

Dominance Measuring Method Performance under Incomplete Information about Weights

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1. INTRODUCTION

In multi-attribute utility theory (MAUT), the functional form of the multi-attribute utility function differs subject to a variety of independence conditions, see Keeney and Raiffa (1976). The *additive model* is considered to be a valid approach in most practical situations for the reasons described in Raiffa (1982) and Stewart (1996). The functional form of this model is

$$u(A_l) = \sum_i w_i u_i(x_i^l),$$

where x_i^l is the performance of alternative A_l in the attribute X_i , $u_i(x_i^l)$ is the utility associated with the aforementioned performance, for $u_i(\cdot)$ is the corresponding component utility function representing the decision maker's (DM) preferences over the possible attribute performances and w_i are the weights representing the relative importance of each attribute. Note that $\sum_i w_i = 1$.

However, complex decision-making problems are usually plagued with uncertainty. Additionally, it is often not easy to elicit precise values for the scaling weights. They are often described within prescribed bounds or as just satisfying certain ordinal relations. Different authors refer to this situation as decision-making with *imprecise information*, with *incomplete information* or with *partial information*.

Several reasons are given in the literature to justify why a DM may wish to provide incomplete information (Weber, 1987; Sarabando and Dias, 2010). Regarding weights, the DM may find it difficult to compare criteria or may not want to reveal his or her preferences in public. Moreover, the decision could be taken in a group decision-making situation, where incomplete information, such as weight intervals, is usually derived from a negotiation process (Jiménez *et al.*, 2005; Mateos *et al.*, 2006).

Many work on MAUT has dealt with incomplete information. Sage and White (1984) proposed the model of imprecisely specified MAUT, where preference information about both weights and utilities is assumed not to be precise. Malakooti (2000) suggested a new efficient algorithm for ranking alternatives when there is incomplete information about the preferences and the value of the alternatives. This involves solving

a single mathematical programming problem many times. Ahn (2003) extended Malakooti's work.

Eum *et al.* (2001) provided linear programming characterizations of dominance and potential optimality for decision alternatives when information about performances and/or weights is incomplete, extended the approach to hierarchical structures (Lee *et al.*, 2002) and developed the concepts of weak potential optimality and strong potential optimality (Park, 2004). More recently, Mateos *et al.* (2007) considered the more general case where imprecision, described by means of fixed bounds, appears in alternative performances, as well as in weights and utilities.

Sarabando and Dias (2010) gave a brief overview of approaches proposed by different authors within the MAUT and MAVT (multi-attribute value theory) framework to deal with incomplete information.

Stochastic multicriteria acceptability analysis (SMAA) was developed for support in discrete group decision-making problems, where there is no weight information (Lahdelma and Salminen, 1998), and is widely used. This situation was also considered by Bana e Costa (1986), Charnetski and Soland (1978), Nijkamp *et al.* (1990) and Voogd (1983). SMAA explores the weight space to describe the ratings that would make each alternative the preferred one and computes *acceptability indices*. Acceptability indices measure the variety of different preferences ranking each alternative best. This information can be used to classify the alternatives as more or less acceptable and unacceptable. However, SMAA ignores information about the other ranks. This can make it difficult to identify good compromise alternatives, particularly when the acceptability is split between extreme alternatives; each of which is ranked best according to a few weights but worst by other weights. The potential compromise alternatives then are the ones that yield a good, but not necessarily the best, rank across many different weights.

Lahdelma and Salminen (2001) proposed the SMAA-2 method to solve this problem. SMAA-2 extends the analysis to the sets of weight vectors for any rank from best to worst for each decision alternative. The rank acceptabilities can then be combined using the so-called meta weights into holistic acceptability indices, describing the overall acceptability of each alternative.

A new approach for dealing with incomplete information about weights is to use information about each alternative's intensity of dominance, known as *dominance measuring methods*.

The first dominance measuring method was proposed in Ahn and Park (2008). They computed both

dominating and dominated measures from a dominance matrix and then derived a *net dominance*. This is used as a measure of the strength of preference in the sense that a greater net value is better. They proposed and compared two alternative approaches with surrogate weighting methods and decision rules by means of a simulation study.

In Mateos *et al.*, (2010), we proposed two new dominance measuring methods aimed at overcoming some drawbacks associated with Ahn and Park's methods. The first method is based on dominating and dominated measures computed from the dominance matrix. They are combined into a net dominance value, but they are all computed differently than Ahn and Park's (2008) measures to resolve deficiencies and improve on Ahn and Park's methods. In the second method, alternatives are ranked on the basis of a *preference intensity measure*. These preference intensity measures are based on the fact that the differences between utilities corresponding to alternatives A_k and A_l are always within the interval whose lower endpoint is the element located at the k th row and l th column of the dominance matrix and whose upper endpoint is located at the l th row and k th column of the dominance matrix with the sign changed. Moreover, alternative A_k is preferred to A_l for the positive values in the above interval, whereas alternative A_l is preferred to A_k for the negative values.

The aforementioned methods considered ordinal relations regarding attribute weights, that is, DMs ranked attributes in descending order of importance. In this paper, we consider different cases with incomplete information about weights. Specifically, we consider weight intervals, weights fitting independent normal probability distributions or weights represented by triangular or trapezoidal fuzzy numbers. Then, a simulation study is performed to compare the proposed methods with the measures reported in Ahn and Park (2008), with classical decision rules and with the SMAA and SMAA-2 methods in the aforementioned situations.

The paper is organized as follows. In Section 2, we examine weight intervals. Pairwise and absolute dominance concepts and derived ranking methods, such as modified classical decision rules and dominance measuring methods, are reviewed, as are the SMAA and SMAA-2 methods. In Section 3, an illustrative example is introduced. Section 4 evaluates and compares the methods in a simulation study and presents the study results. In Section 5, weights fit independent normal probability distributions or are represented by fuzzy numbers. Finally, we outline our conclusions in Section 6.

