Operational Modal Analysis on laminated glass beams

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ABSTRACT: Laminated glass is a sandwich element consisting of two or more glass sheets, with one or more interlayers of polyvinyl butyral (PVB). The dynamic response of laminated glass beams and plates can be predicted using analytical or numerical models in which the glass and the PVB are usually modelled as linear-elastic and linear viscoelastic materials, respectively. In this work the dynamic behavior of laminated glass beams are predicted using a finite element model and the analytical model of Ross-Kerwin-Ungar. The numerical and analytical results are compared with those obtained by operational modal analysis performed at different temperatures.

1 INTRODUCTION

In the last years, the use of laminated glass in building construction has increased considerably, mainly in facades, covers, stairs and windows. Laminated glass consists of two or more sheets of monolithic glass with one or more interlayers of a polymer such as polyvinyl butyral (PVB). The thickness of the PVB layer is usually 0.38 mm or a multiple of this value. The adherence of the glass and PVB layers is provided subjecting the shaped laminate to high temperature and pressure conditions in an autoclave.

The main advantage of laminated glass compared with monolithic glass is the safety provided in case of breakage, because the interlayer holds the fragments together, adhered to the PVB layer, thus attenuating the accident risk. Moreover, the PVB interlayer increases considerably damping, reducing the magnitude of the vibrations due to dynamic loadings.

An experimental characterization of both glass and PVB is needed when the dynamic behaviour of laminated glass elements is simulated by analytical or numerical models. The glass mechanical properties are usually estimated by bending static tests whereas those corresponding to the PVB are estimated by relaxation or creep tests in the time domain or its corresponding dynamic tests in the frequency domain.

PVB is an amorphous thermoplastic which shows linear-viscoelastic behaviour. A fundamental characteristic of viscoelastic materials is that the mechanical properties are frequency (or time) and temperature dependent. This fact makes more difficult the structural analysis of laminated glass elements. On the other hand, glass is usually modelled as a linear elastic material. A high scatter is expected in the mechanical strength because of the superficial micro-defects giving rise from the manufacturing process and further manipulation.

In this paper, the analytical model of Ross, Kerwin y Urgar (Ross et al. 1959) and the finite element method have been used to predict the modal parameters of laminated glass beams in bending with simply supported boundary conditions. The predictions are validated by operational modal tests carried out on small laminated glass specimens.

2 VISCOELASTIC BEHAVIOUR

The polyvinyl butiral (PVB) can be considered as a linear viscoelastic material whose mechanical properties are frequency (or time) and temperature dependent. In the frequency domain, the complex tensile modulus, $E_2^*(\omega)$ is given by:

$$E_2^*(\omega) = E_2^{'}(\omega) + iE_2^{''}(\omega) = E_2^{'}(\omega)(1 + i\eta_2(\omega)) \quad (1)$$

where E'(ω) y E''(ω) are the storage and the loss tensile moduli, respectively. The relation between both moduli is the loss factor $\eta_2(\omega)$, i.e.:

$$\eta_2(\omega) = \frac{E_2^{''}(\omega)}{E_2^{'}(\omega)} \quad (2)$$

Similarly the shear behaviour, the complex shear modulus is given by:

$$G_2^*(\omega) = G_2^{'}(\omega) + iG_2^{''}(\omega) = G_2^{'}(\omega)(1 + i\eta_2(\omega)) \quad (3)$$

where again G'(ω) and G''(ω) are the storage and the loss shear moduli, respectively.

For relating both shear and tensile moduli, the correspondence principle is used (Lee 1960). This principle allows us to derive the expressions for linear viscoelastic materials from those of the linear elasticity theory, but introducing the corresponding complex viscoelastic properties, i.e.:

$$G_2^*(\omega) = \frac{3E_2^*(\omega) \cdot K_2^*(\omega)}{9K_2^*(\omega) - E_2^*(\omega)} \quad (4)$$

where $K_2^*(\omega)$ is the complex bulk modulus.

