# A comparison among different spectrum compatible earthquake simulation methods

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The need for the simulation of spectrum compatible earthquake time histories has existed since earthquake engineering for complicated structures began. More than the safety of the main structure, the analysis of the equipment (piping, racks, etc.) can only be assessed on the basis of the time history of the floor in which they are contained.

This paper presents several methods for calculating simulated spectrum compatible earthquakes as well as a comparison between them. As a result of this comparison, the use of the phase content in real earthquakes as proposed by Ohsaki appears as an effective alternative to the classical methods. With this method, it is possible to establish an approach without the arbitrary modulation commonly used in other methods.

Different procedures are described as is the influence of the different parameters which appear in the analysis. Several numerical examples are also presented, and the effectiveness of Ohsaki's method is confirmed.

## Introduction

As a result of the design of high responsibility structures, such as nuclear reactors, seismic engineering has been developing rapidly during the last few decades.

The analysis requires the use of an analytical model to calculate the structure and, naturally, a definition of the seismic action as close to reality as possible.

These earthquake loads can be defined in two different ways: as an acceleration record, or in a response spectrum form. From the point of view of engineering, the most useful quantitative measurement of an earthquake is, obviously, its acceleration record. From this record, it is possible not only to compute all previous quantities but also to carry out a detailed time-history response analysis, either in the elastic or anelastic domain, with no loss of information.

The difficulty in obtaining reliable records at a given place which satisfy the necessary requirements has been the main reason for the spectacular development of spectral analysis in recent years. The application of this method is quite simple and there are a great many design spectra either with no dependence on soil characteristics[1] or dependent in some way on soil characteristics.[2] These spectra represent the spectral envelope of a large number of acceleration records obtained primarily during the last century.

However, this method is not really suitable as a design tool, so, for the calculation of high responsibility structures, it is necessary to study the response during the period of time in which the load is acting, the computation of the maximum response such as we obtain in a spectral analysis being insufficient. Furthermore, it is sometimes important to compute the response spectrum in certain parts of the structure, and this can be obtained only from the acceleration record of the earthquake. The importance of keeping records, the characteristics of which are as close as possible to the design spectra given by different authors, is thus clear.

Different approaches proposed by different authors can be separated into two major categories.

The first includes such authors as Justo[3], Jennings and Guzman,[4] and Argüelles,[5] etc. If the accumulated experience were extensive, a good approach would be to establish a file of records concerning quantities such as magnitude, intensity, epicentral distance, potential properties and soil characteristics.

The selection of a design earthquake would consist, then, of matching the desired features to the most similar record on file. Useful guidelines can be derived from these exhaustive analyses, but, so far, the scarcity of data means that the conclusions are only tentative.

The second development concerns computer-simulation techniques which allow the computation of a seismic acceleration record with the desired properties, such as magnitude, intensity, phase, etc. In this way, two different approaches have been developed.

The first approach was to produce a time history with a 'white noise' power spectral function (psdf). It relied on Housner's idea of a nearly constant PSV spectrum and on the peaking of transfer functions for most slightly damped structures. The PSV spectrum was scaled to the actual PSV ones; the task is easier because Ravara[6] has demonstrated a relationship between the spectral Housner[7] intensity and the psdf $S_0$.

The second approach concerns the mathematical modelling of physical faults. The use of dislocation theory and numerical methods makes it possible to materialize Reid's rebound theory and to produce synthetic accelerograms.[8-11].

An earlier but excellent attempt was made by Rascon and Cornell[12] where the analytical simulation was based on a probabilistic approach. Although the computer program only included modelled body waves, Rascon also extended the theory to Rayleigh waves.

Another departure point was the regularity shown by spectra of normalized recorded earthquakes. By simulating stochastic processes by computer and adjusting the frequency content time-history amplitudes, spectral intensity, etc., it has been possible to produce accelerograms with features similar to actual earthquakes. Parkus,[13] Ang,[14] Jennings,[15] and Ravara[6] present a selected bibliography on a topic which still admits of further development.

In previously discussed methods, the spectrum is used only as an indirect means of checking the properties of simulated motions.

However, in many cases, as the different rules and guides of different countries prescribe, the excitation is defined by its design spectrum. Therefore, the most important area to be covered is the PSV, and, for this reason, several simple, but important, methods have been created simulating the time-history directly from a specified PSV spectrum.

Several solutions have been proposed[16,17] in this respect, but in general they all use the same process with variations in the parameters which appear throughout the development of the method.

This paper presents a comparison between these methods and, following Ohsaki, proposes a new technique based on the study of phase characteristics of earthquakes.