2. RANKING METHODS BASED ON DOMINANCE MEASURES

Let us consider a group decision-making problem with n attributes ($X_i, i = 1, \dots, n$) and m alternatives ($A_j, j = 1, \dots, m$), where incomplete information about input parameters has been incorporated into the decision-making process as follows:

- Alternative performances under uncertainty ($x_i^j \in [x_i^{jL}, x_i^{jU}], i = 1, \dots, n; j = 1, \dots, m$), where x_i^{jL} and x_i^{jU} are the lower and the upper endpoints of the performance interval of the attribute X_i for the alternative A_j , respectively.
- Imprecision concerning utility function assessment ($u_i(\cdot) \in [u_i^L(\cdot), u_i^U(\cdot)], i = 1, \dots, n$), where $u_i^L(\cdot)$ and $u_i^U(\cdot)$ are the lower and the upper utility functions of the attribute X_i .
- Imprecision concerning weights, which is represented by weights intervals ($w_i \in [w_i^L, w_i^U], i = 1, \dots, n$), where w_i^L and w_i^U are the lower and the upper endpoints of the weight interval for the attribute $X_i, i = 1, \dots, n$.

Given two alternatives A_k and A_l , alternative A_k dominates A_l if $D'_{kl} \geq 0$, D'_{kl} being the optimum value of the optimization problem (Puerto *et al.*, 2000),

$$D'_{kl} = \min \left\{ u(A_k) - u(A_l) = \sum_{i=1}^n w_i u_i(x_i^k) - \sum_{i=1}^n w_i u_i(x_i^l) \right\}$$

$$w_i^L \leq w_i \leq w_i^U, i = 1, \dots, n$$

$$x_i^{kL} \leq x_i^k \leq x_i^{kU}, i = 1, \dots, n$$

$$s.t. \quad x_i^{lL} \leq x_i^l \leq x_i^{lU}, i = 1, \dots, n$$

$$u_i^L(x_i^k) \leq u_i(x_i^k) \leq u_i^U(x_i^k), i = 1, \dots, n$$

$$u_i^L(x_i^l) \leq u_i(x_i^l) \leq u_i^U(x_i^l), i = 1, \dots, n.$$
(1)

Examining the objective function, we find that it can be rewritten as $\sum_i w_i [u_i(x_i^k) - u_i(x_i^l)]$, where $u_i(x_i^k)$ does not depend on weights w_i . Moreover, if we carefully observe the constraints, we discover that variables w_i are independent of the other variables. So, taking into account that weights w_i are non-negative, solving problem (1) is equivalent to solving the optimization problem

$$D'_{kl} = \min \left\{ \sum_{i=1}^n w_i z_i^{kl} \right\}$$

$$s.t. \quad w_i^L \leq w_i \leq w_i^U, i = 1, \dots, n,$$
(2)

where z_i^{kl} are the optimal values of the optimization problem

$$z_i^{kl} = \min \{ u_i(x_i^k) - u_i(x_i^l) \}$$

$$x_i^{kL} \leq x_i^k \leq x_i^{kU}, i = 1, \dots, n$$

$$x_i^{lL} \leq x_i^l \leq x_i^{lU}, i = 1, \dots, n$$

$$s.t. \quad u_i^L(x_i^k) \leq u_i(x_i^k) \leq u_i^U(x_i^k), i = 1, \dots, n$$

$$u_i^L(x_i^l) \leq u_i(x_i^l) \leq u_i^U(x_i^l), i = 1, \dots, n.$$
(3)

Problem (3) can be solved depending on what the characteristics of the utility function for attribute X_i are (Mateos *et al.*, 2007):

- If the utility function is monotonically increasing, then $z_i^{kl} = u_i^L(x_i^{kL}) - u_i^U(x_i^{lU})$.
- If the utility function is monotonically decreasing, then $z_i^{kl} = u_i^L(x_i^{kU}) - u_i^U(x_i^{lL})$.

This concept of dominance is called *pairwise dominance* and leads to the so-called *dominance matrix*:

$$D = \begin{pmatrix} - & D_{12} & \dots & D_{1m-1} & D_{1m} \\ D_{21} & - & \dots & D_{2m-1} & D_{2m} \\ D_{31} & D_{32} & \dots & D_{3m-1} & D_{3m} \\ & & \dots & & \\ D_{m2} & D_{m2} & \dots & D_{mm-1} & - \end{pmatrix}$$
(4)

where $D_{kl} = D'_{kl} / \sum_{i=1}^n w_i$ and D'_{kl} and w_i are the optimum values for the objective function and weights in problem (1), respectively.

Another possibility is to use what is known as *absolute dominance* (Salo and Hämäläinen, 2001). Absolute dominance considers the following linear optimization problems:

$$U_k = \max \sum_{i=1}^n w_i u_i(x_i^k)$$

$$w_i^L \leq w_i \leq w_i^U, i = 1, \dots, n$$

$$s.t. \quad x_i^{kL} \leq x_i^k \leq x_i^{kU}, i = 1, \dots, n$$

$$u_i^L(x_i^k) \leq u_i(x_i^k) \leq u_i^U(x_i^k), i = 1, \dots, n$$

and

$$\begin{aligned}
L_k &= \min \sum_{i=1}^n w_i u_i(x_i^k) \\
w_i^L &\leq w_i \leq w_i^U, i = 1, \dots, n \\
s.t. \quad x_i^{kL} &\leq x_i^k \leq x_i^{kU}, i = 1, \dots, n \\
u_i^L(x_i^k) &\leq u_i(x_i^k) \leq u_i^U(x_i^k), i = 1, \dots, n
\end{aligned}$$

Alternative A_k *absolutely dominates* A_l if $L_k > U_l$, that is, the lower bound of A_k exceeds the upper bound of A_l . Note that if A_k absolutely dominates A_l , then A_k dominates A_l , but the reverse does not hold.

Note that this dominance approach often results in almost no prioritization of alternatives or too many non-dominated alternatives (Kirkwood and Corner, 1993). However, pairwise and absolute dominance values can be used to further prioritize competitive alternatives and hence recommend the best alternative and fully rank alternatives.

The following is an example of how these dominance values have been employed to modify three classical decision rules to operate in an imprecise decision-making context (Puerto *et al.*, 2000; Salo and Hämäläinen, 2001):

- *maximax rule* or *optimist rule (OPT)*: evaluating each alternative for its maximum guaranteed value, that is, $\max\{U_j, j = 1, \dots, m\}$.
- *maximin rule* or *pessimist rule (PES)*: evaluating each alternative for its minimum guaranteed value, that is, $\max\{L_j, j = 1, \dots, m\}$.
- *minimax regret rule (REG)*: evaluating each alternative for the maximum loss of value with respect to a better alternative, that is, $\min\{MR_k, k = 1, \dots, m\}$, where MR_k represents the maximum regret incurred when choosing alternative l , that is, $MR_k = \max\{\max\{u(A_l) - u(A_k) \mid W_{kl}\} \mid l \neq k\}$, where W_{kl} are the constraints of the optimization problem (1).

