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IMPULSIVE MOTIONS OF THE FLOATING ZONE*

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Abstract—A set of problems concerning the behaviour of a suddenly disturbed ideal floating zone is considered.

Mathematical techniques of asymptotic expansions are used to solve these problems. It is seen that many already available solutions, most of them concerning liquids enclosed in cavities, will be regarded as starting approximations which are valid except in the proximity of the free surface which laterally bounds the floating zone.

In particular, the problem of the linear spin-up of an initially cylindrical floating zone is considered in some detail. The presence of a recirculating fluid pattern near the free surface is detected. This configuration is attributed to the interplay between Coriolis forces and the azimuthal component of the viscous forces.

INTRODUCTION

Problems connected with the motion of a free liquid surface are fairly well known, see for example [1]. Here we will deal with a configuration to which very little attention has been paid in the past: the free surface of a floating zone under microgravity conditions.

Floating zone techniques have been used in recent years to prepare high purity materials and, particularly, high purity single crystals of silicon.

The purification process involves (Fig. 1a) vacuum-melting a narrow zone at one end of a heated silicon rod, slowly moving the molten zone along the rod, allowing the silicon to solidify again in the wake of the zone.

Usually the rod is held vertically, the floating zone is kept in position by surface tension forces, and there is no crucible contaminating the silicon. The molten zone refines the silicon by preferentially retaining certain impurities in solution and carrying them along to the end of the rod. Furthermore, the rod can be made into a single crystal by allowing the silicon to slowly solidify around a small seed crystal at one end, the crystal then grows by the addition of atoms from the melt without further crystal nucleating.

The maximum stable length to diameter ratio of a vertically suspended floating zone is controlled by the balance between hydrostatic pressure, which increases with the distance to the top of the molten zone, and surface tension forces.

The floating zone technique is well suited to high melting point materials (above 2000°C); in addition to eliminating crucible contamination, as already mentioned, strains, due to the differential expansion of the crucible and the crystal, are absent.

In spite of these advantages, floating zones which can be naturally stabilized in the terrestrial laboratory are far too short for many applications. Several levitation methods have been suggested to stabilize a slender floating zone but most of them are very

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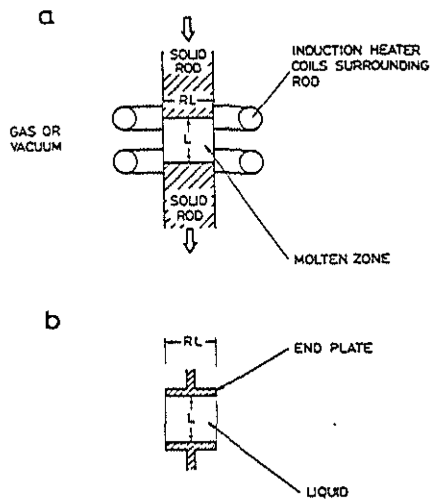


Fig. 1. Floating zone: (a) is a sketch of a floating zone in melting, whereas (b) shows the ideal configuration used for analytical purposes.

limited in scope. The following advantages associated to microgravity are very often quoted:

(1) The restriction imposed by gravity to the zone length to diameter ratio disappears. Unfortunately, zones which are too slender exhibit a tendency to necking because of surface tension and, thence, a maximum stable length to diameter ratio results ($L/D \leq \pi$ for cylindrical zones), as noted by Rayleigh [2]. In addition, microgravity renders the floating zone technique less sensitive to material properties (σ/ρ) than on earth.

(2) Microgravity reduces buoyancy-induced convection (free convection) which results from the changes in fluid density because of the temperature. Very slight temperature gradients could promote convection in a normal gravity field.

Absence of convection is the most widely quoted argument in favour of performing material sciences experiments in space. This absence of convection could be of importance in crystal growth, but convection-inducing mechanism other than buoyancy exist, not to mention that, since diffusion-controlled transport may be inadequate for providing sufficient mixing in the bulk, separate provisions may be required to stir the melt.

The stability of the equilibrium of the floating zone under the disturbances introduced either accidentally or intentionally in microgravity is a matter of concern, thence substantial research should be undertaken to improve our understanding of these problems before exploring the prospects for space materials processing based on the floating zone technique.