## Simulation process

As we have already mentioned, many authors have proposed earthquake simulation methods. Many of them consider the earthquake as the product of a random stationary process, obtained from the filtering of a 'white noise' by an envelope

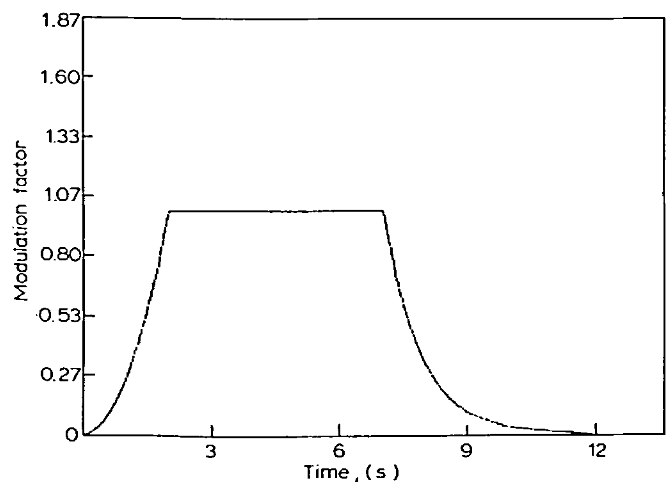


*Figure 1* Modulation function

function which gives a non-stationary character to the record.

This modulation function is similar to *Figure 1* and has the three fundamental intervals in all the acceleration records: a rising interval, which is considered parabolic, a constant interval and an exponential decreasing interval.

The duration of each, and the constant value which defines the exponential time are variables which depend on the type of earthquake, primarily on its period and its magnitude.

A similar method is followed to simulate compatible-spectrum earthquakes. The simplest starts with the modulated Rice expression:

$$\ddot{Z}(t) = \xi(t) R \int_{-\infty}^{\infty} [A(\omega)\, e^{i(\omega t + \phi(\omega))}\, d\omega] \qquad (1)$$

where $\xi(t)$ is the envelope, and the other part is the real part of the inverse Fourier transform of a function $A(\omega)\, e^{i\phi(\omega)}$ in which $A(\omega)$ is prescribed as the Fourier transform of the principal part of the record, or unmodulated part of $\ddot{Z}(t)$, $\ddot{Z}_p(t)$.

$$A(\omega) = \frac{2}{T} \left| \int_{0}^{-T} \ddot{Z}_p(t)\, e^{i\omega t}\, dt \right| \qquad (2)$$

and $\phi(\omega)$, as is usual in Rice, is a random phase angle function with uniform probability density.

Once $A(\omega)$ is generated, it is possible to compute by standard means the $PSV_\zeta$ corresponding to every desired damping ratio $\zeta$ and, then, a comparison is made between the computed $PSV^T(\omega)$ values and the design spectrum target $PSV^T(\omega)$.

If $A$ values do not agree, it is assumed that:

$$A^{i+1}(\omega) = \frac{PSV^T(\omega)}{PSV(\omega)} A^i(\omega) \qquad (3)$$

where $i$ is the actual iteration, $A^i(\omega)$ are the old values and $A^{i+1}(\omega)$ are considered an iterative improvement. The procedures converge well and are stopped when certain specified requirements are fulfilled. For instance, the following conditions are usually established:

(1) For every frequency $\omega_k$:

$$\left| \frac{PSV_k}{PSV_k^T} - 1 \right| \leqslant \epsilon \qquad (4)$$

(2) For the whole record:

$$\frac{1}{N} \sum_{k=1}^{N} \left( \frac{PSV_k}{PSV_k^T} - 1 \right)^2 \leqslant \epsilon \qquad (5)$$

(3) The number of points for which:

$$PSV_k < PSV_k^T \qquad (6)$$

is below a certain limit.

The iteration is started by using the $PSV^T$ for $\zeta = 0$ as a good approximation of the Fourier transform of the record as was shown by Scanlan and Sachs.[17] Therefore, the definition of $A(\omega)$ corresponds to this spectrum.

A different process substitutes the modulation function with a careful choice of the random angles $\phi n$.[18] That is, the nonstationarity is introduced by selection of $\phi n$ according to those observed in real acceleration records.

To do this and to have a suitable criterion for comparison, it is necessary to study the phase angle characteristics of the real records.

## Phase characteristics of the seismic records

A seismic record can be defined as the inverse Fourier transform of a complex function such as:

$$\ddot{Z}(t) = \int_{-\infty}^{\infty} \ddot{Z}(\omega)\, e^{-i(\omega)+\phi(\omega))}\, d\omega \qquad (7)$$

The phase wave is defined as the time-history which results, considering $\ddot{Z} = 1$.

A seismic record and its phase wave are presented in *Figures 2* and *3*. The high frequency content is magnified in the phase wave with respect to the initial record.