Although none of these rules ensures that the best ranked alternative is the same as it would be if precise values were elicited for weights, simulations show that the selected alternative is generally one of the best (Sarabando and Dias, 2009).

A new approach is to use information about each alternative's intensity of dominance employing what are known as *dominance measuring methods*. Ahn and Park (2008) proposed two approaches on the basis of the dominance matrix D . In the first approach, denoted by *AP1* (Ahn and Park 1), alternatives are ranked

according to a *dominating measure* $\alpha_k = \sum_{j=1, j \neq k}^m D_{kj}$.

The higher this dominating measure is, the more preferred the alternative will be because the sum of the intensity of one alternative dominating the others will also be greater. In the second approach, denoted by *AP2* (Ahn and Park 2), alternatives are ranked according to the difference between the dominating measure α_k and a dominated measure $\beta_k = \sum_{l=1, l \neq k}^m D_{lk}$, that is, on the

basis of $\alpha_k - \beta_k$.

A simulation study showed *AP1* to be better than *AP2*. Whereas *AP1* involves just adding the paired dominance values in the k th row of D , *AP2* considers paired dominance values in both the k th row and the k th column of D . The reason why *AP1* is better than *AP2* is that *AP2* uses duplicated information (row and column values). On the other hand, *AP1* only takes into account the dominating measure, leading to a trade-off between positives and negatives. The drawbacks associated with *AP1* and *AP2* are that *AP1* only considers dominating measures (trade-off of positive and negative values), and *AP2* duplicates dominated measures.

Two new methods aimed at overcoming these problems were proposed in Mateos *et al.* (2010). The first one, denoted by *DME1* (Dominance Measuring Extension 1), is based on the same idea as Ahn and Park suggested (Ahn and Park, 2008). First, *dominating* and *dominated measures* are computed from the paired dominance values D_{kl} and then a *net dominance* is derived. This net dominance is used as a measure of the strength of preference. *DME1* computes the positive and negative dominating measures (step 2) and positive and negative dominated measures (step 4). They are used to compute first a proportion representing how strongly one alternative is preferred to the others (step 3) and second, a proportion representing how intensely one alternative is not preferred to the others (step 5). Finally, *DME1* subtracts both proportions (step 6) to compute the intensity of the preference. The ranking of alternatives will be based on this intensity value (step 7).

DME1 can be implemented as follows:

1. Obtain the paired dominance values D_{kl} and the dominance matrix D as in (4).
2. Compute the *dominating measures* α_k , α_k^+ and α_k^- for each alternative A_k :

$$\alpha_k = \sum_{l=1, l \neq k}^m D_{kl}, \alpha_k^+ = \sum_{l=1, l \neq k, D_{kl} > 0}^m D_{kl} \text{ and } \alpha_k^- = \sum_{l=1, l \neq k, D_{kl} < 0}^m D_{kl}, \forall k.$$

3. Compute the proportion

$$P_k^\alpha = \frac{\alpha_k^+}{\alpha_k^+ - \alpha_k^-}.$$

4. Compute the dominated measures β_k , β_k^+ and β_k^- for each alternative A_k :

$$\begin{aligned}\beta_k &= \sum_{l=1, l \neq k}^m D_{lk}, \beta_k^+ = \sum_{l=1, l \neq k, D_{lk} > 0}^m D_{lk} \text{ and } \beta_k^- \\ &= \sum_{l=1, l \neq k, D_{lk} < 0}^m D_{lk}, \forall k.\end{aligned}$$

5. Compute the proportion

$$P_k^\beta = \frac{\beta_k^+}{\beta_k^+ - \beta_k^-}.$$

6. Calculate the preference intensity value P_k for each alternative A_k :

$$P_k = P_k^\alpha - P_k^\beta, k = 1, \dots, m.$$

7. Rank alternatives according to the P_k values, where the best (rank 1) is the alternative for which P_k is a maximum and the worst (rank m) is the alternative for which P_k is the minimum.

The drawback of the *DME1* method is that when the dominance matrix D contains all negative elements, that is, when all the alternatives are non-dominated, the algorithm is unable to rank the alternatives because all P_k are equal to 0. The properties of matrix D were analysed in an attempt to overcome this problem, and we observed that the difference between the utilities corresponding to A_k and A_l , $\sum_{i=1}^n w_i u_i(x_i^k) - \sum_{i=1}^n w_i u_i(x_i^l)$ in W_{kl} , is always within $[D_{kl}, -D_{lk}]$, as demonstrated in Mateos *et al.* (2010). Thus,

- If $-D_{lk} \leq 0 \Leftrightarrow D_{kl} < 0$ and $D_{lk} \geq 0 \Leftrightarrow$ alternative A_l dominates $A_k \Rightarrow$ the intensity with which A_k is preferred to A_l is 0, that is, $PI_{kl} = 0$.
- If $D_{kl} \geq 0 \Leftrightarrow D_{kl} \geq 0$ and $D_{lk} < 0 \Leftrightarrow$ alternative A_k dominates $A_l \Rightarrow$ the intensity with which A_k is preferred to A_l is 1, that is, $PI_{kl} = 1$.
- If $D_{kl} < 0$ and $D_{lk} < 0$, then interval $[D_{kl}, -D_{lk}]$ will contain a positive subinterval with positive values in which alternative A_k is preferred to A_l and a

negative subinterval in which alternative A_l is preferred to A_k . Thus, the intensity with which A_k is preferred to A_l is the proportion of the positive subinterval over the whole $[D_{kl}, -D_{lk}]$.

On the basis of this idea, a second method was proposed in Mateos *et al.* (2010), denoted by *DME2* (Dominance Measuring Extension 2). In *DME2*, paired dominance values D_{kl} are first transformed into *preference intensities* PI_{kl} (step 2) depending on the preference among alternatives A_k and A_l . Then, a *preference intensity measure* (PIM_k) is derived for each alternative A_k (step 3) as the sum of the preference intensities of alternative A_k regarding the other alternatives. This is used as the measure of the strength of preference.

