Dynamic stiffness analysis of bridge abutments

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ABSTRACT. – Strong motion records obtained in instrumented short-span bridges show the importance of the abutments in the dynamic response of the structure. Existing models study the pier foundation influence but not the abutment performance. This work proposes two and three dimensional boundary element models in the frequency domain and studies the dimensionless dynamic stiffness of standard bridge abutments.

1. Introduction

As a branch of Structural Engineering, Earthquake Engineering has mainly focused on providing buildings with the appropiate strength to avoid their collapse in case of being subjected to a "severe" earthquake, and to prevent critical damage when subjected to "minor" quakes, in order to minimize the loss of human lives. As a result, both national and international Seismic Regulations are mainly concerned with actions and detailings to be adopted in building construction. An exception is the area of Nuclear Power Plants, in which a structural failure may lead to very important damage. Along with the improvement of analysis, design and construction, buildings, as a rule, start showing good performance in the prevention of either "total" or "partial" collapse. Studies and investigations are simultaneously carried out on other kind of structures whose failure doesn't directly cause the loss of human lives, but might contribute to it by preventing a prompt evacuation of injured people affected by the quake, or, also, might represent a huge economic damage to repair or to reestablish normal life conditions: bridges, dams, gas, phone or water supply lines, etc..

The failure of bridges may cause important loss of human lives mainly in urban areas where traffic jams are very common.

The study of recent earthquakes has been a very useful tool in the analysis of the behavior of these structures. The Niigata (Japan, 1964) and the Alaska (USA, 1964) Earthquakes produced the total collapse of a great number of bridges due to the loss of lateral support in saturated granular soils by liquefaction. The great amount of Highway Overcrossings collapses produced by the 1971, San Fernando (USA) Earthquake led to an extensive review of the bridges Seismic Regulations. Most of these collapses were complete isostatic spans falling without enough support length. Many of the collapses were produced by wrong reinforcement details, insufficient ductility shear capacity in order to resist inelastic displacements and low anchorage lengths of the bars where the plastic hinges were formed.

The 1989 Loma Prieta (USA) Earthquake has shown the deficiencies in the construction and design of those bridges built prior to the existence of seismic regulations for this kind of structures. Most of those deficiencies were related to the lack of ductility capacity in concrete sections with decisive importance in the global safety of the structure.

Furthermore, the Cypress Viaduct collapse pointed out the importance of the local foundation conditions, and also of the spatial distribution of the seismic motion and its influence on long viaducts.

During the modelling of bridges that have to be analyzed under seismic motions, a great deal of attention is dedicated to the careful representation of the details of the superstructure while the interaction with the soil is usually represented in a less strict way.

This is specially true for the abutments where no much experience is available, while for the pier footings it is possible to use formulas that were developed for other technical areas: machine foundations, nuclear power plants, buildings, etc.

Many interesting studies have been done both with numerical models and analyzing the real response of different structures in order to evaluate the assesment of soil-structure phenomena in the seismic response of bridges.

Ma-Chi Chen and J. Penzien (1979), making use of 2 and 3-dimensional finite elements in the time domain, have evaluated the influence of the soil fill behind the abutments and, also, the pier-foundation interaction. No absorbing boundaries are considered in both abutments and pier foundations, and neither there are any energy dissipation elements by radiation effects. The study points out the importance of dynamic forces in the abutments, the skewness of the deck, the foundation flexibility, and the soil fill in the load transfer between piers and abutments.

D. R. Somaini (1984), J. P. Wolf (1985) and Spyrakos (1990 *a*, *b*) have performed a variety of studies in which the different parameters involved in the soil-structure interaction in pier foundations in short span bridges have been taken into account. A three degree of freedom model has been considered in which material damping in the soil, hysteretic damping in the structure and viscous-type damping in the foundation have been included. The studies conclude that soil-structure phenomena increase vibration period, damping and displacements of the system and reduce the stresses in the structure.

E. Maragakis and P. C. Jennings (1987) have used a model that takes into account the influence of rigid solid plane movements of bridge decks and their impacts against the abutments when they are excited by seismic accelerations, specially the skew ones. The stiffness and damping characteristics of piers and elastomeric bearings are included. Only stiffness parameters are taken into account in the abutments. E. Maragakis (1989) has added the stiffness and damping properties in the soil-foundation system in rectangular bridge decks. Among other results Maragakis found out that even for high accelerations, the non-linear response of the abutments were very low in comparison with the foundations response.

Studies based on the analysis of the response of actual structures making use of system identification techniques, require taking into account large concentrated damping factors in the soil-abutment system in order to achieve adequate matching between actual data and numerical models results. C. B. Crouse, B. Hushmand and G. R. Martin (1987) performed such studies applying forced dynamic loads on structures. Other studies analyze the response of instrumented bridges close to seismic areas such as San Juan Bautista Bridge during Coyote Lake Earthquake, California 1979, studied by J. C. Wilson (1986) or the Meloland Road Overcrossing during 1979 Imperial Valley Earthquake, done by S. D. Werner, J. L. Beck and M. B. Levine (1987).

The above mentioned studies pointed out the importance of soil-structure interaction effects both in pier foundations and abutments. As similar techniques used in other kind structures may be employed in pier foundations effects, we are going to focus in the abutments effects which largely depend on deck-abutment connection and their tipology:

• In simply supported decks these effects are small because the elastomeric bearings act as seismic isolators. However the deck might contact the abutment if seismic buffers are installed, they may act when the displacements are greater than a predefined value or because the expansion joint between the deck and the abutment is not dimensioned to resist strong motions.

• In short span bridges and undercrossing structures in urban areas, the *portal effect* may be used in order to reduce the stresses in the abutments, taking the horizontal loads through the bridge deck from one abutment to the other one. In these cases, knowledge of the stiffness properties of the soil-abutment system is required.

• The use of *integral abutments*, in which the deck is monolithic with the abutments, causes a saving in both installation and maintenance of the expansion joints. Displacements due to thermal and rheological deformations are released through the flexibility of the soil-abutment system usually founded on piles.

• In long span bridges with high piers subjected to strong horizontal accelerations due to live loads, like in railway bridges, or to seismic actions, the deck can be fixed to one of the abutments in order to reduce the stresses on the piers.

The present study will show the analysis of the dynamic stiffnesses in two and three dimensional abutments and their application to the dynamic behavior of bridges.

Fundamental *Soil-Structure Interaction* equations will be formulated and *Boundary Element Method* will be applied to obtain, for both cases, the stiffnesses as a function of dimensionless variables.

2. Dynamic Soil-Structure Interaction

2.1. Formulation

The interaction analysis assumes the existence of a 3-dimensional domain, $\Omega \in \mathbf{R}^3$, which will be divided into two sub-domains, the *structure* Ω_s and the *soil* Ω_g (*Fig.* 1).

$$\Omega \equiv \Omega_s \cup \Omega_g \tag{1}$$

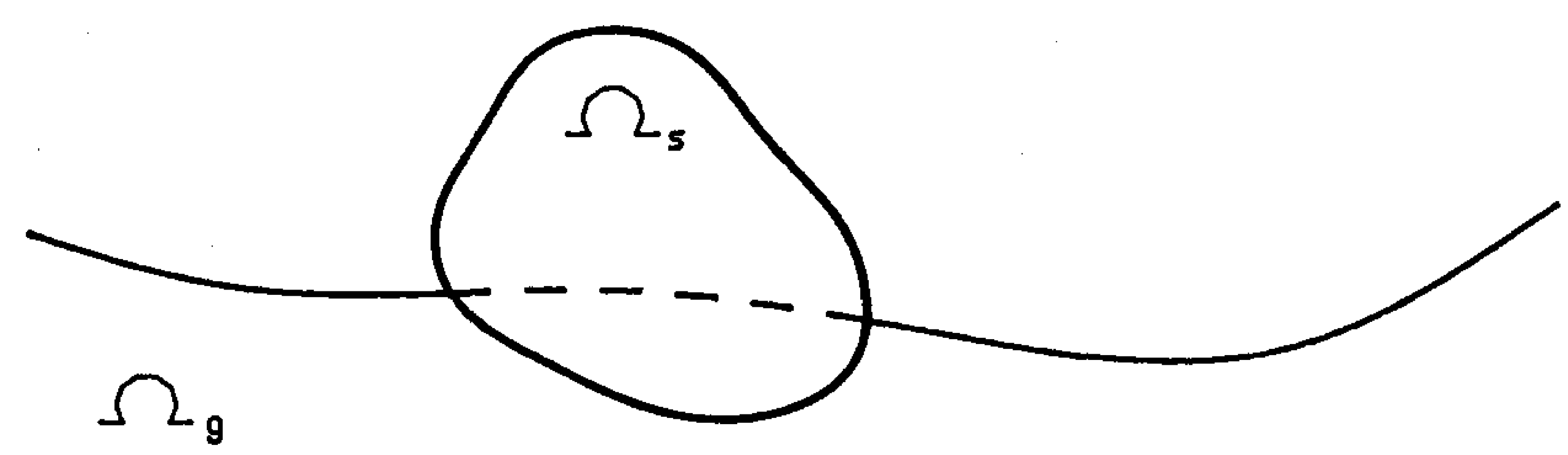


Fig. 1.

For both the soil and structure an elastic or linear viscoelastic behavior will be assumed; the structure is bounded and the soil is a half-space.

The soil subdomain prior to the construction of the structure will be referred as *free field* and the excavation subdomain is that part occupied by the structure.

For this class of problems using finite element or boundary element methods requires discretization of these subdomains. The node notation will be as usual (Fig. 2) (Whitman and Bielak, 1980; Wolf, 1985, 1988).

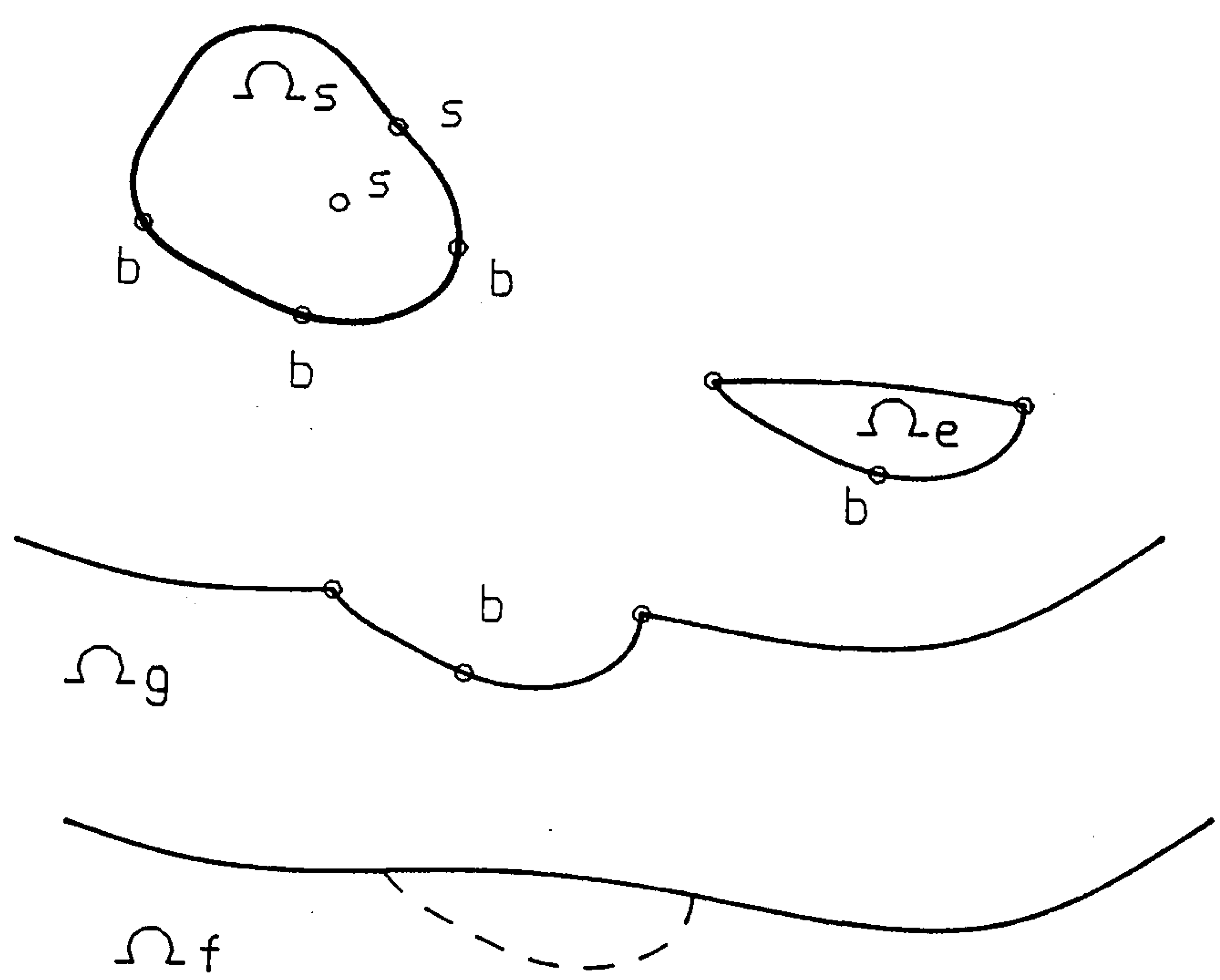


Fig. 2. – Soil-structure interaction: Subdomains.

Subscripts have the following meanings:

s: nodes belonging only to the structure.

b: soil-structure interface nodes.

g: soil domain with excavation.

f: soil domain without excavation.

e: excavated soil domain.