For a stationary wave (*Figure 4*), the distribution of phase angles, as seen in *Figure 5*, is almost uniform and the angle differences, defined as the difference between two consecutive angles when the function $\phi(\omega)$ is discretized in a counter-clockwise direction:

$$\Delta\Psi_k = \Psi_{k+1} - \Psi_k \qquad (8)$$

also have a uniform distribution (see *Figure 6*).

On the other hand, for real records, such as *Figure 2*, the phase angle distribution is not uniform while the phase difference distribution is approximately normal (see *Figures 7* and *8*). Furthermore,[18] the modulation function and the phase difference distribution are very similar.

A careful study of this relationship in the real records makes it possible to find some concrete relationships between them, in such a way that the centre of the phase difference distribution is in practically the same place as

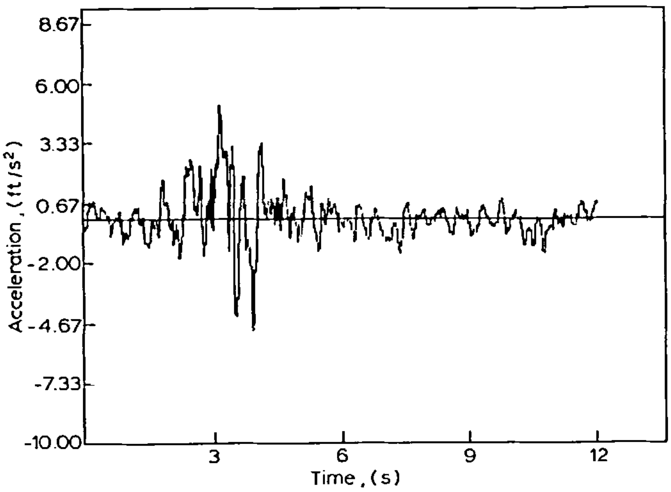

*Figure 2* Acceleration record

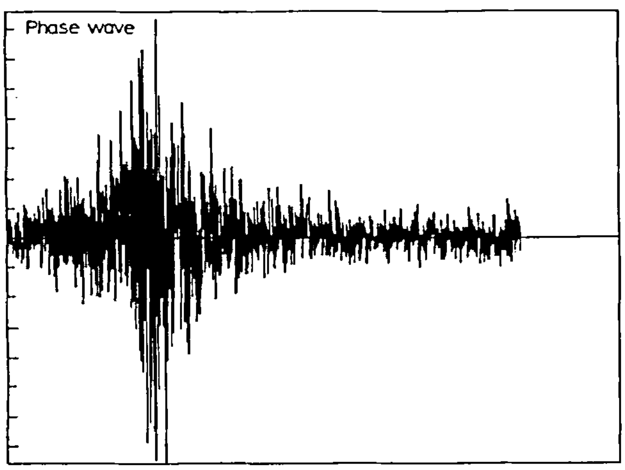

*Figure 3* Phase wave

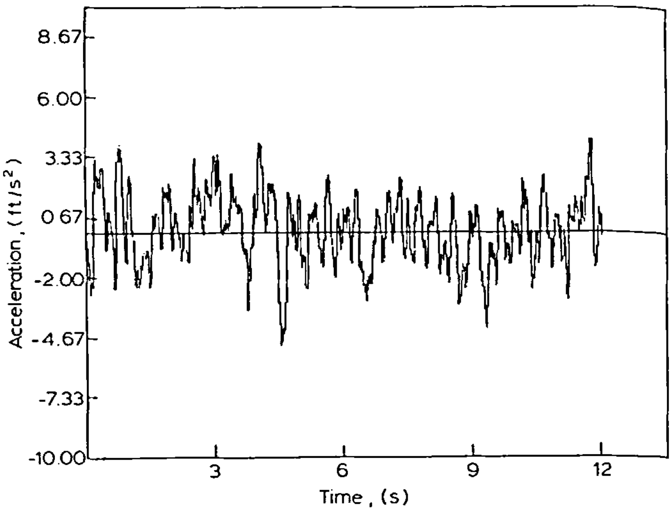

*Figure 4* Acceleration record

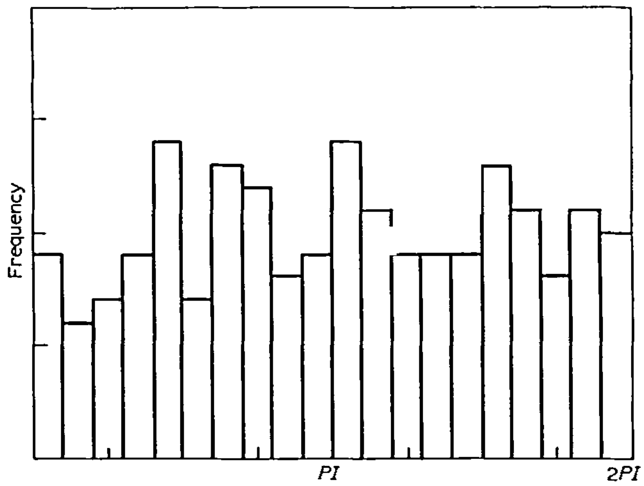

*Figure 5* Stationary process, phase angles

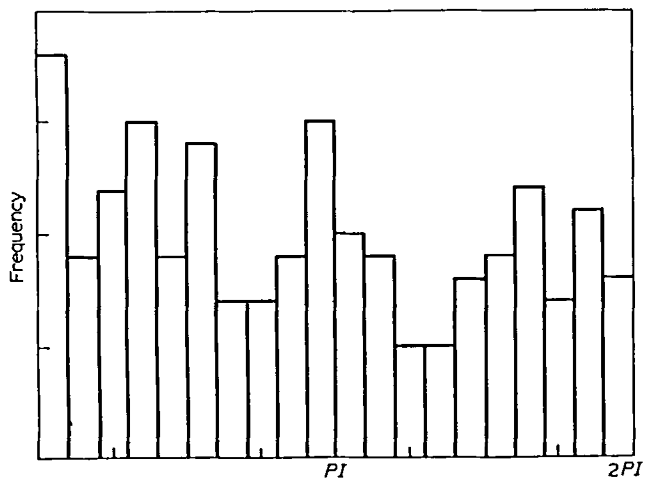

*Figure 6* Stationary process, phase differences

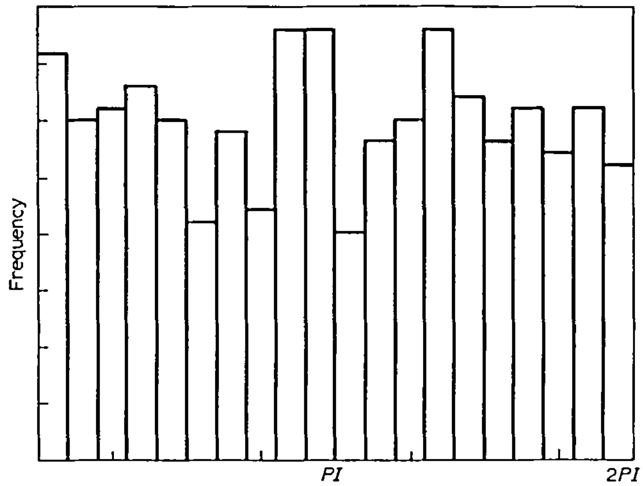


*Figure 7* Nonstationary process, phase angles

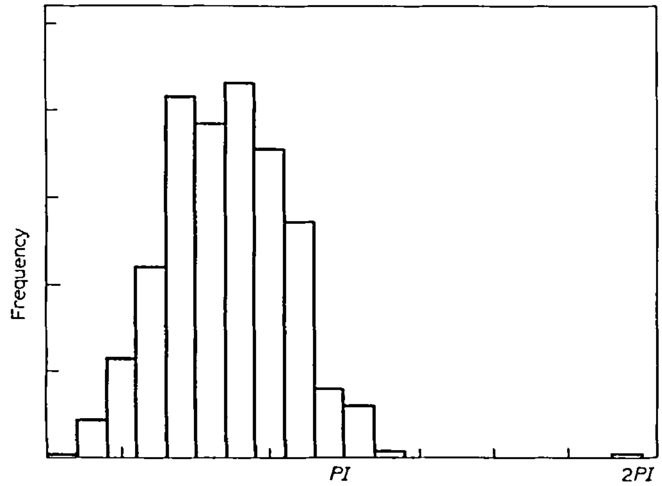


*Figure 8* Nonstationary process, phase differences

the centre of the constant part of the envelope function, when the length of the period of the modulation function and the length of the interval $(0, -2\pi)$ are the same. Also in these axes, the length of the constant part is nearly the same as the interval $(-\sigma, \sigma)$ where $\sigma^2$ is the variance.

In this way, the simulation is intended to produce angles $\phi n$, the differences between which $|\phi_{k+1} - \phi_k|$ follow a Gaussian law defined such that it approximately follows the shape of the envelope function for the earthquake we wish to obtain.

## Accelerogram definition parameters

According to the previous explanation, it will be necessary to define all the parameters which appear in the generation of a spectrum compatible earthquake. Depending on the selection of these values, there are several options for computing the record as well as different methods of generation as demonstrated below.

### *Design spectrum*

Although it is possible to use whatever spectrum one wishes, as an example we have chosen the normalized spectra (lg) proposed by Newmark *et al.*[19] and recommended by NRC as design spectra.

The spectra corresponding to $\zeta = 0\%$ and $\zeta = 2\%$ are shown in *Figures 9* and *10*. The first will be, as indicated, the basis for the initial definition of $A(\omega)$, while we shall try to compute an earthquake with a spectrum as close as possible to the second one since these methods do not approximate all the spectra corresponding to different damping ratios in the same way.