When relaxation tests are used to estimate the viscoelastic behaviour, the experimental master-curve is usually fitted with a generalized Maxwell model (Ferry 1980), that can be represented as a Prony series:

$$E_2(t) = E_{2\infty} + \sum_{i=1}^{n} e_i \cdot \exp\left(-\frac{t}{\tau_i}\right) \quad (5)$$

where e_i and τ_i are coefficients to be estimated and $E_{2\infty}$ is the equilibrium modulus ($t \to \infty$). The storage and loss components of the complex modulus can be obtained directly from the relaxation Prony series coefficients given by (Tschoegl 1989, Fernández 2010):

$$E_2^{'}(\omega) = E_{2\infty} + \sum_{i=1}^{n} \frac{\tau_i^2 \cdot \omega^2 \cdot e_i}{\tau_i^2 \cdot \omega^2 + 1} \quad (6)$$

and

$$E_2^{''}(\omega) = E_{2\infty} + \sum_{i=1}^{n} \frac{\tau_i \cdot \omega \cdot e_i}{\tau_i^2 \cdot \omega^2 + 1} \quad (7)$$

Similar expressions to eqs. (5, 6 and 7) are used to obtain the complex shear modulus.

3 THE ANALYTICAL MODEL OF ROSS, KERWIN AND UNGAR (RKU)

Laminated glass beams can be considered sandwich beams (figure 1). Since the fifties, several analytical models (Mead 1969 and Ross 1959) have been proposed since the fifties to predict the dynamic response of sandwich beams with viscoelastic behaviour.

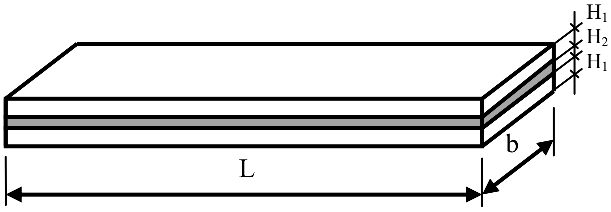

Figure 1. Laminated glass beams.

In this paper, the model proposed by Ross, Kerwin y Urgar (RKU) has been used to estimate the natural frequencies and damping ratios of symmetric laminated glass beams with same thickness for the glass layers. This model assumes that the bending mode shapes of simply supported sandwich beams are represented by sinusoidal functions as in monolithic beams (Ross 1959). Accordingly, the authors propose to use the equations given by the Euler-Bernoulli bending theory to estimate the bending natural frequencies and damping ratios of sandwich beams, through using an apparent or equivalent stiffness EI*, i.e.:

$$\omega_n^{*2} = \omega_n^2\left(1 + i\eta_n\right) = k_n^4 \frac{EI^*}{\overline{m}} \tag{9}$$

where:

- k_n is the wave number of mode n,
- ω_n is the natural frequency of mode n,
- η_n is the damping ratio of mode n,
- $\overline{m}$ is the mass per unit length,
- EI* is the equivalent bending stiffness, given by:

$$EI^* = b \cdot \left[\frac{E_1 \cdot H_1^3}{6} + \frac{E_1 \cdot H_1 \cdot \left(H_1 + H_2\right)^2 \cdot g^*}{1 + 2 \cdot g^*} \right] \tag{10}$$

- H_1 is the glass thickness.
- H_2 is the PVB thickness.

with

- $$g^* = \frac{G_2^* \cdot L^2}{E_1 \cdot H_1 \cdot H_2 \cdot k_n^2}, \tag{11}$$

The wave numbers k_n of the Euler-Bernoulli bending theory can be used for simply supported beams. Correction factors should be used for other boundary conditions (Jones 1996 and Jones 2001).

4 EXPERIMENTAL TESTS

Three laminated glass specimens with the dimensions shown in Table 1 were considered in the investigation under simply supported conditions. ble 1.

Table 1: Dimensions of the laminated glass beams.