The different subdomains are shown in Figure 3 for the case of a bridge structure.

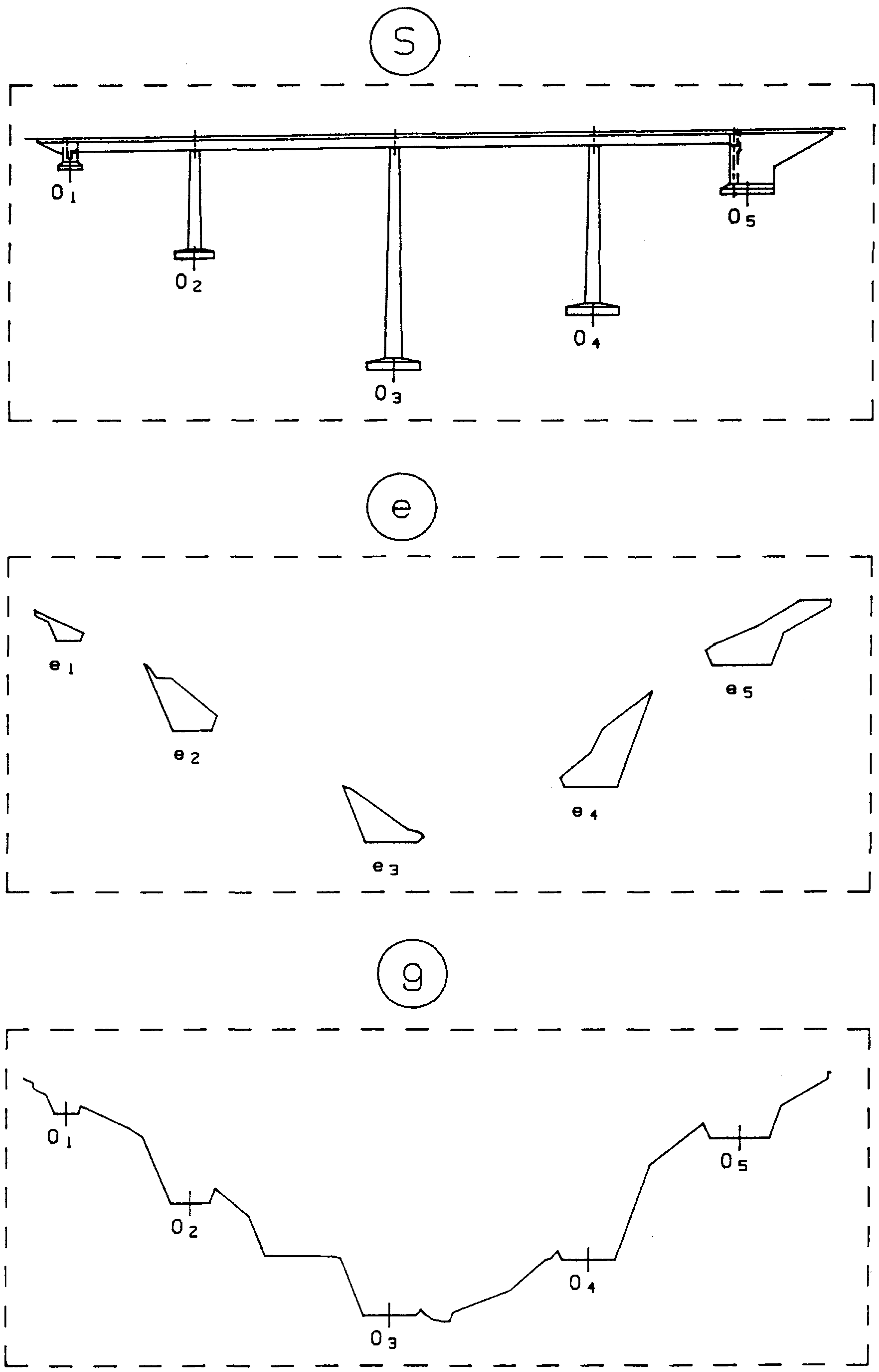


Fig. 3. – Soil and bridge structure subdomains.

2.1.1. *Time-domain equations*

The equations of motion written in the time domain are:

$$\mathbf{M}\ddot{\mathbf{v}}(t) + \mathbf{C}\dot{\mathbf{v}}(t) + \mathbf{K}\mathbf{v}(t) = \mathbf{R}(t)$$

(2) $$\mathbf{v}(0) = \dot{\mathbf{v}}(0) = \mathbf{0}$$

where

$\mathbf{v}(t)$: is the nodal displacement.

$\mathbf{M}$: is the mass matrix.

$\mathbf{C}$: is the damping matrix.

$\mathbf{K}$: is the stiffness matrix.

$\mathbf{R}(t)$: is the time-dependent external load acting on the structure directly or as a soil acceleration.

and the superposed dots indicate derivatives with respect to time.

This system of equations may be solved using techniques such as modal analysis or step by step, implicit or explicit, integration.

2.1.2. *Frequency-domain equations*

The equations of motion can be written in the frequency domain if the external load is harmonic or can be expressed as a harmonic function. Taking the Fourier transform of equation (2) leads to:

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}]\mathbf{u}(\omega) = \mathbf{P}(\omega)$$

(3) $$\mathbf{S}(\omega)\mathbf{u}(\omega) = \mathbf{P}(\omega)$$

where:

$$\mathbf{P}(\omega) = \int_{-\infty}^{\infty} \mathbf{R}(t)e^{-i\omega t}dt$$

and

(4) $$\mathbf{S}(\omega) = -\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K} = \mathbf{K}_{st}(\mathbf{K}^* + i\omega\mathbf{C}^*)$$

is the dynamic stiffness matrix, or impedance matrix. Being $\mathbf{K}_{st}$ the static stiffness and $\mathbf{K}^*$,$\mathbf{C}^*$ the dimensionless frequency dependent stiffnesses.

The differential equation in the frequency domain may be solved using modal superposition techniques from the *transfer function* or the single degree of freedom system response to a harmonic excitation.

In soil-structure interaction problems the damping coefficients vary throughout the system leading to *non-classical damping systems* (Gupta, 1990). A proportional damping matrix is not possible and, therefore the orthogonality of the vibration modes is not achieved in the real numbers field. This leads to a coupled system of differential equations for multi degree of freedom system (Clough and Penzien, 1982). Furthermore, the *exact* modeling of the soil domain leads to frequency dependent stiffness and damping matrices.

2.2. SUBSTRUCTURES METHOD. EQUATIONS OF MOTION

The most widely used technique in linear interaction problems is the *Substructures method.* Due to the different characteristics of the soil and structure domains, it is useful to obtain the dynamic stiffness for both independently and then to perform a coupled analysis. This approach allows the use of different discretizations, different analytical and numerical techniques in each subdomain.

Using Eq. 3, in conjunction with the dynamic equilibrium equations in the structure and soil domains, and the compatibility equations at the interface, the complete soil-structure equations are

$$\begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{sb} \\ \mathbf{S}_{bs} & \mathbf{S}_{bb}^{s} + \mathbf{S}_{bb}^{g} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_s^t \\ \mathbf{u}_b^t \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{S}_{bb}^{g} \mathbf{u}_b^g \end{Bmatrix} \tag{5}$$

where

$$\mathbf{u}^t = \begin{Bmatrix} \mathbf{u}_s^t \\ \mathbf{u}_b^t \end{Bmatrix} \tag{6}$$

is the total displacements vector and $\mathbf{S}_{bb}^{s}$ is the dynamic stiffness matrix of the structure for the nodes in contact with the soil, $\mathbf{u}_b^g$ is the ground motion without the structure and $\mathbf{S}_{bb}^{g}$ is the dynamic stiffness matrix of the soil.

These equations point out the three steps in which every soil-structure interaction problem may be studied:

• *Step 1.* – Soil motion calculation at the foundation level, or in the the soil-structure interface $\mathbf{u}_g$. That motion can be obtained based on the known surface motion, by means of a *deconvolution* process or based on the *scattered motion* calculation from the known motion far away from the surface.

• *Step 2.* – Dynamic stiffness analysis of the soil $\mathbf{S}_{bb}^{g}$, or the stiffness of the free surface of the soil and the indentations produced by the foundation excavations.

• *Step 3.* – Analysis of the structure after calculating the dynamic stiffness matrix obtained in *step 2* and submitted to the motion already obtained in *step 1.*

These equations may be simplified when a *rigid foundation* is assumed. The degrees of freedom in the interface nodes are reduced to six times the number of supports if multiple support excitation is considered, or to only six degrees of freedom if the same motion is considered in all the supports.

2.3. SUBSTRUCTURES METHOD. KINEMATIC AND INERTIAL INTERACTION

The interaction analysis can be separated in two stages:

• *Kinematic interaction.* – The analysis is performed with the massless structure but having its stiffness.

• *Inertial interaction.* – the actual structure (with its mass) is submitted to the inertial forces obtained in the previous stage.

This subdivision is very useful in preferred directions of seismic waves propagation and special soil-structure interfaces in which *kinematic interaction* analysis is trivial.

The displacement decomposition will be:

$$\mathbf{u}_s^t = \mathbf{u}_s^k + \mathbf{u}_s^i$$

$$\mathbf{u}_b^t = \mathbf{u}_b^k + \mathbf{u}_b^i \tag{7}$$

where $\mathbf{u}^k$ are the kinematic interaction displacements and $\mathbf{u}^i$ are the inertial interaction displacements.

2.3.1. *Kinematic interaction*

If a massless structure is assumed, its dynamic stiffness may be expressed as:

$$\mathbf{S}_s(\omega) = \mathbf{K}_s + i\omega\mathbf{C}_s - \omega^2\mathbf{M}_s = \mathbf{K}_s + i\omega\mathbf{C}_s \tag{8}$$

and assuming an *hysteretic* damping type in the structure:

$$\mathbf{S}_s(\omega) = \mathbf{K}_s(1 + 2\zeta i). \tag{9}$$

With these assumptions, Eq. 5 becomes:

$$\begin{bmatrix} (1+2\zeta i)\mathbf{K}_{ss} & (1+2\zeta i)\mathbf{K}_{sb} \\ (1+2\zeta i)\mathbf{K}_{bs} & (1+2\zeta i)\mathbf{K}_{bb}^s + \mathbf{S}_{bb}^g \end{bmatrix} \begin{Bmatrix} \mathbf{u}_s^k \\ \mathbf{u}_b^k \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{S}_{bb}^g \mathbf{u}_b^g \end{Bmatrix} \tag{10}$$

For a general configuration, obtaining the kinematic displacements is as complicated as a general scattered analysis. However for vertical seismic wave propagation and with surface foundations, the kinematic displacements can be obtained with rigid body motion considerations (Wolf, 1985).

2.3.2. *Inertial interaction*

Using the displacement decomposition in Eq. 7, Eq. 5 can be written as

$$\mathbf{Su} = \mathbf{S}(\mathbf{u}^k + \mathbf{u}^i) = \mathbf{P} \tag{11}$$

$$(\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{u}^k + \mathbf{Su}^i = \mathbf{P} \tag{12}$$

using the relation

$$(\mathbf{K} + i\omega\mathbf{C})\mathbf{u}^k = \mathbf{P} \tag{13}$$

the following will hold:

$$-\omega^2\mathbf{Mu}^k + \mathbf{Su}^i = \mathbf{0} \tag{14}$$

this leads to:

$$\begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{sb} \\ \mathbf{S}_{bs} & \mathbf{S}^s_{bb} + \mathbf{S}^g_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^i_s \\ \mathbf{u}^i_b \end{Bmatrix} = \omega^2 \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{M}_{sb} \\ \mathbf{M}_{bs} & \mathbf{M}^s_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^k_s \\ \mathbf{u}^k_b \end{Bmatrix} \tag{15}$$

The displacements corresponding to the inertial interaction are obtained by submitting the soil-structure system to the inertial forces (mass times the acceleration) corresponding to the kinematic interaction. These equations are simplified under the *rigid foundation* assumption.

3. Boundary Element Method

The *Boundary Element Method*, (B.E.M.) is a powerful technique to analyze dynamic problems in unbounded continua. The method applies discretization techniques from the *finite element method* to the integral formulation of elastodynamic problems obtained from the *dynamic reciprocal theorem* and the *fundamental solutions* (Domínguez and Alarcón, 1981; Kobayashi, 1985).

3.1. Dynamic reciprocal theorem

D. Graffi in 1946-1947 obtained the theorem for elastodynamics using the classical *Betti's reciprocal theorem* of elastostatics, and Wheeler and Sternberg (1968) further extended it to unbounded domains

If two different *elastodynamic states* are considered:

$$\mathcal{E}_A \equiv [u_i, t_i, b_i] \quad \mathcal{E}_B \equiv [u_i^{'}, t_i^{'}, b_i^{'}] \tag{16}$$

where u_i, t_i and b_i are the displacements, tractions and body forces vectors respectively.

Let Ω be a regular region with boundary $\Gamma = \partial\Omega$. The reciprocal theorem in the frequency domain may be expressed as:

$$\int_\Gamma t_i u_i^{'} d\Gamma + \int_\Omega \rho b_i u_i^{'} d\Omega = \int_\Gamma t_i^{'} u_i d\Gamma + \int_\Omega \rho b_i^{'} u_i d\Omega \,, \tag{17}$$

where the variables represent the amplitude of the functions in the steady-state situation.