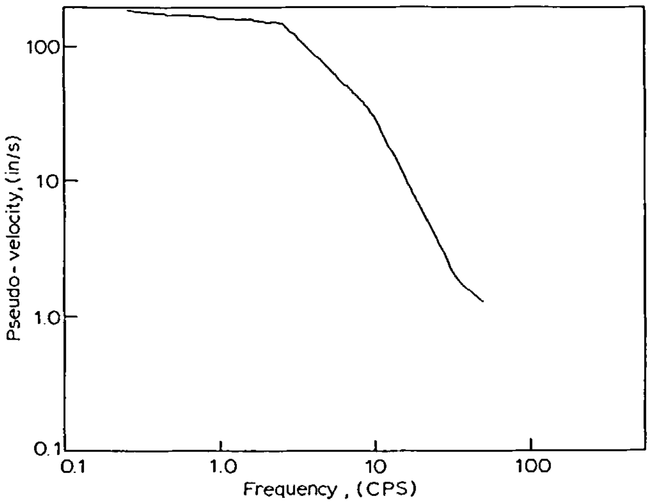


*Figure 9* PSV spectrum (CHI = 0.00) iteration number 0

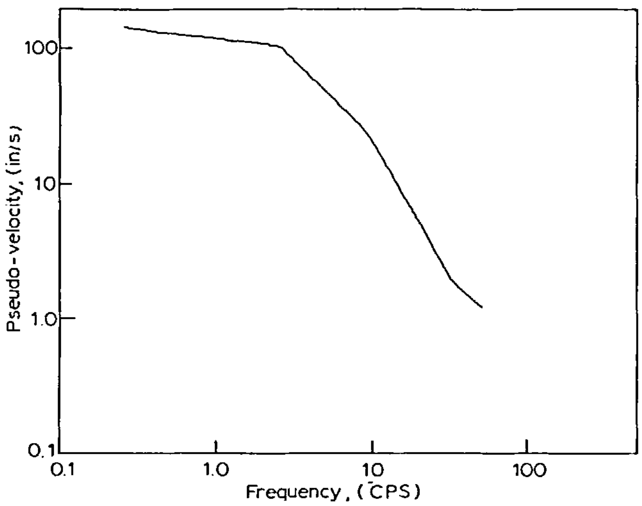


*Figure 10* PSV spectrum (CHI = 0.02) iteration number 0

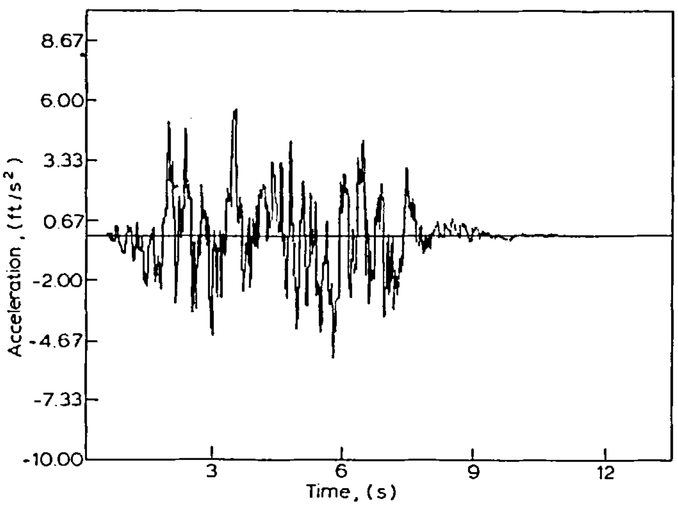


*Figure 11* Acceleration record

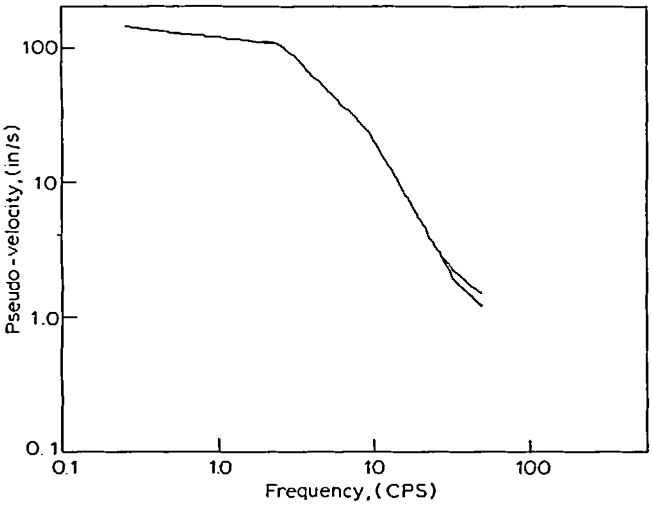


*Figure 12* PSV spectrum (CHI = 0.02) iteration number 4

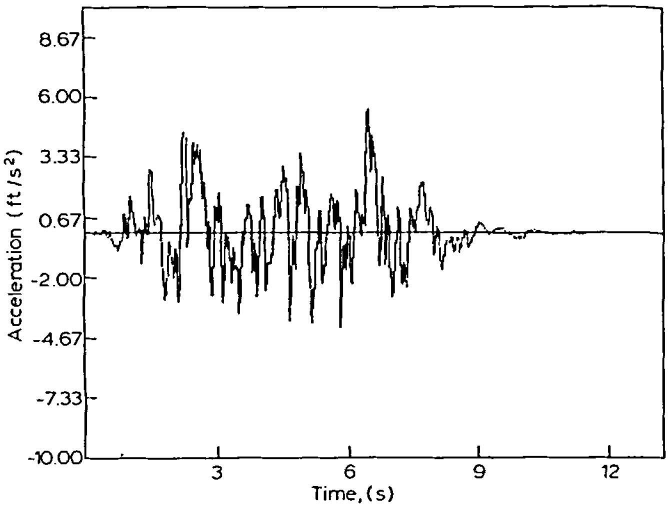


*Figure 15* Acceleration record

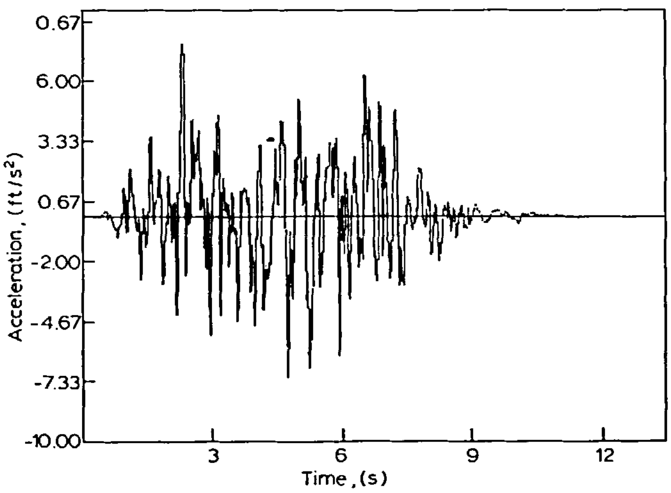


*Figure 13* Acceleration record

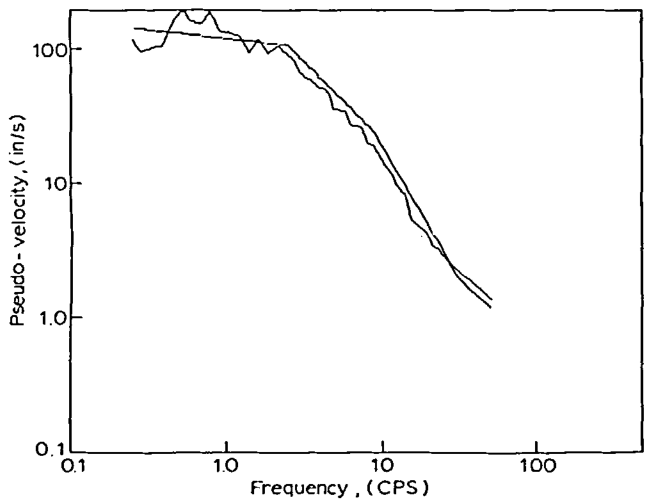


*Figure 16* PSV spectrum (CHI = 0.02) iteration number 4

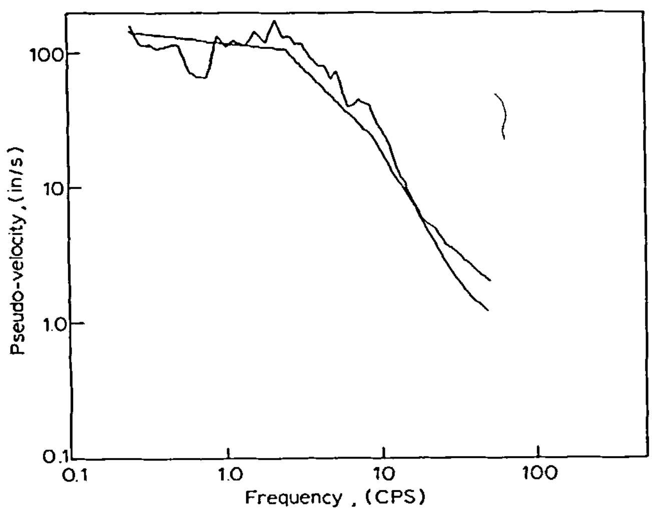


*Figure 14* PSV spectrum (CHI = 0.02) iteration number 4

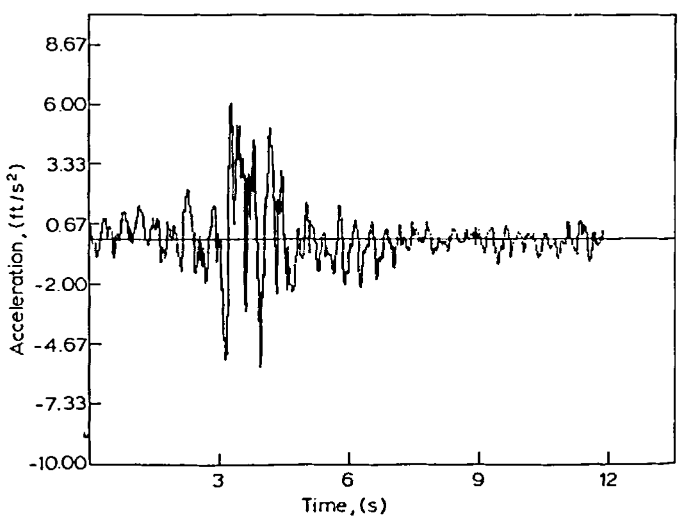


*Figure 17* Acceleration record

There are more sophisticated approaches which allow the simultaneous approximation of two different spectral graphs, which in certain cases, can be interesting, although for the majority of them, it is enough to obtain only one spectral curve.