	Beam 1	Beam 2	Beam 3
Length L (mm)	1000	1000	1400
Width b (mm)	100	100	100
Glass thickness H_1 (mm)	3	4	4
PVB thickness H_2 (mm)	0.38	0.38	0.38

4.1 Material characterization

The glass was modelled as a linear-elastic material with Young modulus E_1=72000 MPa as obtained from bending static tests. The density needed to estimate the modal parameters in analytical and numerical models, was assumed as ρ= 2500 kg/m^3.
The experimental dynamic characterization of PVB was carried out in a DMA RSA3 (T.A. Instruments) subjecting PVB specimens of 0.38 mm thick, to relaxation tests in tensile under constant strain during 10 minutes at different temperatures from -30 ºC to 70 ºC. The relaxation master-curve at 20ºC was calculated from the experimental results at different temperatures applying the time-temperature superposition principle (TTS) (Ferry 1980) by means of the William, Landel y Ferry (WLF) equation (Willians et al. 1955),i.e.:

$$\log(a_T) = -C_1 \frac{T - T_0}{C_2 + (T - T_0)} \tag{8}$$

where the coefficients C_1=12.6027 and C_2=74.46 were estimated for a reference temperature, T_0=20ºC. Once the relaxation master-curve was obtained and fitted with eq. (5), storage and loss components of the complex tensile modulus were calculated with eqs. (6) and (7). Next, the complex shear modulus G*(ω) was derived from E*(ω) using eq. (4) and assuming a constant bulk modulus of 2 GPa (Van Duser 1999). Both complex modulus components are presented in Fig. (2).

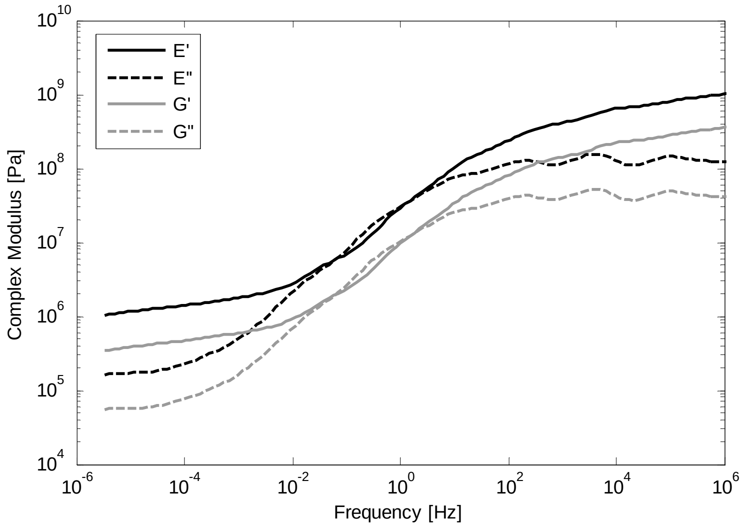

Figure 2. Tensile and shear complex moduli components of PVB.

4.2 Operational Modal Analysis

Operational modal analysis was applied to the laminated glass beams described on Table 1. The responses of the beams were measured using 8 accelerometers with a sensitivity of 100 mV/g, uniformly distributed (Fig. 3) and a National Instruments digital card 4472 PCI. The beams were excited by applying successivelly small hits along the beam. The responses were recorded for approximately 2 minutes using a sampling frequency of 4000 Hz. In Fig. 4, the singular value decomposition of modal tests carried out on beam 1 (1 m length and thickness 3+0.38+3 mm) is shown.

Modal parameters were estimated using both Frequency Domain Decomposition (Brincker et al. 2003) and Stochastic Subspace Identification method (SSI) (Van Overschee et al. 1996). The results obtained with SSI are presented in Table 2. The natural frequencies estimated with OMA include the effect of the weight of sensors (5 g each) and cables. This effect was taken into account in the finite element model but it was not considered in the analytical model. As a consecuence, the experimental natural frequencies are expected to be a little less than those predicted with the analytical model.