3.2. Integral equations

The reciprocal theorem will be applied by taking the actual elastodynamic state $\mathcal{E}_A$ and the $\mathcal{E}_B$ the corresponding to a unit concentrated impulse load along direction i. This leads to

$$\rho b_j^{'} = \delta(\mathbf{x} - \boldsymbol{\xi}) e_j \tag{18}$$

where δ is the *Dirac delta distribution.*

For the $\mathcal{E}_{\mathcal{B}}$ state, zero initial conditions are prescribed. The corresponding displacements and tractions may be written as:

$$u_j^{'} = U_{ij}e_i \quad t_j^{'} = T_{ij}^n e_i = T_{ij}e_i \tag{19}$$

where the expressions for U_{ij} and T_{ij} are known (Domínguez and Abascal, 1987; Manolis and Beskos, 1988).

The following integral equations are obtained:

$$\begin{aligned} C(\boldsymbol{\xi})u_i(\boldsymbol{\xi};\omega) = & \int_\Gamma [U_{ij}(\mathbf{x},\boldsymbol{\xi};\omega)t_j(\mathbf{x};\omega) - T_{ij}(\mathbf{x},\boldsymbol{\xi};\omega)u_j(\mathbf{x};\omega)]d\Gamma(\mathbf{x}) \\ & + \rho\int_\Omega [U_{ij}(\mathbf{x},\boldsymbol{\xi};\omega)b_j(\mathbf{x};\omega)]d\Omega(\mathbf{x}) \end{aligned} \tag{20}$$

where

$$C(\boldsymbol{\xi}) = \begin{cases} 1 & \text{if } \boldsymbol{\xi} \in \Omega \\ \frac{1}{2} & \text{if } \boldsymbol{\xi} \in \Gamma \text{ with } \Gamma \text{ smooth on } \boldsymbol{\xi} \\ 0 & \text{if } \boldsymbol{\xi} \in \Omega_c \end{cases} \tag{21}$$

and Ω_c is the complement of the domain Ω.

Eq. 20 give the displacements, $\mathbf{u}$, at any point in the domain from the displacement functions, tractions and body forces within the domain by applying a concentrated load at that point, called the *collocation point*. These integrals depend only on the boundary displacements, tractions and body forces.

Due to the singularities in the kernels U_{ij} and T_{ij}, the previous equations have been obtained after a limit step. The integrals involving the kernel T_{ij} are strictly written in their *Cauchy's principal value* sense.

3.3. Integral equations discretization: B.E.M.

The *boundary element method* applies the robust domain discretization and variable interpolation techniques from finite elements method to the solution of integral Eq. 20. More details of this technique can be found in texts such as Balas *et al.* (1989), Bonnet (1986), Brebbia *et al.* (1984), Chen and Zhu (1992), Domínguez and Alarcón (1981) and Manolis and Beskos (1988).

In the frequency domain equations, only the discretization of the geometrical variables is required.

The boundary, in 3-D domains, will be discretized into surface elements. The domain will be discretized into solid elements which actually are *integration cells*.

The discretized integral equations lead to a linear system of equations which may be expressed in a matrix form:

$$\mathbf{Hu} - \mathbf{Gt} = \mathbf{F} \tag{22}$$

where $\mathbf{u}$ and $\mathbf{t}$ are the displacements and traction vectors in the boundary, respectively.

If the body forces are not considered, the matrix equation will be reduced to:

$$\mathbf{Hu} = \mathbf{Gt} \tag{23}$$

to solve every mixed boundary value problem.

4. Dynamic Stiffness Analysis

The evaluation of the dynamic stiffness matrix the soil, $\mathbf{S}^g_{bb}$, is the second step in every soil-structure analysis. It is necessary to solve the mixed boundary value problem in the Ω_g subdomain (*Fig.* 4), where the displacements are known in part or all of the soil-structure interface, $\partial\Omega_u$; in the rest of the boundary, $\partial\Omega_t$, the tractions are zero.

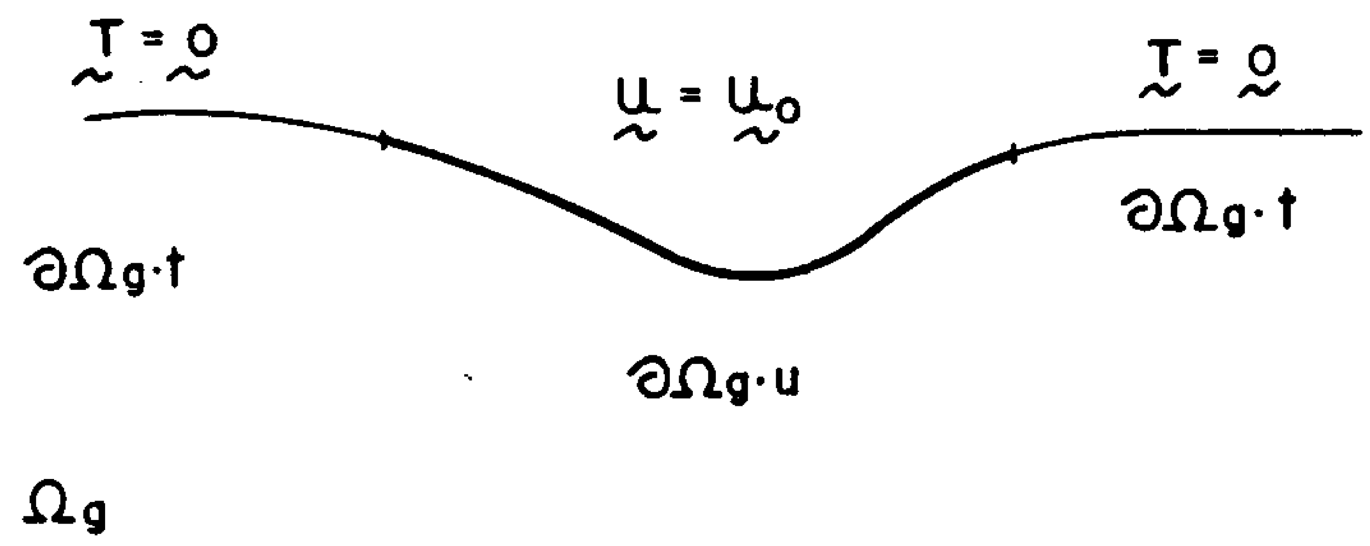


Fig. 4. – Dynamic stiffness analysis. Boundary value problem.

If a flexible contact between the soil and structure is considered, it will be necessary to solve boundary value problems with as many nodes as degrees of freedom existing in the interface. In a rigid contact case, the displacements and the resultant of stresses may be referred to a characteristic point. The number of boundary value problems to solve is the total number of foundations times the number of degrees of freedom considered.

Pier foundations and abutments are soil-structure interfaces in bridges. Research carried out in this area has focused on the dynamic stiffness analysis of foundations, specially surface foundations. These studies comes from areas such as machine foundations or industrial facilities foundations such as nuclear power plants. A recent compilation of different results may be found in Sieffert and Cevaer (1992).

Recent strong motion records obtained in instrumented short span bridges show the importance in the dynamic response of the whole structure. Some models have been proposed to evaluate the dynamic influence of the abutments; these methods are quite different from those used in the analysis of surface foundations.

J. C. Wilson (1988) analyzed the static stiffness of non-skew monolithic bridge abutments. Assuming a rigid behavior of the whole abutment: (front walls, wing walls and foundations), the six degrees of freedom static stiffnesses were obtained. Static solutions for different loads on the elastic half-space have been used.

M. B. Levine and R. F. Scott (1989) obtained the static stiffness of pier foundations and the abutments of the Meloland Road Overpass in order to compare the results of a simple model with the experimental data. A Winkler model of soil-wall interaction has been considered.

J. C. Wilson and B. S. Tan (1990 *a*, *b*) show an interesting study about the embankment-abutment influence in the seismic response of Meloland Road Overpass. In a first part of the study a 2D finite element model is proposed to obtain the vertical and horizontal static stiffness of whole embankment-abutment and its natural frequency.

Using the experimental data and employing system identification techniques, an important reduction of embankment-abutment natural frequency was detected and high damping ratios between 25 and 45% were measured. These concentrated damping ratios represent modal damping ratios from 3 to 12% for certain modes in the whole structure.

The application of *boundary element method* to this particular interaction problem will justify numerically the results obtained experimentally (M. Cutillas and Alarcón, 1994; M. Cutillas *et al.*, 1996).

4.1. Bridge abutments

Bridge abutments are special structures to retain the earth from the embankments at both edges of the bridge and to take the loads from the deck to the foundations.

There are many configurations depending on the embankment height, the deck-abutment connection and the foundations. The main elements of a typical bridge abutment are (*Fig.* 5):

• *Front wall.* – which takes the loads from the bridge deck and their bearings to the foundations and retain the earth from the embankment, longitudinally. Its length depends on the deck width.

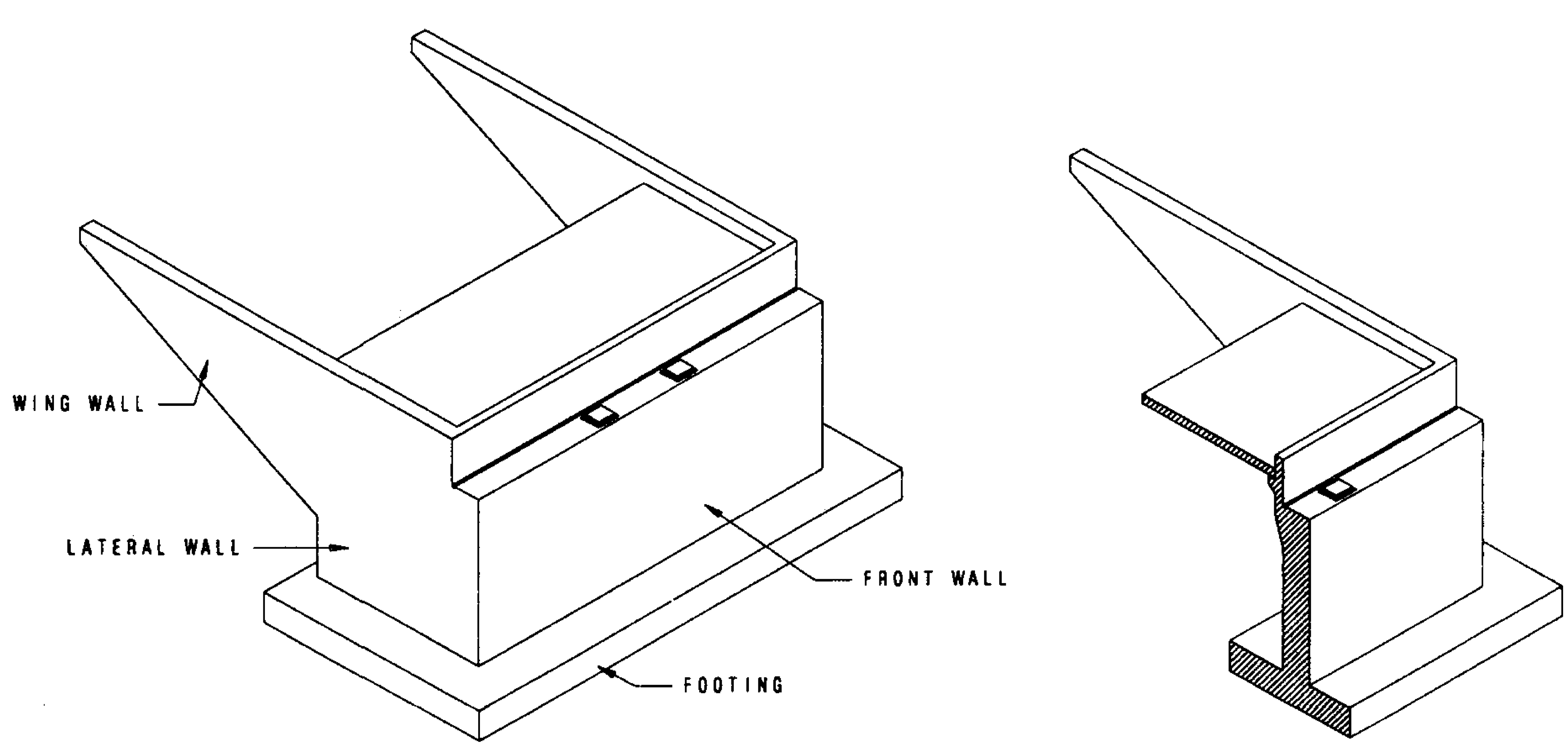


Fig. 5. – Bridge abutment elements.

• *Lateral walls.* – they contain the embankment laterally and their length depend on the spreading of the earth.

• *Wing walls.* – they are lateral walls with a triangular shape to reduce the dimensions of these elements.

• *Foundations.* – depending on the bearing capacity of the soil they can be footings or piles.

In some occasions the abutment is rigidly connected to the deck in what is called an *integral abutment.* Clearly in this case the dynamic response of the bridge is very much affected by the soil-structure interaction.

Depending on the front wall length, the bridge abutment analysis can be considered a two or a three dimensional problem.