### *Phase characteristics*

The selection of phase angles is one of the important decisions for choosing one method or another. We have pointed out two different possibilities. The first consists of the selection of a phase angle set with a uniform distribution and the multiplication of the resultant stationary process by the chosen modulation function.

The second uses a normal distributed phase difference set with appropriate central mean and variance; once this has been done, the FFT of $A(\omega)\, e^{i\phi(\omega)}$ is now a non-stationary process with the typical form of a real record.

*Frequencies*

Although we have said that the stationary part of the record arises from the FFT of a known function, of course this transform has to be made numerically:

$$\ddot{Z}(t) = \sum_{k=1}^{N} A_k \cos(\omega_k E + \phi_k) \tag{9}$$

The selection of frequencies $\omega_k$ is another important step and there are fundamentally two different approaches.

The first one inplies the use of equally spaced frequencies, so $\omega_k = 2\pi k/T$, where $T$ is the total duration of the record. This is the typical Fourier form and allows the computation of the transform using a standard FFT algorithm with an important reduction in computation time.

However, as we use an iterative process, the reduction of iterations is also important and sometimes a different frequency set is used, consisting of nonequally spaced frequencies which constrain the use of an FFT algorithm. This can effectively reduce the number of iterations if these frequencies are appropriately selected. For instance, the frequencies may be placed between the half-power points corresponding to the resonance wave of the previous frequency.

*Modulation function*

This envelope function has already been defined in previous sections and will be used only with a uniform phase angle distribution.

## Different simulation approaches

We have developed computer programs implementing four different methods for purposes of comparison. These are:

*Method 1*

The accelerogram record is defined as:

$$\ddot{Z}(t) = \xi(t) \sum_{k=1}^{N} A_k \cos\left(\frac{2\pi k}{T} t + \phi_k\right) \tag{10}$$