THE FLUID DYNAMICAL APPROACH TO THE PROBLEM

The theoretical study of a floating zone in melting involves a formidable task both because of the material characteristics of the melt, whose properties are strongly

temperature dependent, and because of the complexities associated to the disturbances which could be imposed on the zone, thence several simplifying assumptions must be introduced to hold the analytical study within reasonable bounds.

(1) The first assumption consists in disregarding phase-changes. Several interesting problems connected with the hydrodynamics of phase change are then left aside (solidification and melting fronts, influence of shrinkage forces in promoting convection, etc.). These problems are not specific of the floating zone however.

Once the 'no phase-change' assumption is introduced, the floating zone is assumed to consist of a liquid held between two parallel, coaxial, solid discs (Fig. 1b). The resulting configuration will be of interest to many fluid dynamical applications far aside from the field of crystal growth.

(2) The liquid is assumed to be Newtonian, a hypothesis which is untenable for highly viscous liquids.

(3) Finally, it is assumed that the liquid is pure, exhibits uniform properties, and remains in thermal equilibrium with the environment. This assumption could be relaxed without undue difficulties.

Here we will deal with problems concerning rotation of the zone around its symmetry axis. More precisely, we will investigate the phenomena which result when a cylindrical floating zone, either at rest or under solid rotation, is suddenly disturbed through one or both end discs.

THE FLOATING ZONE UNDER ROTATION

In practical instances rotation is imparted to the floating zone with the aim of making the temperature field uniform Once rotating, the angular moment of inertia and thence the rotation rate of the zone can be changed at will by axially displacing the end discs.

There is an impressive body of work on rotating flows mainly developed by Greenspan and his collaborators, see for example Greenspan [3]. Here the rotating liquid is partially in contact with solid walls and partially contained by surface tension forces.

Figure 2 shows the geometry, coordinate system and fluid velocity components for spinning floating zones.

The governing equations (Navier–Stokes equations) for homogeneous liquids in a reference frame rotating with angular velocity Ω are:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}-\frac{v^2}{r}-2v+w\frac{\partial u}{\partial z}=-\frac{\partial P}{\partial r}+E\mathcal{L}u \qquad (1)$$

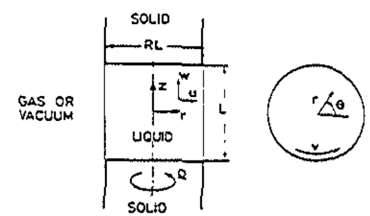


Fig. 2. Geometry, coordinate system and fluid velocity components for spinning floating zones.

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{uv}{r}+2u+w\frac{\partial v}{\partial z}=E\mathscr{L}v, \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+E\left(\mathscr{L}+\frac{1}{r^2}\right)w, \tag{3}$$

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(rw)=0. \tag{4}$$

To write down these equations in dimensionless form, any length has been made dimensionless with $L/2$, any velocity with $\Omega L/2$. Time has been made dimensionless with Ω^{-1}, and gauge pressure with $\rho\Omega^2L^2/4$. E is the Ekman number, $E=\nu/\Omega(L/2)^2$. $P=p-r^2/2$, is the reduced pressure. Finally,

$$\mathscr{L}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}-\frac{1}{r^2}.$$

The direct analytical approach to most fluid flow problems is particularly difficult because of the non-linear character of the convective terms in the Navier–Stokes equations. Thence, we will resort to some linearization method.

There are two main types of linearized rotating-flow problems.

(1) 'Linear spin-up', where the initial state of solid body rotation, with angular velocity, Ω, is suddenly disturbed, through the end discs, which start rotating with the new angular velocity $\Omega(1+\varepsilon)$, $|\varepsilon|\ll 1$. ε is the so-called Rossby number. Obviously the liquid needs some time to reach the new state of solid body rotation with angular velocity $\Omega(1+\varepsilon)$, and the problem consists in studying the flow pattern from the time of the impulsive disturbance onward.

(2) Non-linear spin-up, short times, where the exact solution corresponding to the liquid at rest is suddenly disturbed through the end discs up to $\Omega\sim 1$, but now the small parameter, ε, measures the time from the start of rotation.