Dominance measuring extension 2 can be implemented as follows:

1. Compute dominance matrix D (4) from the paired dominance values D'_{kl} (1).
2. If $D_{kl} \geq 0$, then alternative A_k is preferred to alternative A_l , that is, the intensity with which alternative A_k is preferred to A_l is 1, $PI_{kl} = 1$.

Else ($D_{kl} < 0$),

- If $D_{lk} \geq 0$, then alternative A_l dominates alternative A_k ; therefore, the intensity with which alternative A_k is preferred to A_l is 0, that is, $PI_{kl} = 0$.
- Else, note that alternative A_l is preferred to alternative A_k for those values in W_{kl} that satisfy $D_{kl} \leq \sum_{i=1}^n w_i u_i(x_i^k) - \sum_{i=1}^n w_i u_i(x_i^l) \leq 0$, and A_k is preferred to A_l for those values in W_{kl} that satisfy $0 \leq \sum_{i=1}^n w_i u_i(x_i^k) - \sum_{i=1}^n w_i u_i(x_i^l) \leq -D_{lk} \Rightarrow$ the intensity A_k is preferred to A_l is

$$PI_{kl} = \frac{-D_{lk}}{-D_{lk} - D_{kl}}.$$

3. Compute a preference intensity measure for each alternative A_k

$$PIM_k = \sum_{l=1, l \neq k}^m PI_{kl}.$$

4. Rank alternatives according to the PIM values, where the best (rank 1) is the alternative with greatest PIM and the worst is the alternative with the least PIM .

As mentioned before, *DME1* and *DME2*, same with *API* and *AP2*, considered ordinal relations regarding attribute weights, that is, DMs ranked attributes in descending order of importance. For this scenario, Monte Carlo simulation techniques were carried out to analyse their performance and to compare them with other approaches, such as surrogate weighting methods and adapted classical decision rules (Ahn and Park, 2008; Mateos *et al.*, 2010).

The results showed that *DME2* performs better in terms of the identification of the best alternative and the overall ranking of alternatives than other dominance measuring methods proposed by different authors. Also, *DME2* outperforms the adaptation of classical decision rules and comes quite close to the Rank-Order centroid *ROC* method, which was identified as the best approach.

In Section 4, the results of a new computational study based on Monte Carlo simulation techniques are shown for a scenario where there are weight intervals rather than ordinal relations. Surrogate weighting methods cannot be used for this purpose, so the *API* and *DME2* methods are analysed and compared with modified decision rules and with *SMAA* and *SMAA-2* methods, which are briefly described in the succeeding text.

Stochastic multicriteria acceptability analysis (Lahdelma and Salminen, 1998) is widely used for situations where neither criteria values nor weights are precisely known. Uncertain or imprecise criteria values are represented by stochastic variables ε_{ij} with assumed or estimated joint probability distribution and density function $f(\varepsilon)$ in the space X . Similarly, the DM's unknown or partially known preferences are represented by a weight distribution with density function $f(w)$ in the set of feasible weights W . Total lack of knowledge about weights is represented in 'Bayesian' spirit by a uniform weight distribution in W . The distribution has density function $f(w) = \frac{1}{\text{vol}(W)}$.

The *SMAA* method determines for each alternative the set of favourable weights $W_i(\varepsilon) = \{w \in W : u(\varepsilon_i, w) \geq u(\varepsilon_k, w), \forall k\}$. Then, the *acceptability index* is used to classify the alternatives as more or less acceptable ($a_i > 0$) and unacceptable (a_i zero or near zero), where

$$a_i = \int_X f(\varepsilon) \int_{W_i(\varepsilon)} f(w) dw d\varepsilon.$$

In some cases, some amount of bias may occur in the *SMAA* acceptability indices: a) the scaling of the criteria affects the acceptability indices; b) if the assumed weight distribution does not represent the DMs' preference accurately, classification based on acceptability

indices should be limited to excluding alternatives with zero or near-zero acceptability; and c) *SMAA* ignores information about the other ranks. For this reason, the *SMAA-2* method (Lahdelma and Salminen, 2001) was developed. *SMAA-2* extends the acceptability analysis to the sets of weight vectors for each rank from best to worst for each decision alternative.

The *rank acceptability index* a_i^r is then defined as the expected volume of the set of favourable rank weights. The rank acceptability index is a measure of the variety of different ratings ranking alternative A_i as r ,

$$a_i^r = \int_X f(\varepsilon) \int_{W_i^r(\varepsilon)} f(w) dw d\varepsilon,$$

where $W_i^r(\varepsilon) = \{w \in W : \text{rank}(\varepsilon_i, w) = r\}$ with $\text{rank}(\varepsilon_i, w) = 1 + \sum_k \rho(u(\varepsilon_k, w) > u(\varepsilon_{ki}, w))$ and $\rho(\text{true}) = 1$ and $\rho(\text{false}) = 0$. The problem of comparing the alternatives in terms of their rank acceptabilities can be seen as a 'second-order' multicriteria decision problem. For this reason, Lahdelma and Salminen (2001) proposed a complementary approach of combining the rank acceptabilities into holistic acceptability indices a_i for each alternative using

$$a_i = \sum_r \alpha^r a_i^r,$$

where $\alpha^r = (\sum_{i=r}^m 1/i) / (\sum_{i=1}^m 1/i)$ are centroid weights.

3. AN ILLUSTRATIVE EXAMPLE

To illustrate the methods introduced in the previous section, we consider the data from a decision-making problem consisting of the construction of a new general cargo harbour in Helsinki, Finland (Hokkanen *et al.*, 1999; Lahdelma and Salminen, 2001). Table I shows the criteria performances for $m = 13$ alternatives (where I and II are navigation channels; A, B and C represent roads; and 1, 2, 3 and 4 correspond to railway connections) and $n = 11$ attributes.

Uniform probability distributions for criteria performances in the intervals $[x_{ij} - \Delta_{ij}, x_{ij} + \Delta_{ij}]$ are used. For Δ_{ij} , 10% of the interval between the highest and lowest performance for each criteria is used (i.e. $\Delta_{ij} = (x_j^{\max} - x_j^{\min})/10$). The scaling problem is also outlined here; the criteria performances are scaled by using best and worst criteria values, that is, $u_{ij} = (x_{ij} - x_j^{\text{worst}}) / (x_j^{\text{best}} - x_j^{\text{worst}})$.