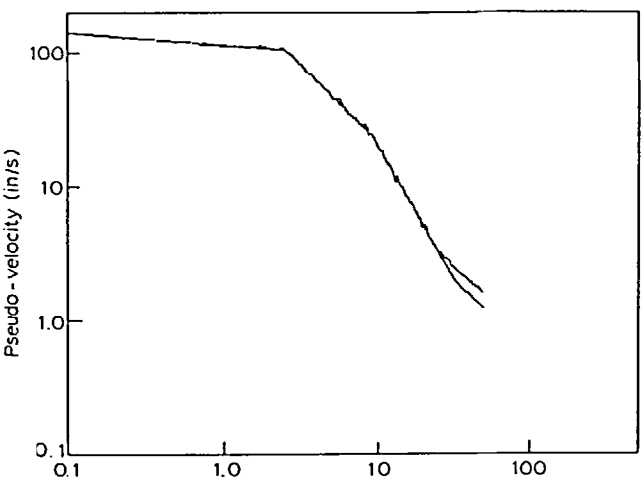


*Figure 18* PSV spectrum (CHI = 0.02) iteration number 4

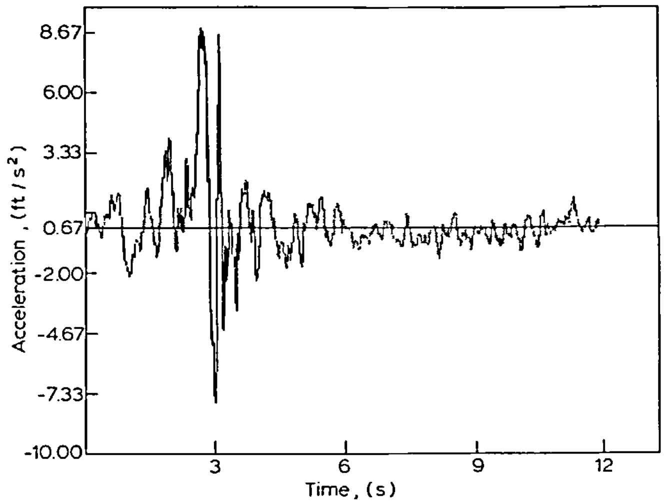


*Figure 19* Acceleration record

where $\omega_k$ are equally spaced frequencies and it is possible to use the FFT algorithm, $\phi$, are elements of a uniform distributed phase angle set between 0 and $\pi$ and $\xi(t)$ is the envelope function. This method was proposed by Scanlan.[17]

*Method 2*

We define the time-history as follows:

$$\ddot{Z}(t) = \xi(t) \sum_{k=1}^{N} A_k(-1)^k \sin(\omega_k t + \phi_k) \tag{11}$$

It is essentially the same as the previous method, although $\phi_k$ is a deterministic phase angle set and $\omega_k$ are semipower frequencies:

$$\omega_{k+1} < (1 + 2\zeta)\, \omega_k \tag{12}$$

*Method 3*

It is exactly the same, but $\phi_k$ are again random angles with a uniform distribution between 0 and $2\pi$:

$$\ddot{Z}(t) = \xi(t) \sum_{k=1}^{N} A_k \sin(\omega_k t + \phi_k) \tag{13}$$

*Method 4*

This method uses the real records phase characteristics and is slightly different from the previous ones. The acceleration is defined as:

$$\ddot{Z} = \sum_{k=1}^{N} A_k \cos\left(\frac{2\pi k}{T} t + \phi_k\right) \tag{14}$$

where again $\omega_k$ are equally spaced frequencies and $\phi_k$ is a phase angle set where $\Delta\phi_k$ (Ohsaki[16]) is a normal distributed function.

## Numerical results

As an example, we have chosen a long duration earthquake (12 s) with maximum acceleration of 0.15 g = 4.83 ft/s and, as the compatible spectrum, the NRC with $\zeta = 2\%$, as shown in *Figure 10*.

The modulation function is shown in *Figure 1* and can be defined as:

$$\xi(t) = t^2 \qquad 0 \leqslant t \leqslant 2$$
$$\xi(t) = 1 \qquad 2 \leqslant t \leqslant 7 \qquad (15)$$
$$\xi(t) = e^{-0.268(t-7)} \qquad 7 \leqslant t \leqslant 12$$

The $A_k$ values have been approximated by 200 values from the $\xi = 0$ spectrum, scaling to 0.15 g. Due to the numerical characteristics of the FFT algorithm, greater errors can be expected for frequencies above $N/T = 200/12 = 16.67$ ccps. This can be observed in the following graphs.

For the fourth method, we have chosen the central mean in such a way that the centre of the constant part of the modulation function is the same as the mean centre in the normal distribution function between 0 and $2\pi$:

$$\bar{\phi} = \frac{-2\pi}{12} + \left[2 + \frac{(7-2)}{2}\right]^2 = 2.3562$$

and the variance is:

$$\sigma^2 = \left[\frac{-2\pi}{12}\left(\frac{7-2}{2}\right)\right]^2 = 1.71347$$

The following was chosen as the convergence criterion:

$$\frac{PSV_k}{PSV_k^T} - 1 \leqslant \epsilon \qquad (16)$$

The following figure shows the results for four iterations. We can see that, for the first and fourth methods, as we have explained, the error is greater for frequencies $\omega_k \geqslant$ 20 cps. However, this problem does not occur with the other methods because the FFT algorithm is not used.

We can also see that the approximation for random angles (methods 1, 3 and 4) is better than the deterministic approach (method 2), obtaining a slightly closer approximation for the semipower method with two iterations than for the equally spaced one with four.

## Conclusions

A comparison between different methods of simulating compatible spectrum earthquakes is presented.

The results suggest that Ohsaki's technique, with normally distributed phase differences, is a good approach, more in accordance with reality, demonstrating good stability and offering quick convergence.

The computation time for one iteration proves that the FFT algorithm is much faster and also simpler than the semipower standard process. Therefore, and despite the fact that it is possible to obtain an iteration reduction with methods 2 and 3, it is better to use methods 1 and 4, the latter seeming to be valid and perhaps more closely approximating a real situation than the first method.

Finally, *Figure 19* presents the initial record for method 4 obtained from the initial $A_k$ from the $\zeta = 0$ spectrum, showing the important variation in this record during the process.

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