4.2. 2D PROBLEMS

Many soil-structure interaction problems may be considered as two dimensional, such as undercrossing structures in urban areas, walls in the basements of buildings. The boundary value problem to be solved for these cases is shown in Figure 6 and the assumptions used are:

• Linear viscoelastic behavior of the soil being the main parameters: *density ρ, shear modulus G, Poisson's ratio ν and damping ratio ζ.*

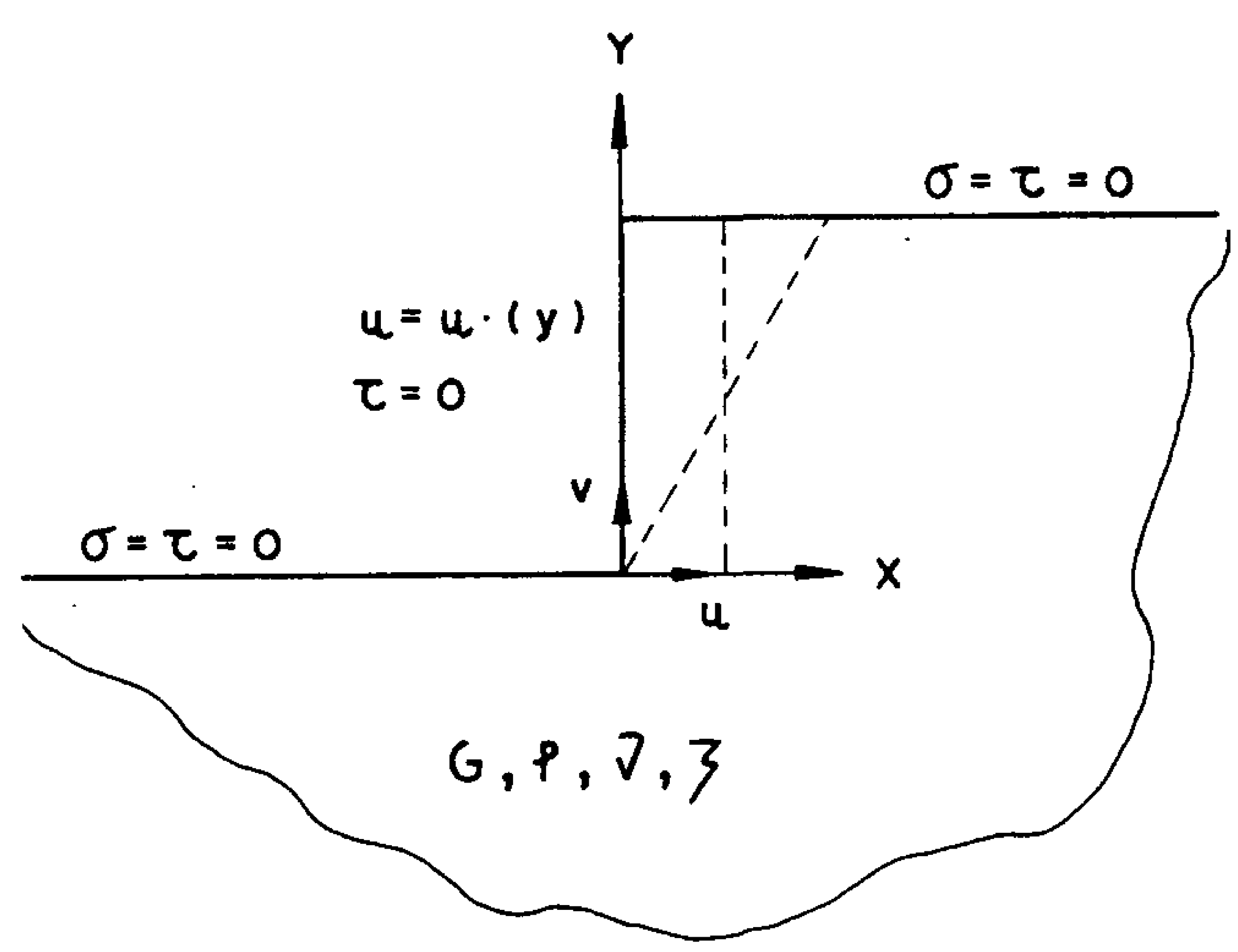


Fig. 6. – Two dimensional model in the half-plane.

• Rigid behavior of the wall.

The boundary conditions are:

• The vertical boundary is the interface between the front wall and the soil. An unbonded contact between the soil and the wall is assumed so the shear stress is zero. Horizontal displacements are known, they will be uniform or a rotation around the origin.

• Horizontal sides are free and unbounded boundaries. Normal and shear stresses are zero.

As a first approach to the problem only the front wall influence has been considered. The foundation influence has not been taken into account. Only the horizontal and rocking stiffnesses have been obtained because the vertical depends on the type of foundation and on the contact between the soil and the front wall.

The dynamic stiffness matrix will be referred to the coordinate system in Figure 6 with point O coordinate origin:

$$\mathbf{S}^g_{OO}(\omega) = \begin{bmatrix} S_x & S_{x,zz} \\ S_{zz,x} & S_{zz} \end{bmatrix} \tag{24}$$

Existing analytical and semianalytical solutions in dynamic earth pressure field have been used to compare our numerical results. Important results have been obtained by J. Wood (1973) in bounded domains and by H. Tajimi (1973) in unbounded domains. Good agreement between B.E.M. results and analytical solutions are shown in M. Cutillas *et al.* (1992) and M. Cutillas (1993).

The boundary element mesh used in the study is shown in Figure 7. An *adaptive mesh* with frequency is employed in the unbounded boundaries. The discretized length is always a quarter of the wave length and around the corners, on the top and bottom part of the wall, equally sized smaller elements have been used.

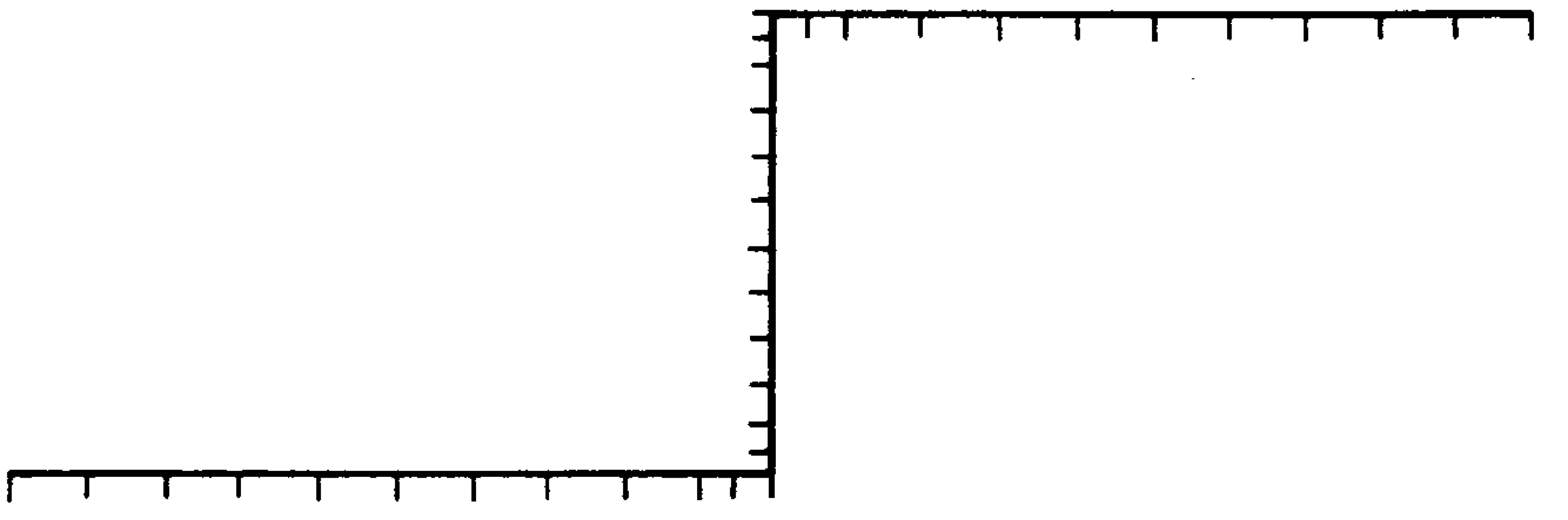


Fig. 7. – Wall in the half-plane. Boundary element mesh.

As two dimensional static stiffnesses are zero, the parametric study of Poisson's ratio influence will be done with the following dimensionless variables (k_x, c_x, k_{zz}, c_{zz}):

$$S_x(\omega) = K_x(\omega) = G[k_x + ia_0 c_x] \tag{25}$$

$$S_{zz}(\omega) = K_{zz}(\omega) = GH^2[k_{zz} + ia_0 c_{zz}] \tag{26}$$

where $a_0 = \omega H / c_s$ and c_s the shear wave velocity of the soil ($c_s = \sqrt{G/\rho}$). Coupled $S_{x,zz}$ stiffness has not been considered because of its relative small values.

In Figures 8 and 9, dimensionless stiffness variation with Poisson's ratio is shown. Although analytical expressions have not been obtained a $\frac{1}{1-\nu}$ dependence can be observed if a least squares approximation is performed.

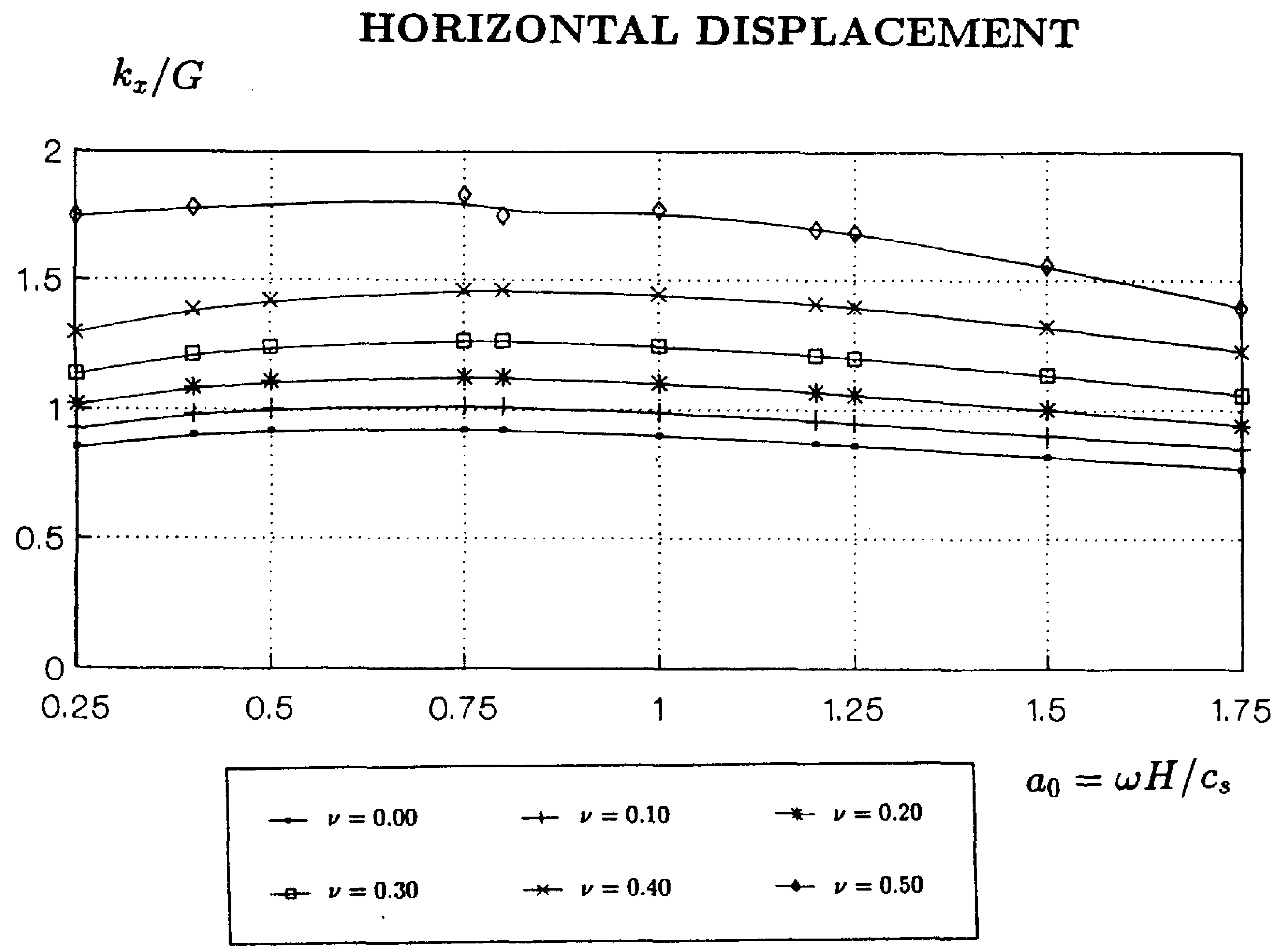
These results can also be considered as the longitudinal stiffnesses in a three-dimensional problem. Transverse stiffnesses have been obtained according to the two-dimensional boundary value problem from Figure 10. (Alarcón *et al.*, 1992; M. Cutillas, 1993).

4.3. 3D Problems

Although the field of application of two dimensional models is quite large, most bridge abutments have a three dimensional behavior. The abutment used in this study has a vertical front wall and two lateral walls perpendicular to the first one (*Fig.* 11).