The equations for the linear spin-up in a rotating reference frame are the following:

$$\frac{\partial u}{\partial t}-2v=-\frac{\partial P}{\partial r}+E\mathscr{L}u, \tag{5}$$

$$\frac{\partial v}{\partial t}+2u=E\mathscr{L}v, \tag{6}$$

$$\frac{\partial w}{\partial t}=-\frac{\partial P}{\partial z}+E\left(\mathscr{L}+\frac{1}{r^2}\right)w, \tag{7}$$

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(rw)=0. \tag{8}$$

In addition to the usual boundary conditions at the discs, namely the kinematic condition $(w=0)$ and the no-slip conditions $(u=0, v=r)$, the following boundary conditions at the free surface, whose equation is $r=R[1+l(z,t)]$, must be fulfilled:

$$p-\frac{2E}{1+R^2(\partial l/\partial z)^2}\left[\frac{\partial u}{\partial r}+R^2\left(\frac{\partial l}{\partial z}\right)^2\frac{\partial w}{\partial z}-R\frac{\partial l}{\partial z}\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)\right]$$
$$=\frac{1}{C}\left[\frac{1}{R(1+l)[1+R^2(\partial l/\partial z)^2]^{1/2}}-\frac{R(\partial^2 l/\partial z^2)}{[1+R^2(\partial l/\partial z)^2]^{3/2}}\right], \tag{9}$$

where $C=\rho\Omega^2L^3/8\sigma$ is the so-called rotation parameter.

$$\frac{\partial v}{\partial r}-\frac{v}{r}-R\frac{\partial l}{\partial z}\frac{\partial v}{\partial z}=0. \tag{10}$$

$$\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}+\frac{2R(\partial l/\partial z)}{1-R^2(\partial l/\partial z)^2}\left(\frac{\partial u}{\partial r}-\frac{\partial w}{\partial z}\right)=0. \tag{11}$$

Equation (9) expresses the balance of forces normal to the free surface, whereas eqns. (10) and (11) express that viscous forces must vanish there.

In addition, the kinematic condition yields,

$$R\frac{\partial l}{\partial t}-u+wR\frac{\partial l}{\partial z}=0. \tag{12}$$

Furthermore, it is assumed that the free surface remains anchored to the disc edge, thus

$$l(\pm 1, t)=0. \tag{13}$$

This assumption seems to be substantiated by the experimental evidence, at least for moderate spinning rates.

TYPICAL PROBLEMS

The solution of many interesting problems can be undertaken at this stage. The additional assumption $E\ll 1$ will be introduced. This seems to be the case of greatest mathematical interest (and difficulty), although very viscous liquids would be present in real floating zones.

Mathematical techniques of asymptotic expansions can be used to solve those problems which exhibit one or several small parameters. In order to apply these techniques we will look for solutions which are valid sufficiently far from the free surface (central core solutions). Many solutions of this type are already available in the literature, a selection of some of them is presented in Fig. 3. None of them fulfils the boundary conditions at the free surface (nor were intended to do so) and this indicates that several of the terms which were negligible in the central core will change in the outer layer(s) steeper than it was anticipated. This will give the clue in the determination of the outer layer thickness and of the order of magnitude of the controlling terms in this layer.

Let us consider, as an example, the case 3 in Fig. 3. The 'central core solution' has been obtained by Greenspan and Howard [5], who also considered the case of arbitrary axisymmetric rigid containers, and the particular case of the cylindrical container which is relevant here. The complete solution for times of the order of the so called Ekman spin-up time, $t_E=1/\Omega\sqrt{E}$, is given in the mentioned paper. For a qualitative description of the phenomena involved see Benton and Clark [9].