Table I. Criteria performances

Alternative	1 (min)	2 (min)	3 (min)	4 (max)	5 (max)	6 (min)	7 (max)	8 (max)	9 (max)	10 (max)	11 (max)
IA1	4	1.0	985	30	166	705	25	4.5	4.2	15.1	1.75
IA2	4	2.5	985	30	166	765	25	4.5	4.1	15.3	1.69
IA3	4	1.5	985	30	166	705	25	4.5	4.3	12.7	1.75
IA4	4	1.5	985	30	166	705	25	4.5	4.3	12.2	1.65
IIB1	4	1.5	985	35	177	705	25	4.5	4.4	15.1	1.68
IIB2	4	2.5	985	35	177	765	25	4.5	4.3	15.3	1.62
IIB3	4	2.0	985	35	177	705	25	4.5	4.5	12.7	1.68
IIB4	4	2.0	985	35	177	705	25	4.5	4.5	12.2	1.58
IIC1	4	1.0	985	35	166	705	25	4.5	4.6	14.8	1.72
IIC2	4	2.5	985	35	166	765	25	4.5	4.5	15	1.66
IIC3	4	1.5	985	35	166	705	25	4.5	4.7	12.4	1.72
IIC4	4	2.0	985	35	166	705	25	4.5	4.7	11.9	1.62
ZERO	1	0.0	1300	50	266	4200	0	2.0	1.0	18.8	1.00

Table II shows the dominance matrix. Note that, in this example, a simple operation solves the optimization problem (2). For example,

$$D_{12} \min\{0 - 0.1\hat{6}, 0.5 - 0.1\hat{6}, 0.57 - 1, 0 - 0.1\hat{6}, 0 - 0.1\hat{6}, 0.8\hat{3} - 0.99, 0.8\hat{3} - 1, 0.8\hat{3} - 1, 0.72 - 0.86, 0.45 - 0.59, 0.8\hat{3} - 0.9\hat{3}\} = -0.43.$$

Rows 1 and 2 of Table III show the results output by Ahn and Park's method and the corresponding ranking, respectively. Each element in the first row is computed by adding the values of the corresponding row in Table II.

To apply the *DME2* method, we first have to compute a preference intensity matrix from the dominance matrix and then the preference intensity measure for each alternative. Table IV shows the preference intensity matrix, whereas Table V lists the preference intensity measures for each alternative and the resulting ranking.

Note that the first value in Table IV is computed as follows: $P_{kl} = \frac{0.6}{0.6+0.43} = 0.607$. Values shown in Table V are computed by adding the values in the corresponding row of Table IV (the first row is used for the first value, and so on).

Finally, Table VI shows the results output by the *SMAA* and *SMAA-2* methods, see Lahdelma and Salminen (2001).

Figure 1 illustrates the alternative rankings included in Tables III and V together with the results of the *SMAA* and *SMAA-2* methods.

Looking at Figure 1, we find that

- The best alternative is IIC3 for all four methods, whereas the second-ranked is the same for the

DME2, *SMAA* and *SMAA-2* methods (IIC1), but this alternative is ranked as seventh by the *API* method.

- The best two and the worst three alternatives are the same for the *DME2* method, analysing the set of favourable weights (*SMAA* and *SMAA-2* methods). This similarity is stronger for the *DME2* and *SMAA-2* methods, which also output the same alternatives for the third, fourth, fifth and sixth rankings, whereas IIA4 and IIB1 just switch positions (seventh and eighth). Looking at Tables V and VI, we find that the difference between IIA4 and IIB1 is minimum, although IIA4 was possibly a little better. The same applies to the tenth-ranked and eleventh-ranked alternatives.
- The main difference between the ranking output by the *SMAA* method and *DME2* and *SMAA-2* methods is that alternative ZERO is ranked fourth by *SMAA*, whereas the *DME2* and *SMAA-2* methods rank it as the ninth and tenth, respectively.
- In any case, the results output by the *DME2*, *SMAA* and *SMAA-2* methods are very similar.

Generally, *DME2* is very easy to apply and computationally efficient. On the other hand, the *SMAA-2* method shows more information. In our opinion, the *DME2* approach could be used to output the ranking of alternatives, and the *SMAA-2* method could be used to study the robustness of this ranking.

4. COMPUTATIONAL STUDY

We propose to conduct a simulation study of the dominance measuring methods described in the previous

Table II. Dominance matrix

	IA1	IIA2	IIA3	IIA4	IIB1	IIB2	IIB3	IIB4	IIC1	IIC2	IIC3	IIC4	ZERO
IA1		-0.43	-0.46	-0.52	-0.43	-0.43	-0.46	-0.52	-0.43	-0.43	-0.49	-0.55	-1
IIA2	-0.6		-0.5	-0.54	-0.5	-0.38	-0.48	-0.54	-0.6	-0.38	-0.52	-0.58	-1
IIA3	-0.3	-0.16		-0.23	-0.37	-0.38	-0.38	-0.38	-0.38	-0.38	-0.38	-0.38	-1
IIA4	-0.3	-0.21	-0.27		-0.37	-0.38	-0.38	-0.38	-0.38	-0.38	-0.38	-0.38	-1
IIB1	-0.3	-0.17	-0.46	-0.52		-0.16	-0.46	-0.54	-0.3	-0.19	-0.49	-0.55	-1
IIB2	-0.6	-0.24	-0.5	-0.54	-0.5		-0.48	-0.54	-0.6	-0.21	-0.52	-0.58	-1
IIB3	-0.5	-0.17	-0.3	-0.3	-0.3	-0.16		-0.23	-0.5	-0.16	-0.3	-0.26	-1
IIB4	-0.5	-0.28	-0.35	-0.3	-0.3	-0.21	-0.27		-0.5	-0.25	-0.3	-0.21	-1
IIC1	-0.2	-0.16	-0.42	-0.48	-0.26	-0.26	-0.42	-0.48		-0.16	-0.46	-0.52	-1
IIC2	-0.6	-0.2	-0.5	-0.50	-0.5	-0.26	-0.4	-0.50	-0.6		-0.5	-0.54	-1
IIC3	-0.3	-0.16	-0.2	-0.19	-0.26	-0.26	-0.26	-0.26	-0.3	-0.16		-0.23	-1
IIC4	-0.5	-0.24	-0.3	-0.3	-0.3	-0.26	-0.26	-0.26	-0.5	-0.21	-0.3		-1
ZERO	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	

Table III. Results and corresponding ranking output by Ahn and Park's method

	IA1	IIA2	IIA3	IIA4	IIB1	IIB2	IIB3	IIB4	IIC1	IIC2	IIC3	IIC4	ZERO
<i>APL</i>	-6.2	-6.7	-4.7	-4.8	-5.2	-6.4	-4.3	-4.6	-4.8	-6.3	-3.6	-4.6	-12
Rank	9	12	5	6	8	9	11	2	4	7	10	3	13

APL, Ahn and Park 1.