The main assumptions which are similar to the two dimensional problems are:

• Linear viscoelastic behavior of the soil whose main parameters are: *density ρ, transversal modulus of elasticity G, Poisson's ratio ν and damping ratio ζ.*



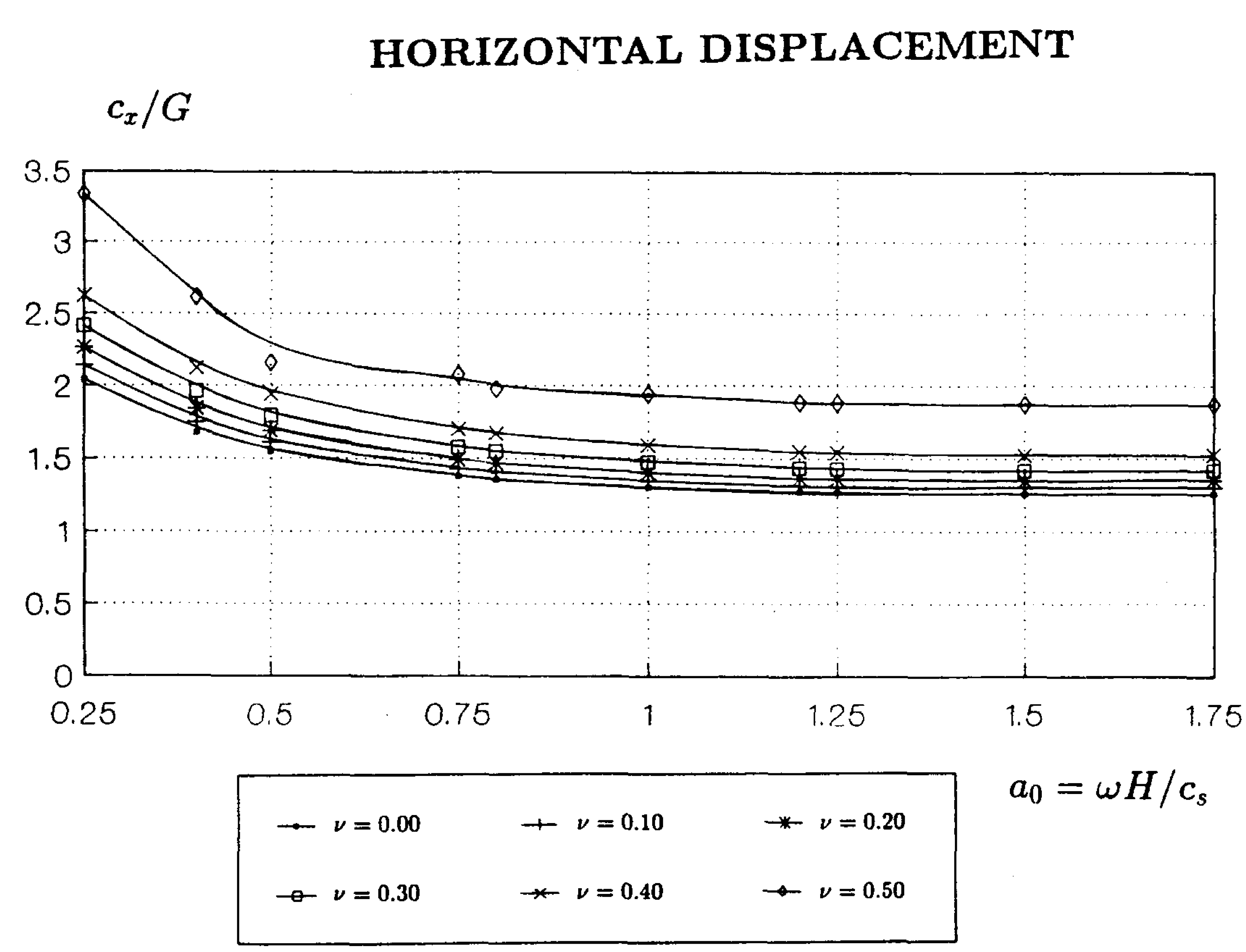
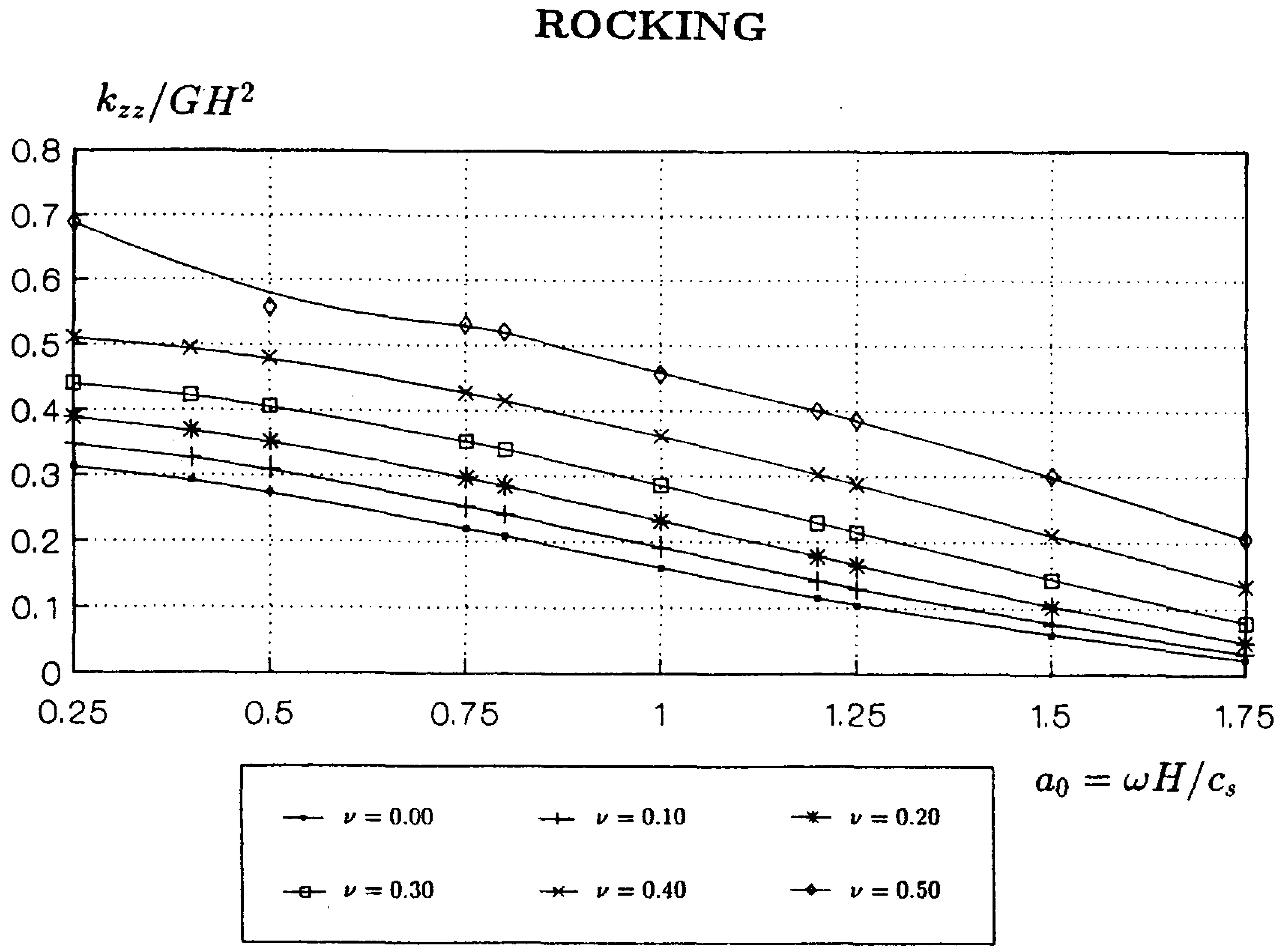


Fig. 8. – Wall in the half-plane. K_x stiffness variation with Poisson's ratio.

• Rigid behavior of the walls.

Some special assumptions have been made for the abutment-embankment geometry to reduce the variables involved:



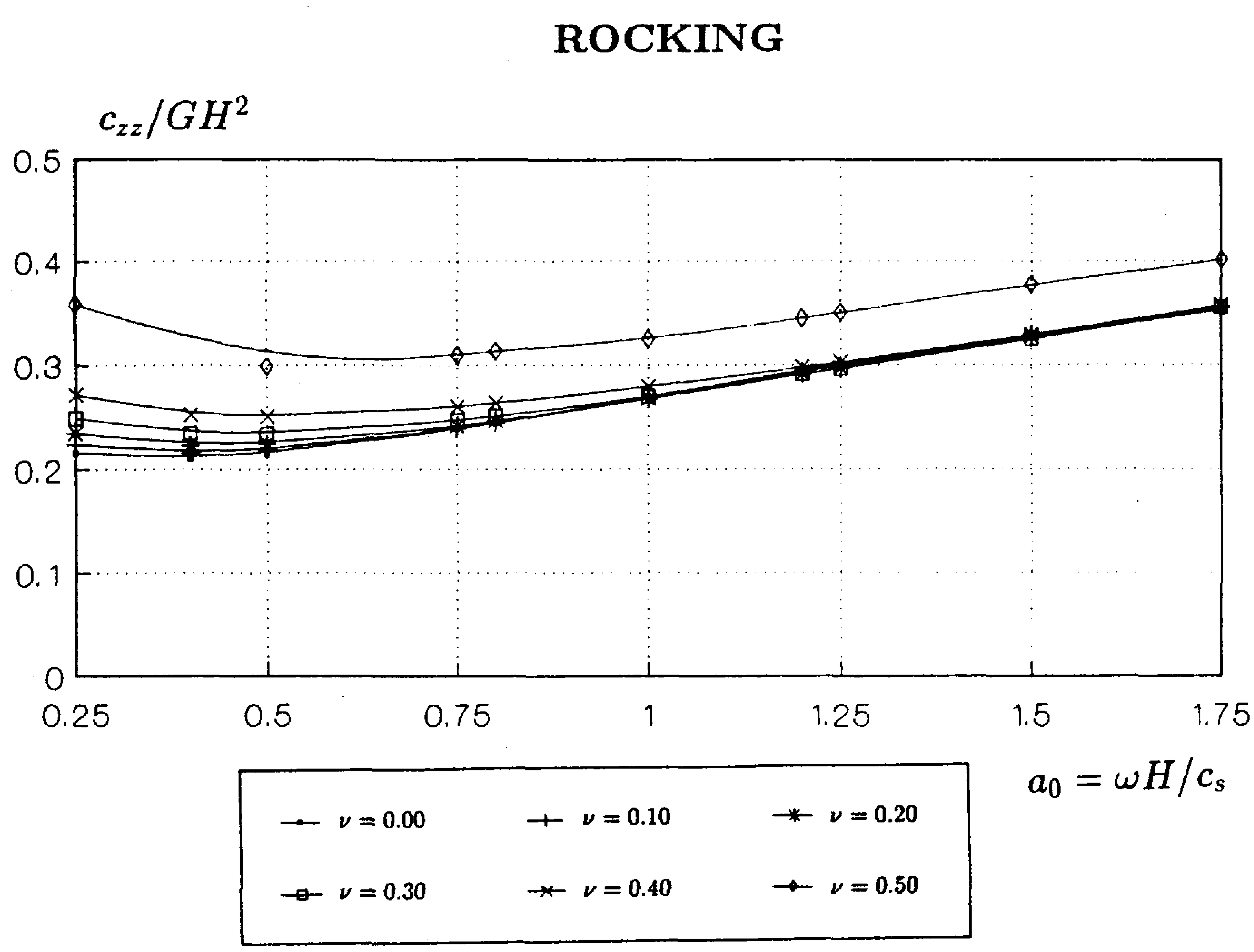


Fig. 9. – Wall in the half-plane. K_{zz} stiffness variation with Poisson's ratio.

• The approach embankments, behind the wall, are considered *horizontal and unlimited.* Next to the wall the grade of the embankment is small because it is usually located in a vertical parabolic alignment close to its vertex. The small gradings produce embankments

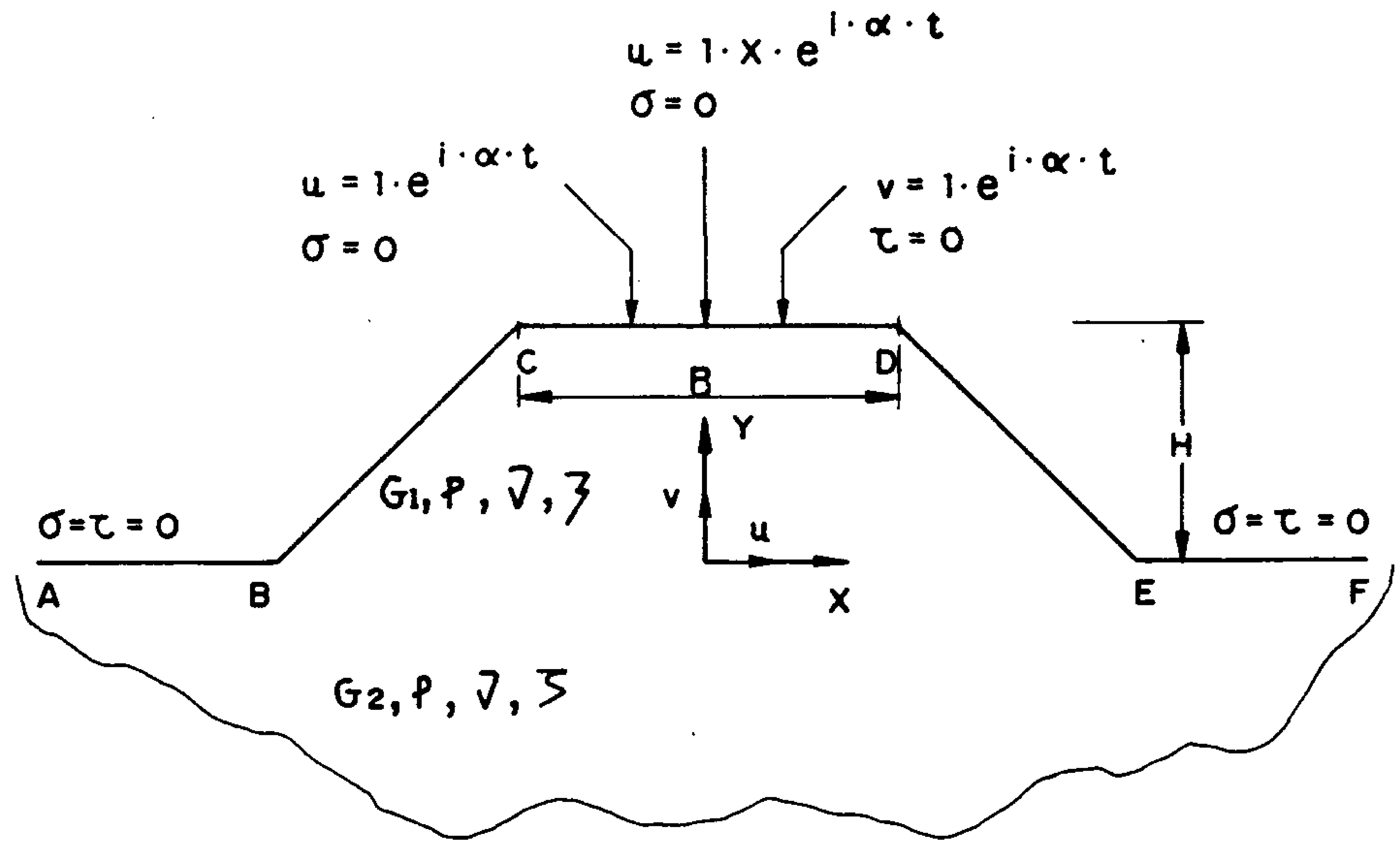


Fig. 10. – Transverse model. Boundary value problem.