The fluid pattern consists of six main regions (Fig. 4). In region D_1 (whose characteristic lengths, both radial and normal to the discs, are of order one) viscous forces are negligible, whereas inertial, Coriolis and pressure forces balance. Oppositely directed axial fluid masses reach the two Ekman layers near the discs (regions D_2, whose thicknesses are of order $E^{1/2}$) in order to replace, according to the continuity requirements, the fluid which is propelled radially outwards in these layers. Neither u nor v depend on z in this central region. The 'central core solution' satisfies neither the kinematic $(u=0)$ nor the no-slip $(v=r, w=0)$ boundary conditions at the rigid side

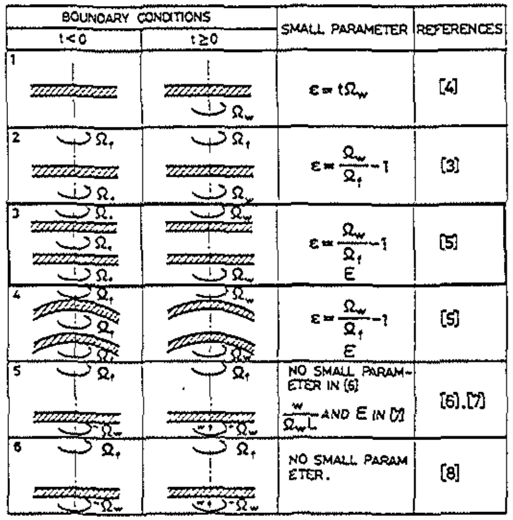
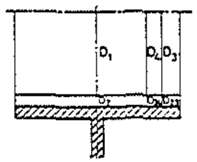


Fig. 3. Several 'central core' solutions.

wall and, thence, a set of two axial shear layers (the so-called Stewartson layers) arises to make the required adjustment. In the first of these layers, D_4, of thickness $E^{1/4}$, the radial mass flux is brought to rest, the azimuthal velocity of the interior flow is joined smoothly to the appropriate value at the wall, and the mass flux flowing radially outward in the Ekman layer turns into the axial direction. Viscous forces only appear in the azimuthal momentum balance. A thinner transition region, thickness $E^{1/3}$, is



BALANCING FORCES IN THE DIFFERENT REGIONS (FIRST ORDER)

REGION	INERTIA			CORIOLIS		VISCOUS			PRESSURE	
	r	θ	z	r	θ	r	θ	z	r	z
D_1		•		•	•				•	
D_2				•	•	•	•		•	
D_4		•		•	•		•		•	
D_{24}				•	•	•	•		•	
D_3				•	•		•		•	
D_{23}				•	•	•	•		•	

Fig. 4. Main regions in the linear spin-up of an initially cylindrical floating liquid zone. Times of order t_E.

imbedded between the D_4 region and the outer surface. The function of this layer is, in the case of the rigid container, to reduce the axial velocity to its zero value at the wall.

SPIN-UP OF A CYCLINDRICAL FLOATING ZONE

It can be seen that the above-discussed solutions for the regions D_1, D_2, D_4 and D_{24} are also valid in the case of the linear spin-up of an initially cylindrical floating zone. The function of zone D_3, whose thickness is again of order $E^{1/3}$, since this thickness is controlled by the structure of the differential equations, is now to fulfil the boundary condition of zero azimuthal component of the viscous stress tensor at the free surface. This is achieved through an interplay between Coriolis forces, radial an axial pressure gradients (which are mainly induced by the distortion of the free surface), and the azimuthal component of the viscous forces. Let us look closely at the relevant equations.

The independent variables, of order one, τ, η and z, are defined in this region as:

$$t = \tau/\sqrt{E}; \qquad r = R + E^{1/3}\eta; \qquad z = z.$$

The differential equations (5)–(8) become, once higher order terms have been neglected,

$$-2v = -E^{-1/3}\frac{\partial P}{\partial \eta} + E^{1/3}\frac{\partial^2 u}{\partial \eta^2}, \tag{14}$$

$$2u = E^{1/3}\frac{\partial^2 v}{\partial \eta^2}, \tag{15}$$

$$0 = -\frac{\partial P}{\partial z} + E^{1/3}\frac{\partial^2 w}{\partial \eta^2}, \tag{16}$$

$$0 = E^{-1/3}\frac{\partial u}{\partial \eta} - \frac{\partial w}{\partial z}. \tag{17}$$

Time-dependent terms have been deleted since they are, for each equation, obviously smaller than at least one of the written terms.