Table IV. The preference intensity matrix

	IA1	IIA2	IIA3	IIA4	IIB1	IIB2	IIB3	IIB4	IIC1	IIC2	IIC3	IIC4	ZERO
IA1													
IIA2	0.393		0.422	0.392	0.436	0.607	0.523	0.492	0.317	0.607	0.403	0.475	0.5
IIA3	0.578	0.75	0.25	0.281	0.262	0.395	0.270	0.348	0.2	0.347	0.244	0.297	0.5
IIA4	0.608	0.719	0.450	0.550	0.549	0.571	0.470	0.487	0.528	0.571	0.348	0.470	0.5
IIB1	0.564	0.738	0.451	0.420	0.580	0.591	0.470	0.470	0.562	0.574	0.337	0.470	0.5
IIB2	0.393	0.605	0.428	0.409	0.25	0.75	0.422	0.392	0.437	0.725	0.344	0.376	0.5
IIB3	0.477	0.730	0.529	0.529	0.578	0.742	0.257	0.281	0.279	0.549	0.333	0.309	0.5
IIB4	0.508	0.652	0.513	0.529	0.608	0.719	0.450	0.550	0.457	0.727	0.437	0.495	0.5
IIC1	0.683	0.8	0.471	0.438	0.563	0.720	0.543	0.510	0.490	0.664	0.437	0.549	0.5
IIC2	0.393	0.652	0.428	0.426	0.274	0.450	0.272	0.336	0.2	0.8	0.422	0.492	0.5
IIC3	0.596	0.756	0.652	0.663	0.656	0.667	0.563	0.563	0.578	0.75	0.25	0.281	0.5
IIC4	0.525	0.702	0.529	0.529	0.624	0.691	0.505	0.450	0.508	0.719	0.405	0.595	0.5
ZERO	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

section to analyse their performance. For a decision-making problem with m alternatives and n attributes, the process would be as follows:

1. Randomly generate component utilities for each alternative in each attribute from a uniform distribution in $[0,1]$, leading to an $m \times n$ matrix. Normalize the columns in this matrix to make the smallest value 0 and the largest 1 and remove dominated alternatives.
2. Generate attribute weights representing their relative importance. Note that these weights are the TRUE weights, and the derived ranking of alternatives will be denoted as the TRUE ranking. To generate the TRUE weights, we first select $n-1$ independent random numbers from a uniform distribution on $(0,1)$ and then rank these numbers. Suppose the ranked numbers are $1 \geq r_{n-1} \geq \dots \geq r_2 \geq r_1 > 0$. The differences between adjacently ranked numbers are then used as the desired weights: $w_n^T = 1 - r_{n-1}$, $w_{n-1}^T = r_{n-1} - r_{n-2}$, \dots , $w_1^T = r_1$. The resulting weights will sum 1 and be uniformly distributed in the weight space.
3. To derive the corresponding weight intervals, add and subtract the same quantity to precise values, leading to the lower and upper endpoints of the weight intervals. We used the quantities, q , of 0.1/2, 0.2/2, 0.3/2, 0.4/2 and 0.5/2 that represent 10%, 20%, 30%, 40% and 50% imprecision, respectively. In other words, $[w_i^T, w_i^U] = [w_i^T - q, w_i^T + q]$. If $w_i^T - q < 0$, then $w_i^T - q = 0$, and if $w_i^T + q > 1$, then $w_i^T + q = 1$ is considered. Throughout the simulation process, weights will be randomly generated from these weight intervals, $[w_i^T - q, w_i^T + q]$.
4. Compute the ranking of alternatives for each method according to their procedures and compare with the TRUE ranking, computed in step 2. We use two measures of efficacy, *hit ratio* and *rank-order correlation* (Ahn and Park, 2008; Barron and Barrett, 1996; Winkler and Hays, 1985). The hit ratio is the proportion of all cases in which the method selects the same best alternative as in the TRUE ranking. Rank-order correlation represents how similar the overall alternative-ranking structures are in the TRUE and the method-driven rankings. It is calculated using Kendall's τ (Mateos *et al.*, 2009): $\tau = 1 - 2 \times (\text{number of pairwise preference violations}) / (\text{total number of pair preferences})$.

Following (Ahn and Park, 2008; Mateos *et al.*, 2010; Winkler and Hays, 1985), four different levels of alternatives ($m = 3, 5, 7, 10$) and five different levels of

Table V. Results and corresponding ranking output by *DME2* method

	IA1	IIA2	IIA3	IIA4	IIB1	IIB2	IIB3	IIB4	IIC1	IIC2	IIC3	IIC4	ZERO
<i>DME2</i>	5.7	3.79	6.37	6.33	6.12	4.60	6.75	6.62	6.94	4.46	7.54	6.69	6
Rank	10	13	6	7	8	11	3	5	2	12	1	4	9

DME2, dominance measuring extension 2.

Table VI. *SMAA* and *SMAA-2* results

	a^1	a^2	a^3	Holistic a	a^1 -rank	a^2 -rank	a^3 -rank	Holistic rank
IA1	5	11	18	28	8	7	8	9
IIA2	0	0	0	7	13	13	13	13
IIA3	5	12	22	34	7	6	6	6
IIA4	3	9	17	30	10	10	9	8
IIB1	4	11	20	33	9	8	7	7
IIB2	0	1	2	14	11	11	11	12
IIB3	9	21	34	42	5	4	4	3
IIB4	8	18	29	39	6	5	5	5
IIC1	18	33	45	49	2	2	2	2
IIC2	0	1	2	14	12	12	12	11
IIC3	27	48	64	61	1	1	1	1
IIC4	11	22	34	42	3	3	3	4
ZERO	10	11	12	15	4	9	10	10

SMAA, stochastic multicriteria acceptability analysis.