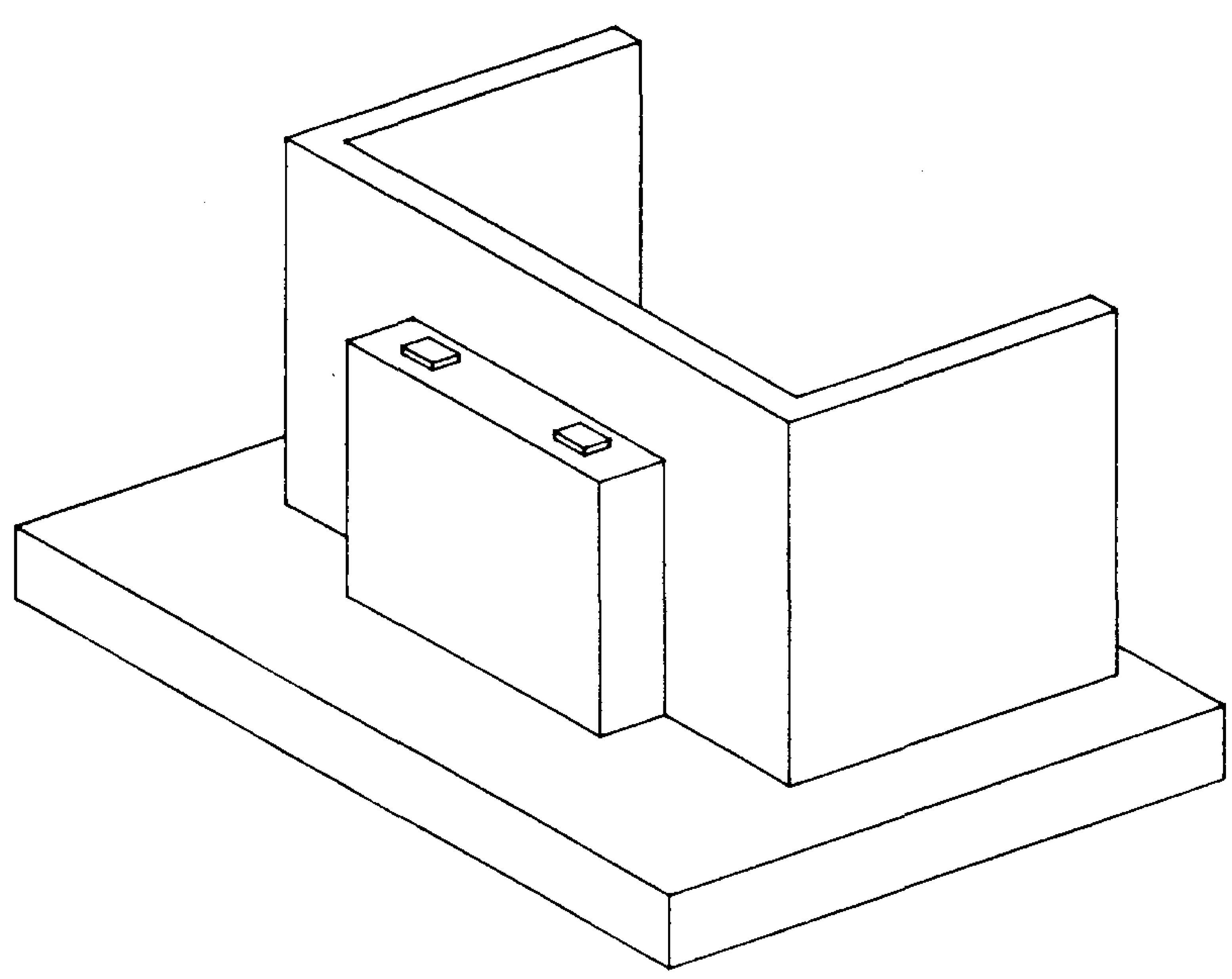


Fig. 11. – Typical three dimensional abutment.

lengths from twenty to thirty times the wall height which can be considered unlimited for this study purposes.

• *The influence of the lateral slopes of the embankments has not been considered.* A granular type material is usually employed to build the approach embankments so some planes and transition cones are needed to make them stable. As these parts of the embankments are not really well compacted, their capacity to resist an stress increment is very small. To neglect the slopes influence may not affect to the stiffness component although it could underestimate the damping component evaluation.

• As a first approach to the problem the foundation of the walls has not been considered in order to isolate the walls influence. The foundations have an important influence in the vertical component of the dynamic stiffness which has a small importance in the dynamic response of the bridges.

Under these assumptions the boundary value problem to be solved is shown in Figure 12. The geometry is a rectangular prism on the half space which represent the approach embankments with the following boundary conditions:

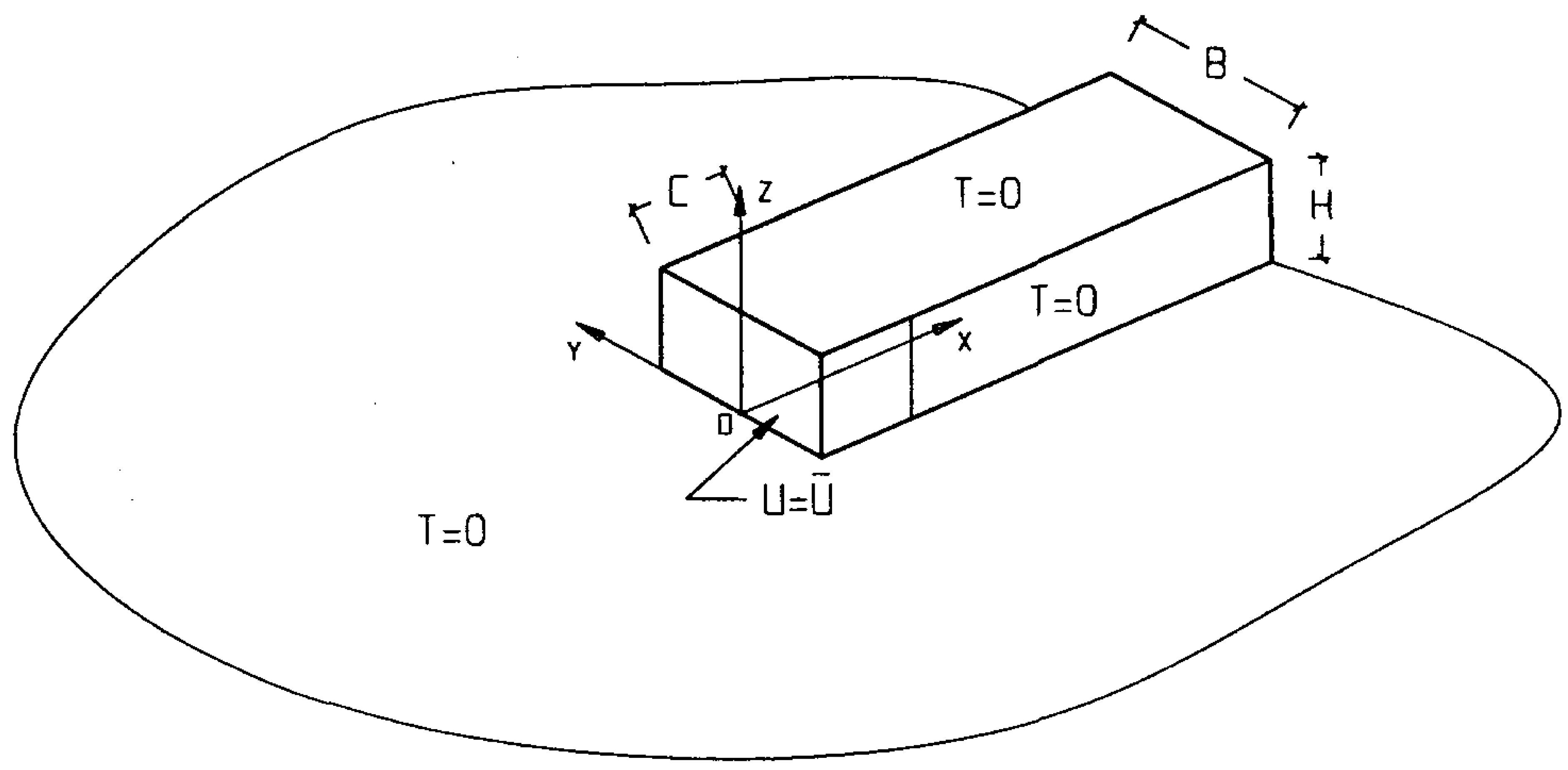


Fig. 12. – Boundary value problem.

• The horizontal planes, $z = 0$ and $z = H$, are the free boundaries of the half space and the embankment, respectively. The tractions in these planes are zero.

• The vertical plane $x = 0$, is the plane contact between the embankment and the front wall of the abutment. The displacements are known in order to evaluate the stiffness of the whole system.

• The vertical planes $y = B/2$ $y = -B/2$ contact the lateral walls of the abutment, $0 \leq x < C$, (where the displacements are known), and a free boundary, $x \geq C$.

The structure of the dynamic stiffness matrix of the standard abutment may be expressed referred to the coordinate system (O;XYZ) in the following way:

$$(27) \qquad \mathbf{S}^g_{OO}(\omega) = \begin{bmatrix} S_x & 0 & S_{xz} & 0 & S_{x,yy} & 0 \\ 0 & S_y & 0 & S_{y,xx} & 0 & S_{y,zz} \\ S_{zx} & 0 & S_z & 0 & S_{z,yy} & 0 \\ 0 & S_{xx,y} & 0 & S_{xx} & 0 & S_{xx,zz} \\ S_{yy,x} & 0 & S_{yy,z} & 0 & S_{yy} & 0 \\ 0 & S_{zz,y} & 0 & S_{zz,xx} & 0 & S_{zz} \end{bmatrix}$$

There are null terms because of the plane of symmetry XZ. Direct stiffnesses will be specially studied, the main diagonal terms, because the coupled stiffnesses have smaller values.

The stiffness terms may be expressed as usual:

$$(28) \qquad S(\omega) = K(\omega) = K_{st}[k(\omega) + ia_0 c(\omega)]$$

where $a_0 = \omega H/c_s$.

The variables involved in the discretization of the geometry like the free discretized surface and the characteristic size of the elements have been studied in M. Cutillas (1993). The last one of these two variables is the most important for the frequency range studied. The different boundary element meshes used to compare the results are shown in Figure 13.