The sought solution must match, in an asymptotic sense, with the solution in D_4, and this supplies the required boundary conditions for very large negative values of η. Such conditions are (see Greenspan and Howard [5] for the relevant equations in D_4):

$$E^{-1/2}2u = v - R = E^{1/12}\frac{Re^{-\tau}}{\sqrt{\pi\tau}}\eta, \tag{18}$$

$$\frac{\partial P}{\partial \eta} = E^{1/3}2R. \tag{19}$$

Now the aim is to slightly change the azimuthal velocity, v, since that given by the D_4 solution does not fulfil the boundary condition at the free surface (which is $\partial v/\partial \eta = 0$, as we will see very soon). Equation (18) suggests the following asymptotic expansion for v.

$$v = R + E^{1/12}v_1(\eta, z; \tau) + \dots . \tag{20}$$

According to the azimuthal momentum equation (eqn. 15), the radial velocity u must be of order $E^{5/12}$, instead of being of order $E^{7/12}$, as it could be (wrongly) deduced by matching with u in region D_4 (eqn. 18). This could come as a surprise although it can be justified as follows: Coriolis forces are the only available forces for changing the azimuthal velocity, v; nevertheless, they are too small in region D_4 to bring $\partial v/\partial\eta$ to its correct value at the free surface. Thence,

$$u = E^{5/12}u_0(\eta, z; \tau) + \dots \quad (21)$$

The order of magnitude of the axial velocity is deduced from eqns. (17) and (21),

$$w = E^{1/12}w_0(\eta, z; \tau) + \dots , \quad (22)$$

whereas eqns. (16) and (19) yield:

$$P = \bar{P} + E^{1/3}2R\eta + E^{5/12}P_2(\eta, z; \tau) + \dots \quad (23)$$

THE FREE SURFACE

Now let us consider the linearized boundary conditions at the free surface, whose equation is, to first order:

$$r = R[1 + E^n l_1(z, \tau)], \quad (24)$$

where n is unknown beforehand, but it can be deduced from the boundary condition expressing the normal balance of forces.

Equation (9), after neglecting higher order terms, becomes:

$$\bar{P} + E^{5/12}P_2 + \tfrac{1}{2}R^2(1 + 2E^n l_1) = \frac{1}{CR}(1 - E^n l_1) - \frac{R}{C}E^n\frac{\partial^2 l_1}{\partial z^2}, \quad (25)$$

from which we deduce:

$$\bar{P} + \tfrac{1}{2}R^2 = \frac{1}{CR}, \quad (25a)$$

$$n = \tfrac{5}{12}, \quad (25b)$$

$$\frac{\partial^2 l_1}{\partial z^2} + \left(RC + \frac{1}{R^2}\right)l_1 = -\frac{C}{R}P_2. \quad (25c)$$

Boundary conditions (10), (11) and (12) become respectively

$$\frac{\partial v_1}{\partial \eta} = 0, \quad (26)$$

$$\frac{\partial w_0}{\partial \eta} = 0, \quad (27)$$

$$u_0 = 0. \quad (28)$$

The condition at the disc edge cannot be fulfilled at this stage because of reasons which will be indicated later.

SEVERAL RESULTS

Once solved the problem, the expressions for the velocity field and the stream function X are, in the first order approximation:

$$u = E^{5/12}u_0(\eta, z; \tau) + \dots ,$$

$$u_0 = -\frac{1}{2}\frac{Re^{-\tau}}{\sqrt{\pi\tau}}\sum_{k=1}^{\infty}(-1)^{k-1}\sqrt[3]{2\pi k}\cos\pi kz$$

$$\times\left[\exp(\sqrt[3]{2\pi k}\eta) + 2\exp(\sqrt[3]{2\pi k}\eta/2)\cos\left(\frac{\sqrt{3}}{2}\sqrt[3]{2\pi k}\eta + \frac{2\pi}{3}\right)\right]; \quad (29)$$

$$v = R + E^{1/12}v_1(\eta, z; \tau) + \dots ,$$

$$v_1 = \frac{Re^{-\tau}}{\sqrt{\pi\tau}}\left\langle \eta - \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{\sqrt[3]{2\pi k}}\cos\pi kz\right.$$