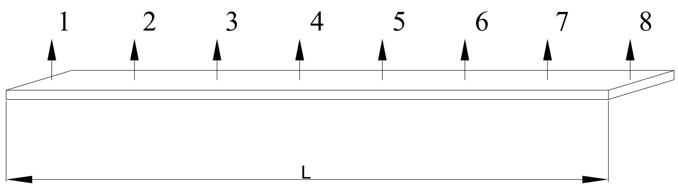

Figure 3. Location of the measurement points.

As regards the damping, the loss factor η is shown in Table 2. It has been assumed that loss factor η and the modal damping ζ, are related by:

$$\eta = 2\zeta \tag{12}$$

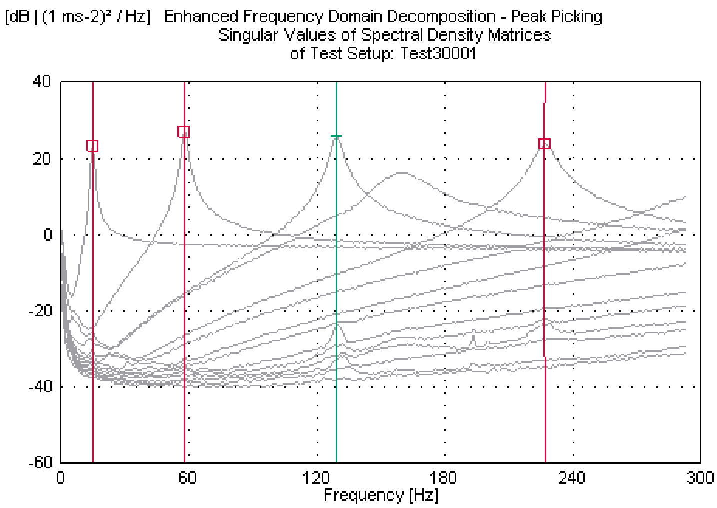

Figure 4 :Singular value decomposition of modal tests on the simply supported beams

Table 2. Natural frequencies and damping ratios for the simply supported beams.

Beam	Mode	T [ºC]	fn [Hz]			Damping ratio η(%)		
			OMA	RKU MODEL	FEM MODEL	OMA	RKU MODEL	FEM MODEL
1	1	25.1	14.8	14.78	14.52	2.8	0.35	0.36
	2	25.1	57.9	58.89	57.85	2.5	0.61	0.64
	3	25.1	129	131.71	129.03	3.3	1.04	1.05
	4	25.1	226.7	232.59	228.95	3.9	1.51	1.81
2	1	24.4	19.9	19.63	19.29	1.3	0.35	0.36
	2	24.4	78.4	78.12	76.71	2.2	0.67	0.65
	3	24.4	173.7	174.57	172.86	3.3	1.18	1.56
	4	24.4	304.2	307.83	303.93	4.1	1.59	1.87
3	1	24.9	10.6	10.03	9.94	1.07	0.28	0.26
	2	24.9	40.9	39.97	39.62	1.91	0.49	0.52
	3	24.9	91.0	89.56	88.17	2.56	0.78	0.76
	4	24.9	160.2	158.41	156.83	3.09	1.17	1.22

5 ANALYTICAL AND NUMERICAL PREDICTIONS

The RKU model was applied to predict the natural frequencies and damping ratios of the simply supported beams described in Table 1. The results provided by eq. (9), using the material properties described in section 4.1, are presented in Table 2.

On the other hand, a 3D solid finite element model has been developed to simulate the dynamic tests. In the context of dynamic analysis, linear elastic behavior is assumed for glass, while PVB is considered a linear viscoelastic material. The viscoelastic properties of PVB have been introduced in the model through the Prony series coefficients. The temperature dependence of PVB is accounted for using the WLF equation.