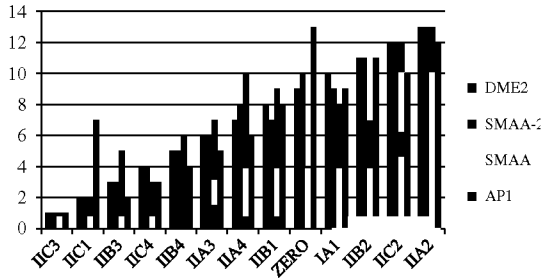


Figure 1. Rankings output by dominance measuring methods (Ahn and Park 1 (*API*) and dominance measuring extension 2 (*DME2*)) and the stochastic multicriteria acceptability analysis (*SMAA*) and *SMAA-2* methods.

attributes ($n=3,5,7,10,15$) were considered to validate the output results. Also, 20 000 trials were performed for each of the 20 design elements (alternatives \times attributes).

The results of similar computational studies when ordinal information about weights is available are shown in Mateos *et al.* (2009). Surrogate weighting methods, methods that modified classical decision rules (*REG*, *PES* and *OPT*, see Section 2) and dominance measuring methods (*API*, *DME1*, *DME2*) were considered. The findings were as follows:

1. Regarding methods that modified classical decision rules, the *REG* method appears to be better than the *PES* method. *PES* outperforms the *OPT* method. They are all outperformed by dominance measuring methods. Moreover, the results for *DME1* and *DME2* methods are similar, but both outperform *API*.
2. The average hit ratio decreases the more attributes there are for all the methods under consideration and any given number of alternatives. This decrease is more pronounced than if the number of alternatives is increased for the same number of attributes.

Surrogate weighting methods are also compared with dominance measuring methods when ordinal information about weights is available in Mateos *et al.* (2010). *DME2* outperforms all the surrogate weighting methods except the *ROC* method, which is the best in terms of the hit ratio and the rank-order correlation.

In the simulation study carried out in this work, we compare the *API* and *DME2* methods with methods that analyse the set of favourable weights (*SMAA* and *SMAA-2* methods). Additionally we use intervals rather than ordinal relations for weights. The *ROC* method cannot be applied in this scenario.

Figure 2 shows the average hit ratio for each of the 20 design elements when the interval length is 0.1, that is, the average values of 20 000 trials. The 20 design elements are represented on the abscissa, where the first value indicates the number of alternatives and the second the number of attributes.

The hit ratio decreases the more attributes there are for any given number of alternatives. Additionally, the number of alternatives also affects the hit ratio.

The *DME2*, *SMAA* and *SMAA-2* methods clearly outperform the *API* method. On the other hand, though the performance of the *DME2*, *SMAA* and *SMAA-2* methods is very similar, *SMAA* and *SMAA-2* slightly outperform *DME2* for a small number of alternatives when the number of attributes is increased.

Looking at the rank-order correlation, see Figure 3, then

1. as the rank-order correlation decreases, the more attributes there are for any given number of alternatives. However, the number of alternatives does not affect the correlation.
2. the *SMAA-2* method slightly outperforms the *DME2* method, which also outperforms the *API* method.

Note that *SMAA* was not considered when analysing the rank-order correlation because it is not always possible to derive a ranking of alternatives from this method.

The simulation was also run for the other listed percentages of imprecision (20%, 30%, 40% and 50%), and the results were similar, that is, the *DME2*, *SMAA* and *SMAA-2* methods clearly outperform the *API* method. *DME2*, *SMAA* and *SMAA-2* output very

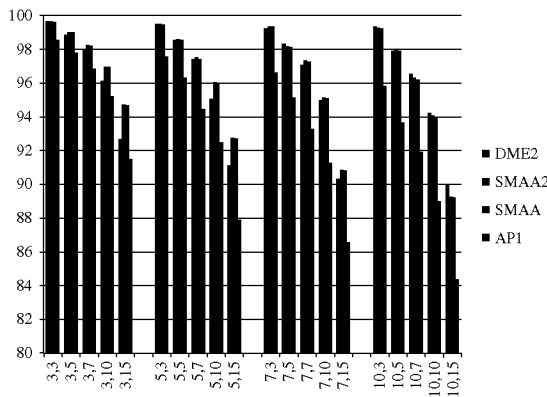


Figure 2. Hit ratio for the four methods. *DME2*, dominance measuring extension 2; *SMAA*, stochastic multicriteria acceptability analysis; *API*, Ahn and Park 1.

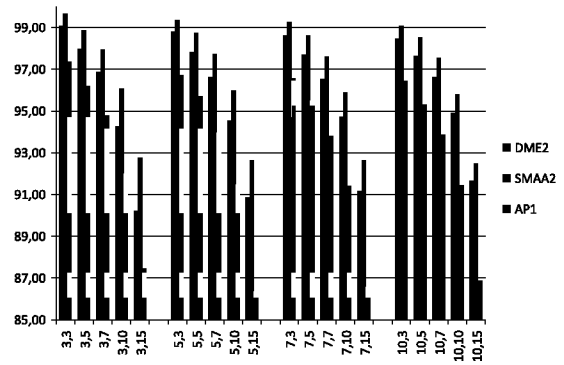


Figure 3. Rank-order correlation for the three methods. *DME2*, dominance measuring extension 2; *SMAA*, stochastic multicriteria acceptability analysis; *API*, Ahn and Park 1.

similar results in terms of hit ratio, but *SMAA-2* slightly outperforms *DME2* on rank-order correlation.

5. WEIGHTS FITTING INDEPENDENT NORMAL DISTRIBUTIONS OR WEIGHTS REPRESENTED BY FUZZY NUMBERS

Now, we consider that the uncertainty about weights is represented by independent normal distributions with means (μ_1, \dots, μ_n) and variances $(\sigma_1^2, \dots, \sigma_n^2)$, that is, $(w_i \sim N(\mu_i, \sigma_i^2), i = 1, \dots, n)$.