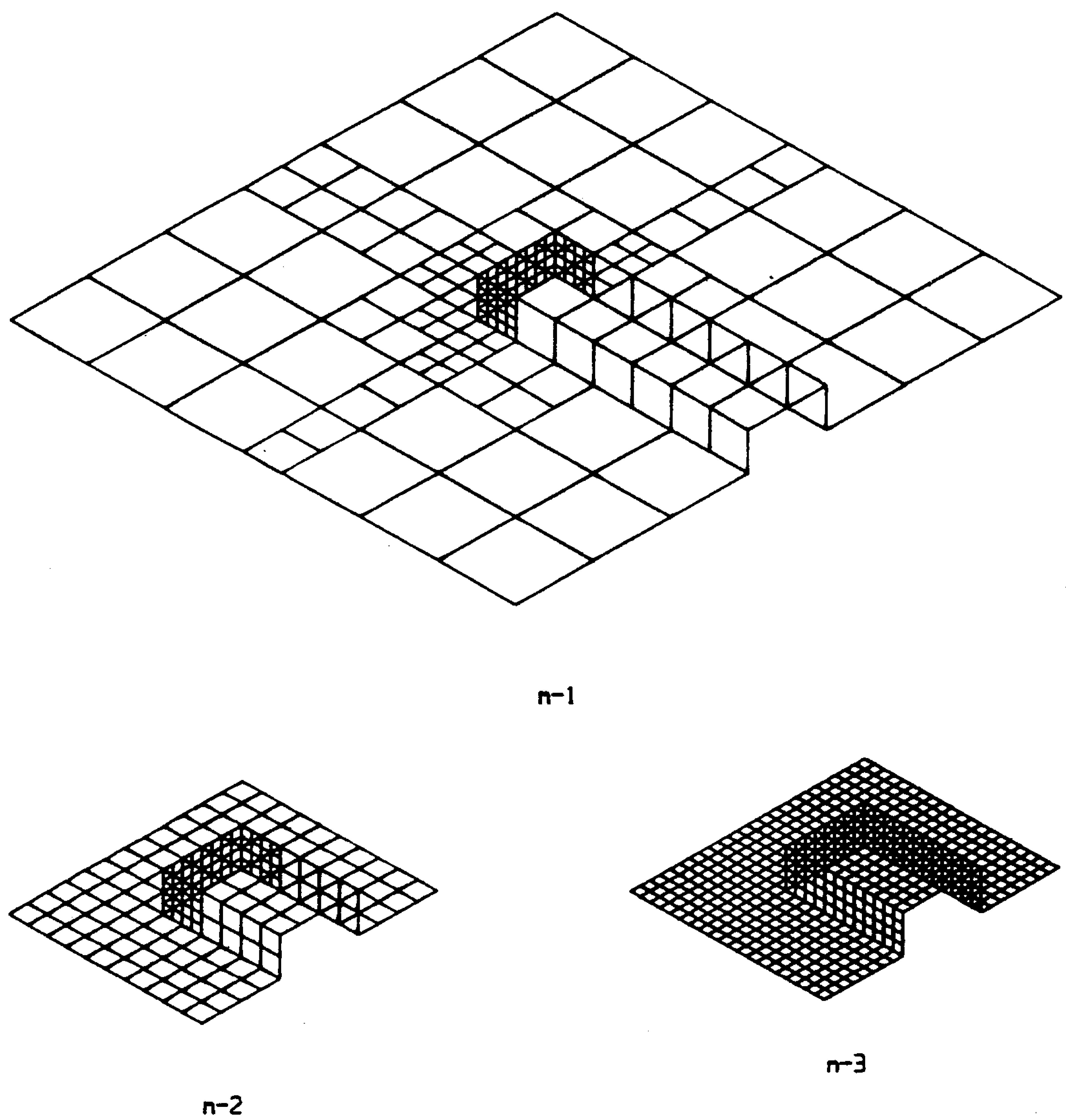


Fig. 13. – Three dimensional model. Boundary element meshes.

For the dimensionless ratios $B/H = 2$ and $C/H = 1$ the static stiffnesses may be expressed as:

$$K_x = 6.07\frac{GH}{2-\nu} \qquad K_{xx} = 5.69\frac{GH^3}{2-\nu}$$

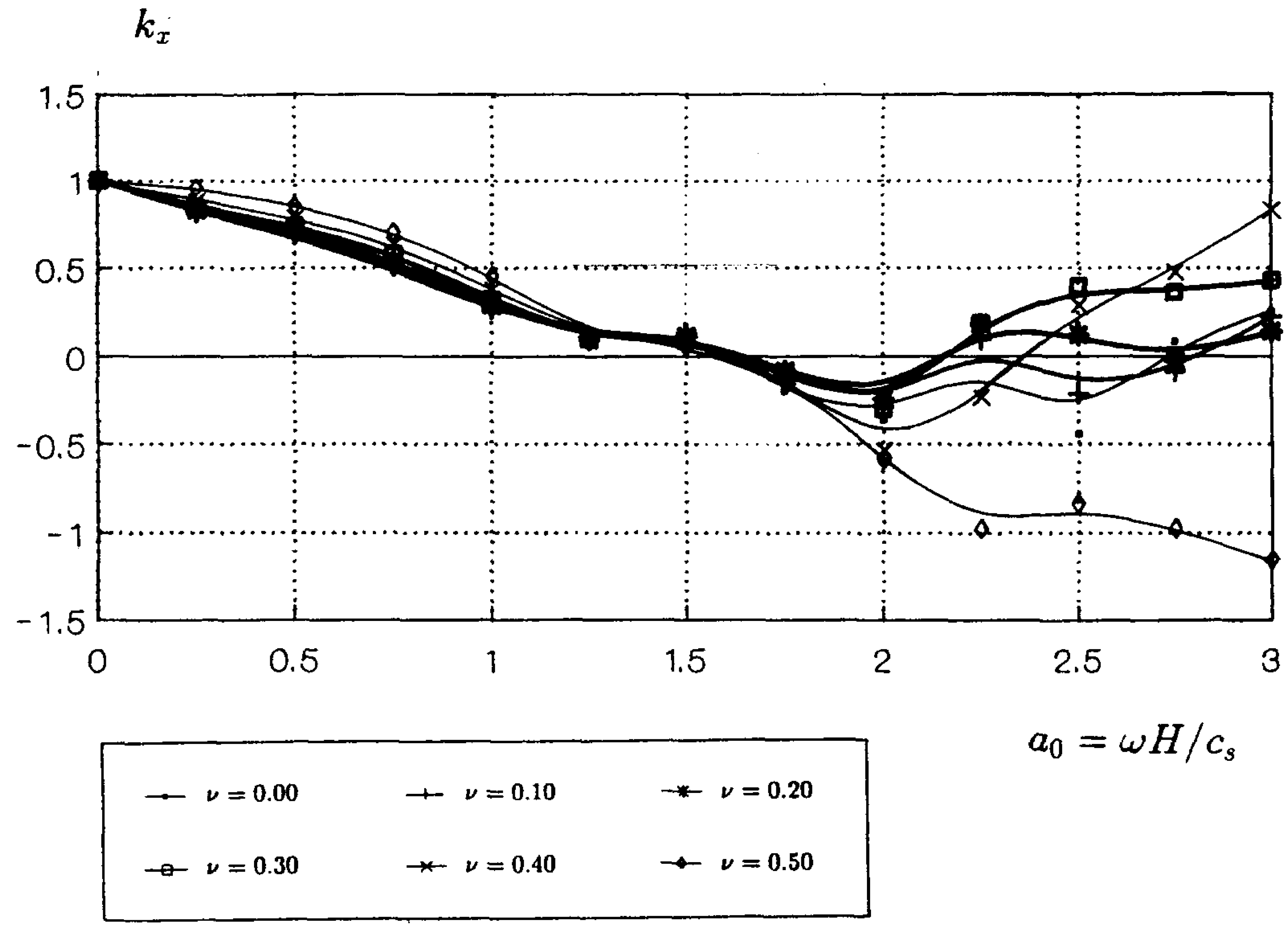
(29)
$$K_y = 4.90\frac{GH}{2-\nu} \qquad K_{yy} = 5.10\frac{GH^3}{2-\nu}$$

$$K_z = 6.08\frac{GH}{2-\nu} \qquad K_{zz} = 9.27\frac{GH^3}{2-\nu}$$

These values have been obtained numerically by least squares approximation techniques taking ν as a variable.

The Poisson's ratio dependence of the dimensionless dynamic stiffness in the form (28), is shown in Figures 14, 15 and 16, which are the longitudinal and transverse displacements and the rotation in a vertical plane.

k_x STIFFNESS



c_x STIFFNESS

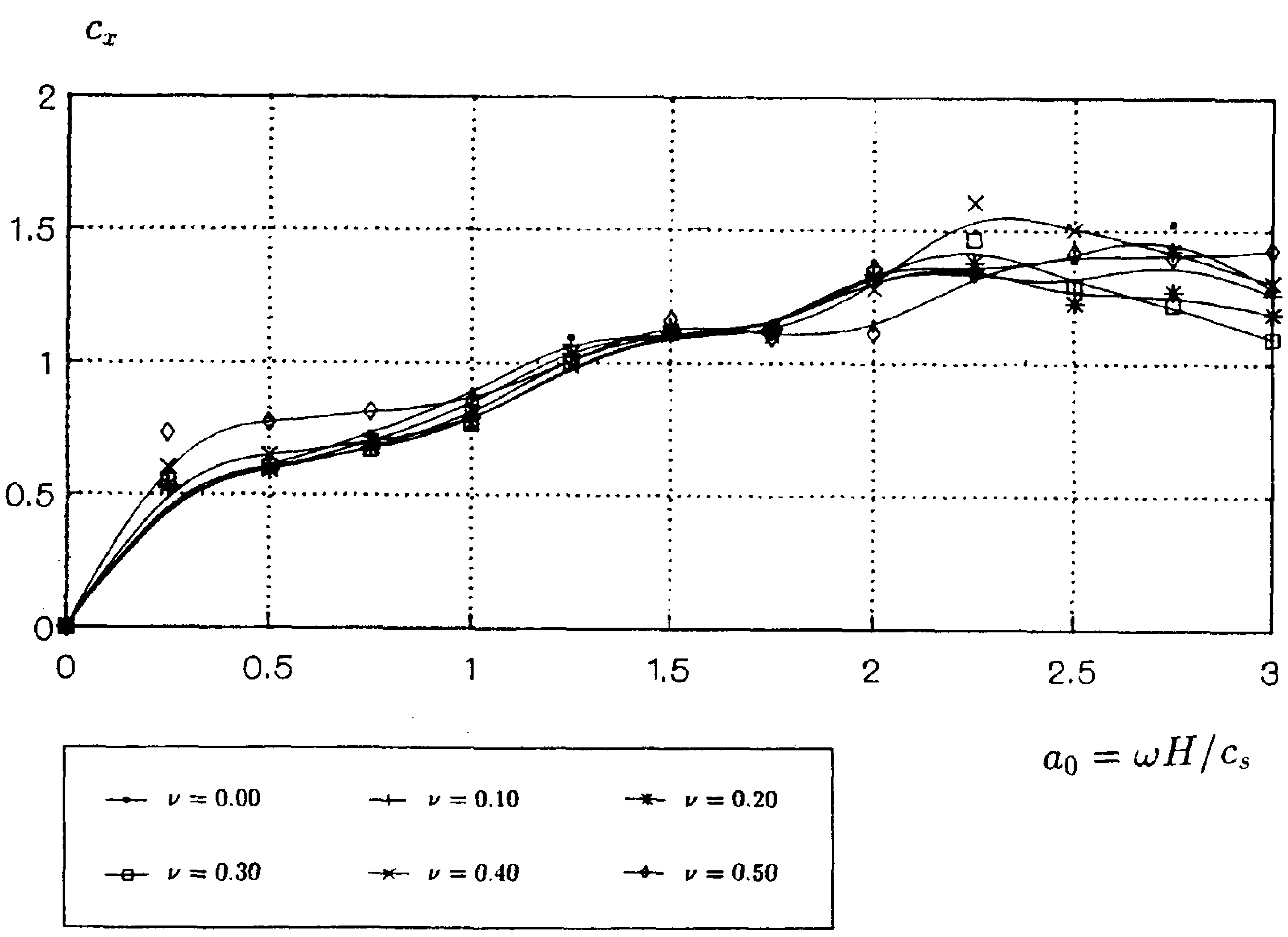


Fig. 14. – Abutment on the half-space. Dynamic stiffnesses K_x. ν dependence.

The displacement stiffness components k_x, k_y and k_z have a similar ν dependence. For a dimensionless frequency a_0 less than 2 ($a_0 < 2$), there is not any Poisson's ratio dependence. For larger values this dependence could be very important, specially for k_x and k_z components and great ν values ($\nu > 0.4$).

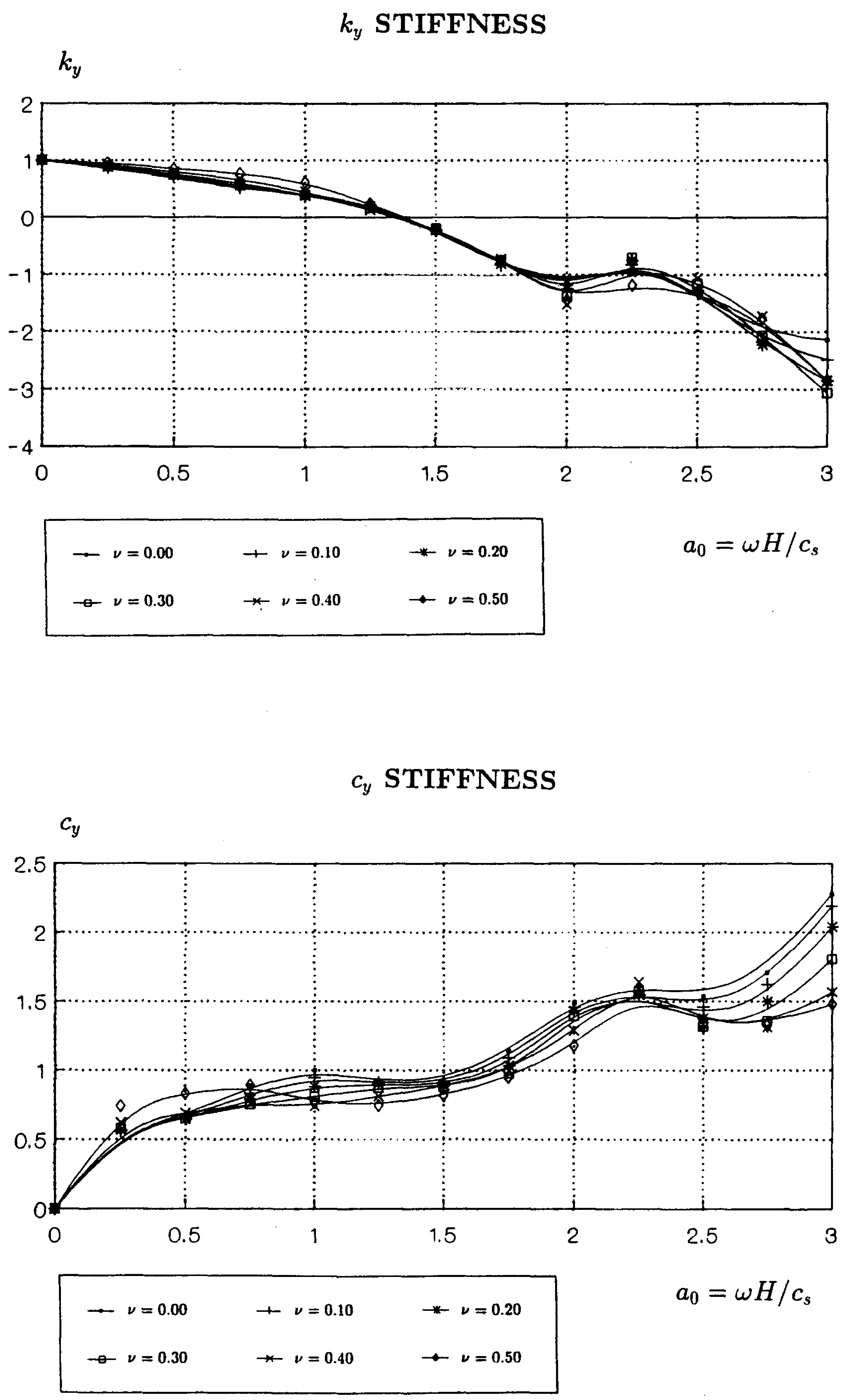


Fig. 15. – Abutment on the half-space. Dynamic stiffnesses K_y. ν dependence.