The mesh chosen for the model is strongly influenced by the reduced thickness of the PVB layer. In order to get reasonably well shaped elements, three elements are used for each glass layer (Fig. 5) in the thickness direction. A three-dimensional twenty node solid element exhibiting quadratic displacement behaviour is chosen. Each node has three degrees of freedom: translations in the nodal directions x, y and z.

Structural mass elements were used to take the mass sensors into account. They are represented by four small black points on the symmetry axis of the ¼ model as can be seen in figure 5.

The test setup chosen allows us exploiting the symmetry conditions in order to reduce the calculation time. It proves to be faster to analyse two models, one for each plane of symmetry or anti-symmetry, than using only one model without taking advantage of the symmetry conditions. The number of degrees of freedom is reduced by a factor which outweighs the cost of the additional run.

The time step and total time for the transient simulation have been chosen for each mode in accordance to the recommendations of Barredo et al. 2010. The estimation of the properties of each mode from a transient analysis is preferably performed by using at least 30 points per period and 100 cycles.

The displacements obtained in the model were analyzed in the frequency domain isolating the modes by means of band pass filters. The frequency is estimated employing a peak picking procedure while the damping ratio is obtained using the logarithmic decrement method (Fig. 6). Table 2 summarizes the results obtained with the finite element model.

From Table 2 can be observed that the analytical and numerical predictions for the natural frequencies are close to the experimental values obtained by OMA, being the error less than 6%. As regards the predictions with the RKU model it gives accurate results at minimal cost while it takes several hours to obtain similar results using the FE model.

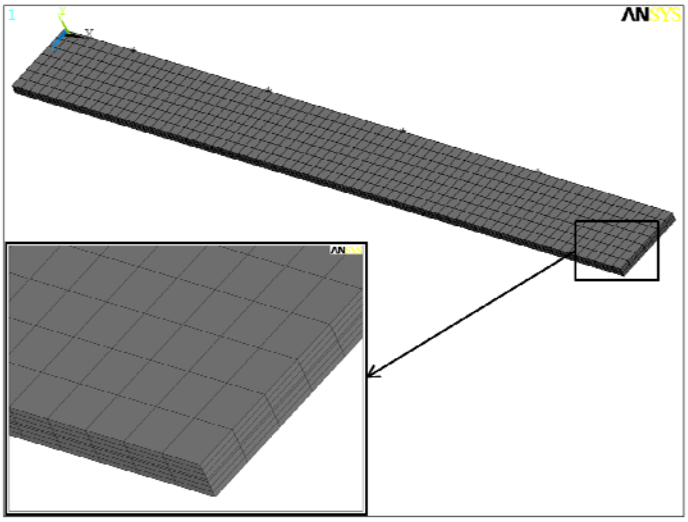

Figure 5. Mesh of the laminated glass FE model.

