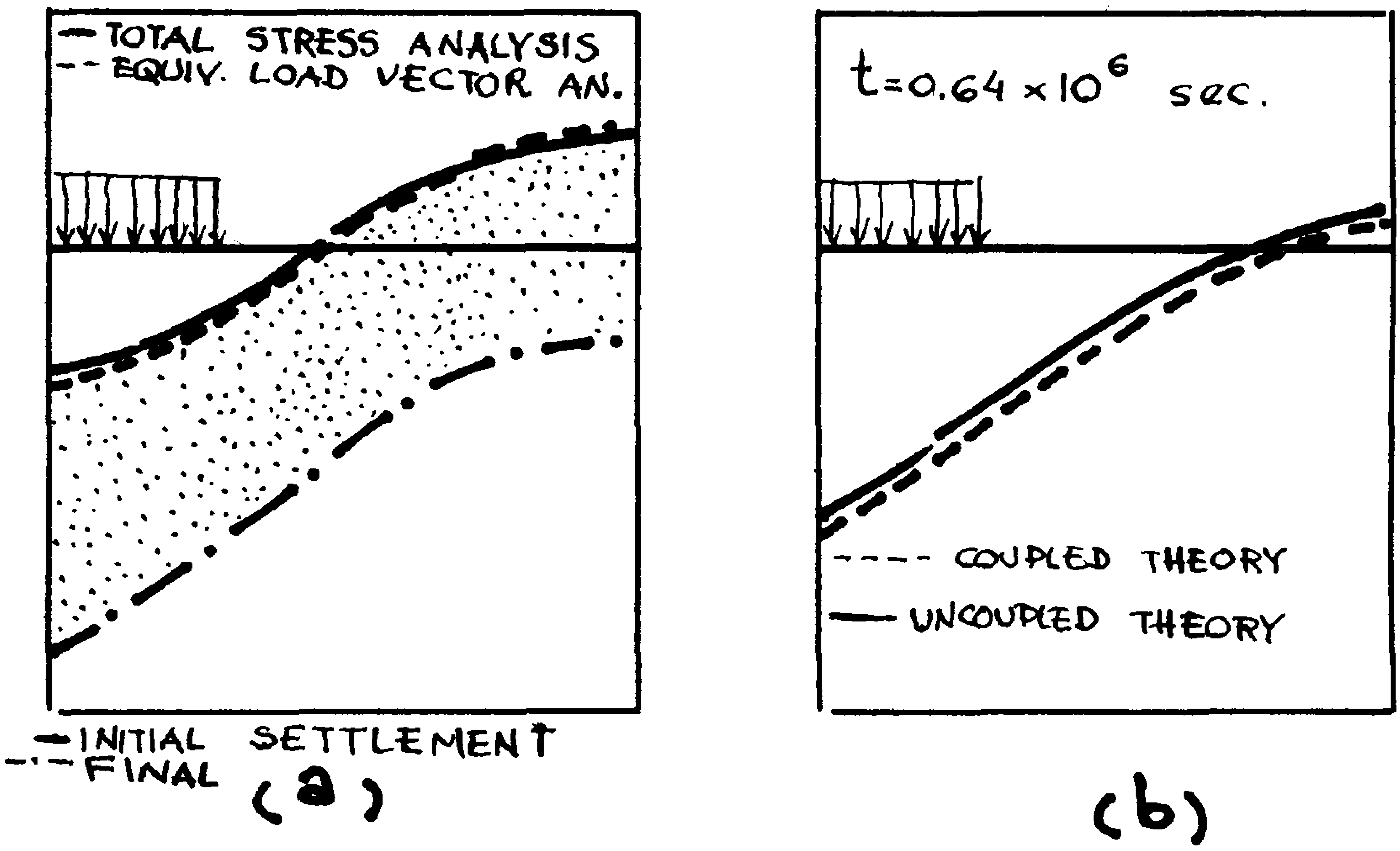
where the first integral in the r.h.s. refers to the domain except a ball of radius ϵ around the pole where the fundamental solution is being applied and the second integral is extended over the boundary. It is important to notice that, in principle, we do not need the σ_{ij} value, but the octaedral one. It is possible then to show that the first integral dissapears (Gª Suarez, 1982), and then it is only neccesary to reduce the second integration to