In this case, problem (2) can be represented by

$$D_{kl} = \sum_{i=1}^n w_i z_i^{kl} \quad (5)$$

$$\text{s.t.} \quad w_i \sim N(\mu_i, \sigma_i^2), i = 1, \dots, n.$$

Our aim is to take advantage of normal distribution properties, such as closure under linear combinations. For example, any linear combination of a number of independent normal distributions also follows a normal distribution. Therefore, it is well known that if $w_i \sim N(\mu_i, \sigma_i^2), i = 1, \dots, n$, then D_{kl} has a normal distribution with mean $\sum_{i=1}^n z_i^{kl} \mu_i$ and variance $\sum_{i=1}^n (z_i^{kl})^2 \sigma_i^2$ because D_{kl} is a linear combination of w_i .

The approach is based on the same idea as method *DME2*, where only weight intervals are considered to represent weights.

The new approach, denoted by *DME2-Normal*, can be implemented as follows. In the first step, the method computes the optimal solution of the optimization problems (3) for each pair of alternatives A_k and A_l

($k, l = 1, \dots, m$). In the second step, the intensity with which alternative A_k is preferred to the others is computed. In the third step, a *preference intensity measure* for each alternative A_k is computed. This measure represents a preference intensity measure of A_k . The third step ranks the alternatives considering the preference intensity measure. Alternative A_k is better than alternative A_l if the preference intensity measure of alternative A_k is greater than the preference intensity measure of alternative A_l .

1. Compute z_i^{kl} for alternatives A_k and A_l ($k, l = 1, \dots, m$) and each attribute X_i ($i = 1, \dots, n$) following (3).
2. Compute

$$PI_{kl} = \int_0^\infty f_{kl}(x) dx,$$

where $f_{kl}(x)$ is the density function of the variable D_{kl} , that is, $f_{kl}(x)$ is the density function of a normal distribution variable with mean $\sum_{i=1}^n z_i^{kl} \mu_i$ and variance $\sum_{i=1}^n (z_i^{kl})^2 \sigma_i^2$.

3. Compute a preference intensity measure for each alternative A_k :

$$PIM_k = \sum_{\substack{l=1 \\ l \neq k}}^m P_{kl}.$$

4. Rank alternatives according to the PIM_k values, where the best (rank 1) is the alternative with the greatest PIM_k and the worst is the alternative with the least PIM_k .

If the weights are represented by triangular or trapezoidal fuzzy numbers, adaptation is direct because the product of a scalar multiplied by a triangular or trapezoidal fuzzy number and the sum of triangular or trapezoidal fuzzy numbers is another triangular or trapezoidal fuzzy number. For example, if weights w_i are represented by triangular fuzzy numbers (a_i, b_i, c_i) , $\forall i = 1, \dots, n$, then problem (5) is

$$D_{kl} = \sum_{i=1}^n w_i z_i^{kl}$$

$$\text{s.t.} \quad w_i \equiv (a_i, b_i, c_i), i = 1, \dots, n.$$

To compute the components of D_{kl} , we must take into account that

$$w_i z_i^{kl} \equiv \begin{cases} (a_i z_i^{kl}, b_i z_i^{kl}, c_i z_i^{kl}), & \text{if } z_i^{kl} > 0 \\ (c_i z_i^{kl}, b_i z_i^{kl}, a_i z_i^{kl}), & \text{if } z_i^{kl} < 0 \end{cases}$$

and

$$(a_i, b_i, c_i) + (a_j, b_j, c_j) = (a_i + a_j, b_i + b_j, c_i + c_j).$$

Thus, $D_{kl} \equiv (a_{kl}, b_{kl}, c_{kl})$ and is a fuzzy number.

The algorithm for fuzzy numbers and independent normal probability distributions is similar, except that PI_{kl} is defined in step 2 as follows:

$$PI_{kl} = \begin{cases} \frac{\frac{c_{kl} - a_{kl}}{2} - \frac{(-a_{kl})}{2} \frac{(-a_{kl})}{b_{kl} - a_{kl}}}{\frac{c_{kl} - a_{kl}}{2}} = \frac{(c_{kl} - a_{kl})(b_{kl} - a_{kl}) - (a_{kl})^2}{(c_{kl} - a_{kl})(b_{kl} - a_{kl})}, & \text{if } a_{kl} \leq 0 \leq b_{kl} \\ \frac{\frac{c_{kl}}{c_{kl} - b_{kl}}}{\frac{c_{kl} - a_{kl}}{2}} = \frac{(c_{kl})^2}{(c_{kl} - b_{kl})(c_{kl} - a_{kl})}, & \text{if } b_{kl} \leq 0 \leq c_{kl} \end{cases}$$

6. CONCLUSIONS

In real complex decision-making problems, it is not easy to elicit precise values for the weights representing the relative importance of criteria, which often leads to information being incomplete. A recent approach for dealing with such situations is to use information about each alternative's intensity of dominance employing what are known as *dominance measuring methods*.

We have reviewed the features of existing dominance measuring methods, all applied when there are ordinal relations regarding attribute weights, that is, DMs ranked attributes in descending order of importance. Simulation studies have been conducted in this scenario to compare the methods with each other and with other approaches, such as *surrogate weighting methods* and *modified decision rules*. The results showed that *DME2* performs better than other dominance measuring methods and the adaptation of classical decision rules and comes quite close to the *ROC* method, which was identified as the best approach.

However, incomplete information about weights can be expressed in other ways, for example, by weight intervals fitting independent normal probability distributions or represented by triangular or trapezoidal fuzzy numbers. Moreover, the *ROC* method cannot be applied in these situations.

A simulation study has been run on the aforementioned scenarios to analyse the *DME2* method, and again, compare it with the methods suggested by Ahn and Park, with modified decision rules and with *SMAA* and *SMAA-2* methods.

The results show that *DME2* and *SMAA-2* outperform the other methods in terms of the identification of the best alternative and the overall ranking of alternatives. Different situations of imprecision were analysed (10%, 20%, 30%, 40% and 50% imprecision), leading to the same conclusion. Furthermore, all these methods are sensitive to the number of attributes. An increase in the number of attributes implies the reduction of the proportion of well-ordered alternatives. On the other hand, the increase in the number of alternatives does not significantly affect the goodness of the method.

Although *SMAA-2* slightly outperforms *DME2*, *DME2* can be used when incomplete information about weights is expressed not just as weight intervals but also as weights satisfying linear or non-linear constraints, weights represented by fuzzy numbers or weights fitting normal probability distributions. However, the *SMAA-2* method cannot consider fuzzy numbers, and simulation techniques could be hard to apply considering constraints on weights.

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