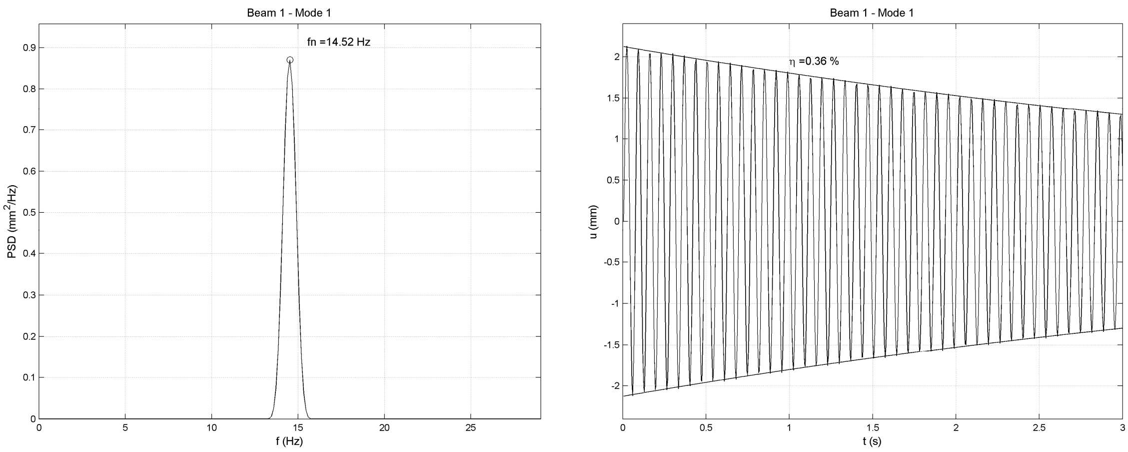

Figure 6. Response of one mode.

As far the analytical and numerical predictions of damping ratios are concerned, they are much lower than the corresponding experimental values, indicating that some additional dissipative mechanisms, acting during the tests, are not allowed for in the models.

6 CONCLUSSIONS

- Operational modal analysis has been used to estimate the modal parameters of simply supported laminated glass beams. The results have been used to validate the predictions obtained with the analytical model of Ross, Kerwin and Ungar and a finite element model.
- The analytical and numerical predictions for the natural frequencies are in good agreement with the experimental values obtained by OMA.
- The predicted damping ratios are much lower than the corresponding experimental values, indicating that some additional dissipative mechanisms, acting during the tests, were not considered in the analytical and numerical models.
- The analytical RKU model provides similar results to the finite element model at minimal cost.

REFERENCES

Brincker, R., Ventura, C. and Andersen, P. 2003. Why Output-Only Modal Testing is a Desirable Tool for a Wide Range of Practical Applications. *In Proc. Of the International Modal Analysis Conference (IMAC) XXI*, paper 265.

Tschoegl, N.W. 1989 . The Phenomenological Theory of Linear Viscoelastic Behaviour. Sringer-Verlag, Berlin.

Fernández, P., Rodríguez, D., Lamela, M.J. and Fernández Canteli, A. 2010. Study of the interconversión between viscoelastic behaviour functions of PMMA. Mechanics of Time-Dependent Materials. DOI 10.1007/211043-010-9128-3.

Ferry, J. D. 1980. *Viscoelastic Properties of Polymers*. John Wiley & Sons. 3ª Edition.

Jones, D.I.G. 1996. Reflections on damping technology at the end of the twentieth century. *Journal of Sound and Vibration*,190 (3), p. 449-462

Jones, D.I.G. 2001. *Handbook of viscoelastic vibration damping.* John Wiley&Sons, Ltd.

Mead, D.J., and Markus, S. 1969. The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions. *Journal of Sound and Vibration.* 10(2), 163-175.

Ross, D., Ungar, E.E., Kerwin, E.M. Jr. 1959. Damping of plate flexural vibrations by means of viscoelastic laminate. *ASME Structural Damping*. p. 49-88.

Lee, E.H., 'Viscoelastic stress analysis', in *Structural Mechanics*, J.N. Goodier and N.J. Hoff (eds), Pergamon Press, Oxford, 1960, 456–482.

Williams, M.L., Landel, R.F. and Ferry, J.. 1955. The Temperature Dependence of Relaxation Mechanisms in Amorphous Polymers and Other Glass-forming Liquids. *Journal of the American Chemical Society* 77, p. 3701-3707.

Van Duser, A., Jagota, A., Benisson, S.J. 1999. Analysis of glass/polyvinylbutyral laminates subjected to uniform pressure. *Journal of Engineering Mechanics*, 125(4), pp 435–442.

Van Overschee, P., De Moor, B., 1996. *Subspace identification for linear systems – Theory, Implementation, Applications*, Kluwer Academic Publisher.

Barredo, J., Soriano, M., Hermanns, L., Fraile, A., López, M. and Gómez, M.S. 2010. Comparison of different finite element models for transient dynamic analysis of laminated glass in structural applications. In proceedings of the 10th International Conference on Computational Structures Technology. Ed. B. Topping.