$$I_2=-(\emptyset.\delta_{ij}/2-2\nu) \quad \ldots(15)$$

And so

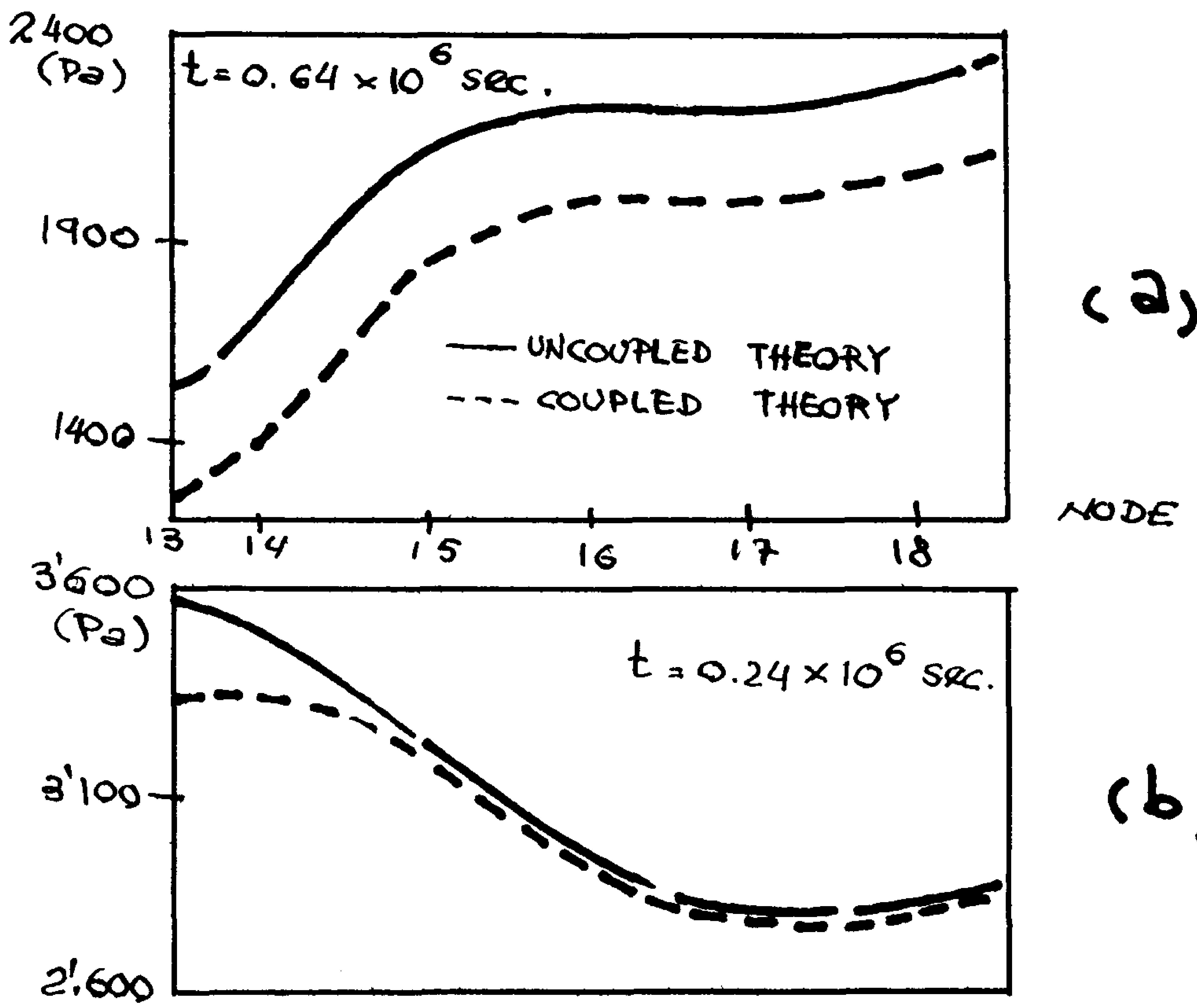
$$\sigma'_{oct}=-\emptyset/2(1-\nu) \quad \ldots(16)$$

Needless to say that the most important part in the computations are the volume integrals. We have tried several possibilities that are described in other part of this volume. (Gª Suárez et al.). In fig 5 we have collected some results for a classical consoli



TOTAL STRESS ANALYSIS
EQUIV. LOAD VECTOR AN.
INITIAL SETTLEMENT
FINAL
(a)
t=0.64×10^6 sec.
COUPLED THEORY
UNCOUPLED THEORY
(b)

FIG. 4



2400
(Pa)
t=0.64×10^6 sec.
1900
1400
UNCOUPLED THEORY
COUPLED THEORY
(a)
NODE
13
14
15
16
17
18
3'600
(Pa)
t=0.24×10^6 sec.
3'100
2'600
(b)

FIG. 5

dation of a stratum 20 m.depth by 20 m. width loaded in the center of the upper surface by a uniformly distributed pressure 6 m. width of 100000 N/m². Fig.4 a. shows the comparison between the initial and final settlements,comparing the results of a total stress analysis with other in effective stresses with an equivalent load vector such

$$\int_{\Omega} \varnothing_{ij} U_{,i} \qquad \ldots(17)$$

Finally,it is possible to say that,apparently,the coupled consolidation theory is fastest than the one obtained by the Terzaghi-Rendulic's assumption. Fig.4b. displays both results for a particular time while Figs.5 a & b. show the distribution of neutral pressures along the axis of symmetry in two different times.The same phenomenon can be observed here.

REFERENCES

Alarcon,E.& Martí,P.(1983)Discussion to Bielak's note.To be published.Jour.Geot.Eng.Div.ASCE.

Bielak,J.(1982) Initial Pore Pressure from vertical Surface Loads.Jour.Geot.Div.ASCE.

GªSuárez,C.(1982)The BEM in consolidation problems (in spanish).Anales de Ingeniería Mecánica.Madrid

GªSuárez,C.&Alarcón,E.(1982)Consolidation problems in Boundary Element Methods in Engineering.Ed.C.Brebbia.Springer Verlag.

GªSuárez,C.&Alarcón,E.(1983)A new scheme for domain integration in Boundary Element problemsThis volume

Onishi,K.,Kuroki,T.&Tomoko,I.(1982)Boundary element methods in Biot's linear consolidation.App.Math.Modelling.Vol2,n.2

Poulos,H.G.&Davis,E.H.(1974)Elastic solutions for rock and soil mechanics.J.Wiley

Scott,R.F.(1965)Principles of soil mechanics.Addisson Wesley Pub.

Sundaram,P.N.(1980)Initial Excess Pore Pressure in soils.Jour.Geot.Div.ASCE.Vol.100,n GT4

Verruijt,A.(1977)Generation and dissipation of pore-water pressures.In Finite element in Geomechanics Ed.by Gudehus.J.Wiley