$$\left.\times\left[\exp(\sqrt[3]{2\pi k}\eta) + 2\exp(\sqrt[3]{2\pi k}\eta/2)\cos\frac{\sqrt{3}}{2}\sqrt[3]{2\pi k}\eta\right]\right\rangle; \quad (30)$$

$$w = E^{1/12}w_0(\eta, z; \tau) + \dots ,$$

$$w_0 = \frac{1}{2}\frac{Re^{-\tau}}{\sqrt{\pi\tau}}\sum_{k=1}^{\infty}(-1)^{k-1}\left(\frac{4}{\pi k}\right)^{1/3}\sin\pi kz$$

$$\times\left[\exp(\sqrt[3]{2\pi k}\eta) - 2\exp(\sqrt[3]{2\pi k}\eta/2)\cos\frac{\sqrt{3}}{2}\sqrt[3]{2\pi k}\eta\right]; \quad (31)$$

$$X = E^{5/12}X_0(\eta, z; \tau) + \dots ,$$

$$X_0 = -\frac{1}{2}\frac{Re^{-\tau}}{\sqrt{\pi\tau}}\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\sqrt[3]{2\pi k}}{\pi k}\sin\pi kz$$

$$\times\left[\exp(\sqrt[3]{2\pi k}\eta) + 2\exp(\sqrt[3]{2\pi k}\eta/2)\cos\left(\frac{\sqrt{3}}{2}\sqrt[3]{2\pi k}\eta + \frac{2\pi}{3}\right)\right]. \quad (32)$$

The expression for the stream function, X, allows the representation of the streamlines in the secondary recirculation zone as indicated in Fig. 5. The upper part of this figure, from Benton and Clark [9], shows the meridional circulation pattern for the linear spin-up of a homogeneous liquid in a circular cylinder with rigid walls. No particular details in zone D_3 are shown. The lower part of the figure corresponds to the

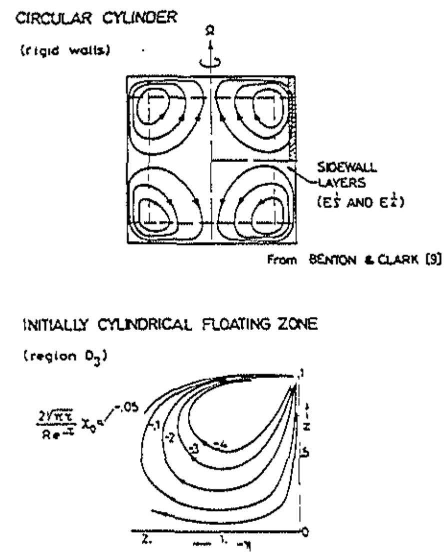


Fig. 5. *Meridional circulation pattern for the spin-up. The upper part shows a liquid enclosed in a rigid circular cylinder. The lower part shows the secondary recirculation which appears near the free surface in a floating liquid zone.*

shaded region in the upper part, and shows the predicted secondary recirculation near the free surface.

In order to obtain the shape the normal balance of forces at the free surface (eqn. 25) is used, as already mentioned. To this aim the pressure field must be calculated up to order $E^{5/12}$. This can be achieved through the radial and axial momentum equations (eqns. 14 and 16), which yield:

$$P_2(0, z; \tau) = -2\frac{Re^{-\tau}}{\sqrt{\pi\tau}} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\sqrt[3]{2\pi k}}{\pi k} \cos \pi kz. \qquad (33)$$

Bringing this expression of P_2 to the right hand side of eqn. (25c), and taking into account the following additional conditions of symmetry (eqn. 34) and of volume invariance (eqn. 35),

$$l_1(z, \tau) = l_1(-z, \tau), \qquad (34)$$

$$\int_{-1}^{1} l_1(z, \tau)\, dz = 0, \qquad (35)$$

the following equation for the free surface results:

$$l_1(z, \tau) = -2\sqrt[3]{2}\frac{e^{-\tau}}{\sqrt{\pi\tau}} CR^2 \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(\pi k)^{2/3}} \frac{1}{\pi^2 k^2 R^2 - 1 - R^3 C} \cos \pi kz. \qquad (36)$$

Several representative results are given in Fig. 6. It should be noticed that the displacement of the free surface becomes infinitely large when the dimensionless radius of the floating zone, R, is related to the rotation parameter, C, as follows:

$$1/R = \pi/\sqrt{1 + R^3 C}. \qquad (37)$$

This equation gives the maximum stable length to diameter ratio of a cylindrical zone in terms of the rotation parameter C (recall that R is the zone radius made dimensionless with half the zone length). Equation (37) was first obtained by Gillis [10].