The rotation stiffness components k_{xx}, k_{yy} and k_{zz} and all the damping components c, have a very homogeneous ν dependent behavior for the range of frequencies studied.

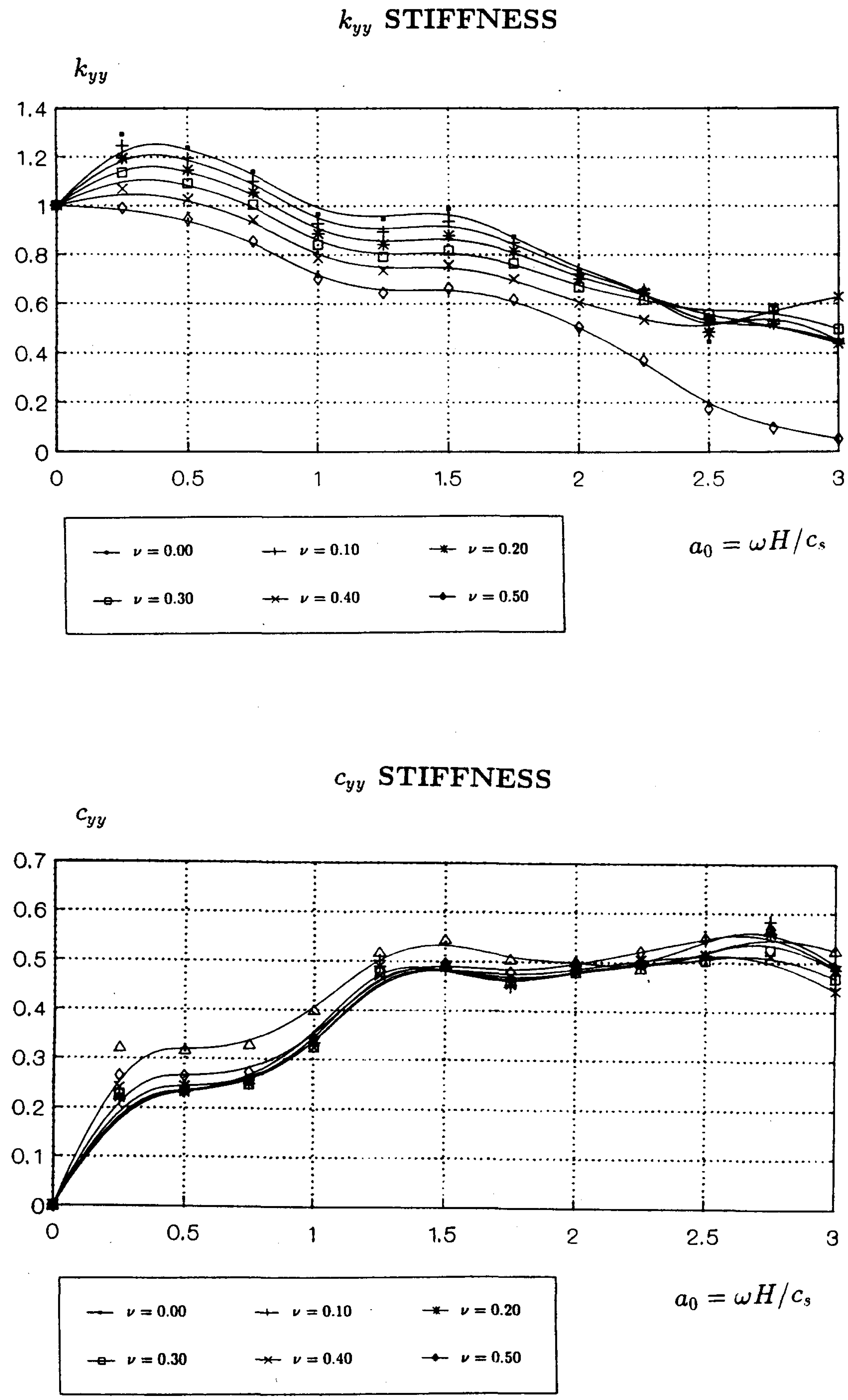


Fig. 16. – Abutment on the half-space. Dynamic stiffnesses K_{yy}. ν dependence.

This dependence may be considered homotetic from an average stiffness variation with frequency.

This variation is similar to those obtained by Veletsos for superficial circular footings (Domínguez and Abascal, 1987).

It is worth mentioning the decreasing values in the stiffness components, even reaching negative values. Negative values show that inertial effects are larger than stiffness ones for such frequencies.

As the real part of the dynamic stiffness is:

$$K(\omega) = \Re[S(\omega)] = K - \omega^2 M \tag{30}$$

for some ω values the term $\omega^2 M$ may be greater than K and the dynamic stiffness will be negative.

In a different way, damping components have increasing values with frequency for the range studied.

6. Conclusions

The main conclusions of this work may be summarized as follows:

• Recent earthquakes have shown the importance of soil-structure interaction in the dynamic response of bridges. Bridge structures and soil interact through piers foundations and abutments.

• High damping ratios have been detected in the dynamic response of instrumented structures subjected to strong seismic motion or subjected to forced motions in *in situ* experiments. These values are obtained for special modes of response both in pier foundations and abutments. These modes of response may be produced by the contact between deck and abutment. This contact will be caused accidentally or by design requirements.

• General equations of linear soil-structure interaction may be applied to the analysis of bridges. Different methods of analysis may be employed in each substructure: soil and bridge.

• Boundary Element Method (B.E.M.) is the most powerful technique in the analysis of dynamical problems in unbounded domains.

• A two dimensional approach to the analysis of dynamic stiffness of bridge abutments may be employed in many situations like *underpass structures* in urban areas.

• Frequency dependent dynamic stiffness of rigid walls has been obtained in dimensionless form. In a first approach, the foundation of the abutment has not been modelled.

Parametric studies have been performed to analyse the influence of Poisson's ratio and the depth of a rigid base.

• A three dimensional bridge abutment with frontal and lateral walls has also been analyzed.

The size of the elements employed in the discretization is the main parameter to be taken into account in a 3D mesh.

• Parametric studies analysing the influence of Poisson's ratio and the depth of a rigid base, both in the static and dynamic case, have been performed.

REFERENCES

ALARCÓN E., MARTÍNEZ A., GÓMEZ M. S., 1992, Dynamic stiffness of bridge abutments, *Proc. 10th. World Conf. on Earth. Eng.*, Madrid.

BALAS J., SLADEK J., SLADEK V., 1989, *Stress Analysis by Boundary Element Methods*, Elsevier Amsterdam.

BONNET M., 1986, *Méthode des équations integrales régularisées en élastodynamique*, These École Nat. des Ponts & Chaussées, Paris.

BREBBIA C. A., TELLES J. C. F., WROBEL L. C., 1984, *Boundary Element Techniques*, Springer Verlag, Berlin and New York.

CHEN M. C., PENZIEN J., 1979, Soil-structure interaction of short highway bridges, *Proceedings of a Workshop on Earthquake Resistance of Highway Bridges*, Applied Technology Council, 434.

CHEN G., ZHU J., 1992, *Boundary Element Methods*, Academic Press. Hancourt Brace.

CLOUGH R. W., PENZIEN J., 1982, *Dynamics of Structures*, Mc Graw-Hill.

CROUSE C. B., HUSHMAND B., MARTIN G. R., 1987, Dynamic soil-structure interaction of a single-span bridge, *Earth. Eng. & Struct. Dyn.*, **15**, 711-729.

DOMÍNGUEZ J., ALARCÓN E., 1981, Elastodynamics, Chapter 7, *Progress in Boundary Element Methods*, ed. Brebbia C. A., Pentech Press, London, **1**, 213-257.

DOMÍNGUEZ J., ABASCAL R., 1987, Dynamics of foundations, *Topics in boundary element research*, **4** ed. C. A. Brebbia, Springer-Verlag, Berlin, 27-75.

GUPTA A. K., 1990, *Response Spectrum Method in Seismic Analysis and Design of Structures*, Blackwell Scientific Publications. Boston.

KOBAYASHI S., 1985, Fundamentals of boundary integral equation methods in elastodynamics, *Topics in boundary element research*, **2** ed. C. A. Brebbia, Springer-Verlag, Berlin, 1-54.

LEVINE M. B., SCOTT R. F., 1989, Dynamic response verification of simplified bridge-foundation model, *J. Geotechnical Eng., ASCE*, **115**, 1246-261.

MANOLIS G. D., BESKOS D. E., 1988, *Boundary Element Methods in Elastodynamics*, Unwin Hyman. London.

MARAGAKIS E. A., JENNINGS P. C., 1987, Analytical models for the rigid body motions of skew bridges, *Earth. Eng. & Struct. Dyn.*, **15**, 923-944.

MARAGAKIS E. A., THORNTON G., SAHDI M., SIDDHARTHAN R., 1989, A simple non-linear model for the investigation of the effects of the gap closure at the abutments joints of short bridges, *Earth. Eng. & Struct. Dyn.*, **18**, 1163-1178.

MARTÍNEZ CUTILLAS A., ALARCÓN E., GÓMEZ-LERA M. S., CHIRINO F., 1992, Application of Boundary Element Method to the analysis of bridge abutments, *14th. International Conference on Boundary Element Methods*, Sevilla.

MARTÍNEZ CUTILLAS A., 1993, *Dynamic stiffness analysis of bridge abutments*, PhD. Thesis. (in Spanish) Universidad Politécnica de Madrid.

MARTÍNEZ CUTILLAS A., ALARCÓN E., 1994, Simplified modelling of bridge abutments-soil interaction under seismic effects, *Proc. 10th. European Conf. on Earth. Eng.*, Vienna.

MARTÍNEZ CUTILLAS A., GÓMEZ-LERA M. S., ALARCÓN E., 1996, Abutment influence in the dynamic response of bridges, *European Earthquake Engineering*, (To be published)

SIEFFERT J. G., CEVAER F., 1992, *Handbook of impedance functions. Surface foundations*, Quest Editions. Presses Academiques, Nantes.

SOMAINI D. R., 1984, Parametric study on soil-structure interaction of bridges with shallow foundations, *Proc. 8th. World Conf. on Earth. Eng.*, San Francisco.

SPYRAKOS C. C., 1990 *a*, Assessment of SSI on the longitudinal seismic response of short span bridges, *Eng. Struct.*, **12**, 60-66.

SPYRAKOS C. C., 1990 *b*, Methods of dynamic analysis of structural systems, *Structural Dynamics, Kratzig et al (eds).*, Balkema, Rotterdam, 771-778.

TAJIMI H., 1973, Dynamic earth pressures on basement wall, *Proc. 5th. World Conf. on Earth. Eng.*, Rome.

WERNER S. D., BECK J. L., LEVINE M. B., 1987, Seismic response evaluation of Meloland road overpass using 1979 Imperial Valley Earthquake records, *Earth. Eng. & Struct. Dyn.*, **15**, 249-274.

WHEELER L. T., STERNBERG E., 1968, Some theorems in classical elastodynamics, *Arch. Rational Mech. Anal.*, **31**, 51-90.

WHITMAN R. V., BIELAK J., 1980, Foundations, Chapter 7, *Design of Earthquake Resistant Structures*, ed. Rosenblueth E., Pentech Press, London, 223-260.

WILSON J. C., 1986, Analysis of the observed seismic response of a highway bridge, *Earth. Eng. & Struct. Dyn.*, **14**, 339-354.

WILSON J. C., 1988, Stiffness of non-skew monolithic bridge abutments for seismic analysis, *Earth. Eng. & Struct. Dyn.*, **16**, 867-883.

WILSON J. C., TAN B. S., 1990 *a*, Bridge abutments: formulation of simple model for earthquake response analysis, *J. Engrg. Mech., ASCE*, **116**, 1828-1837.

WILSON J. C., TAN B. S., 1990 *b*, Bridge abutments: assessing their influence on earthquake response of Meloland Road Overpass, *J. Engrg. Mech., ASCE*, **116**, 1838-1856.

WOLF J. P., 1985, *Dynamic Soil-Structure Interaction*, Prentice-Hall.

WOLF J. P., 1988, *Soil-Structure Interaction Analysis in Time Domain*, Prentice-Hall.

WOOD J., 1973, *Earthquake-induced soil pressures on structures*, PhD. Thesis. Caltech. Pasadena.