The zones D_4 and D_3 merge with the Ekman layer, D_2, near the discs. The resulting solution, which can be obtained without difficulty, is not uniformly valid in a small

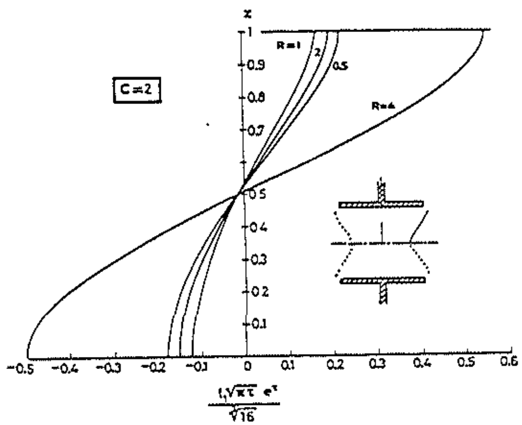


Fig. 6. Shape of the free surface for several values of the dimensionless radius, R. $C = 2$.

zone, the 'corner region', placed near the free surface, whose height and radial thickness are both of order $E^{1/2}$. The relevant differential equations in this zone (which may be important in the study of the conditions at the disc edge, and in the control of crystal imperfections) become of elliptic type and, thence, its solution is not readily accessible. The mathematical difficulties associated to this region have been discussed, in connection with the case 1 in Fig. 3, by one of the present authors [11].

CONCLUSIONS

Since rotation will be imparted to a floating zone for homogenizing purposes, some knowledge of the phenomena associated with the onset of rotation or to changes in the rotation rate would be of interest.

A very simple problem has been analyzed in the present paper: the linear spin-up of an initially cylindrical floating liquid zone.

The first, fairly obvious, conclusion is that the inner flow pattern does not differ too much from that corresponding to a liquid enclosed in a cylindrical container of circular cross-section. Rotating cylindrical configurations are moderately insensitive to changes in the boundary conditions.

On the other hand, the secondary recirculation near the free surface comes somewhat as a surprise, although several antecedents are found in the literature. This secondary recirculation could be important in those situations where interface phenomena control the fluid motion.

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NOMENCLATURE

C rotation parameter $C = \rho\Omega^2 L^3/8\sigma$
E Ekman number, $E = \nu/\Omega(L/2)^2$
L distance between end discs [m]
$\mathcal{L}$ differential operator,

$$\mathcal{L} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{r^2}.$$

P dimensionless reduced pressure, $P = p - (r^2/2)$
$\bar{P}$ dimensionless gage pressure at the free surface of the floating zone in solid rotation
R disc radius, made dimensionless with $L/2$
X Greenspan stream function,

$$u = \frac{\partial X}{\partial z};\ w = -\frac{\partial X}{\partial r} - \frac{X}{r}$$

l radial deviation of the free surface from its undisturbed position, $l(z, t) = (r - R)/R$
p dimensionless gage pressure
r dimensionless radial distance, $r \leq R$
t dimensionless time
t_E Ekman spin-up time, $t_E = 1/\Omega\sqrt{E}$ [s]
u dimensionless radial velocity
v dimensionless azimuthal velocity
w dimensionless axial velocity
z dimensionless axial distance to the mean plane, $|z| \leq 1$
Ω angular velocity [rad · s^{-1}]
η radial coordinate of order one in region D_3, $\eta = E^{-1/3}(r - R)$.
ν liquid kinematic viscosity [m^2 · s^{-1}]

ρ	liquid density $[kg \cdot m^{-3}]$
σ	liquid–gas surface tension $[N \cdot m^{-1}]$
τ	dimensionless time, $\tau = t\sqrt{E}$

Subscripts

0, 1, 2	indicate zero, first or second order of approximation
f	fluid
w	wall

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