# 3x3 Multibeam Network for a Triangular Array of Three Radiating Elements 

Design and Measurement

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#### Abstract

A multibeam antenna study based on Butler network will be undertaken in this document. These antenna designs combines phase shift systems with multibeam networks to optimize multiple channel systems. The system will work at 1.7 $\mathbf{G H z}$ with circular polarization. Specifically, result simulations and measurements of 3 element triangular subarray will be shown. A 45 element triangular array will be formed by the subarrays. Using triangular subarrays, side lobes and crossing points are reduced.


Keywords-component; multibeam, Butler, triangular subarray.

## I. Introduction

GEODA is a smart, conformal multi-array antenna system designed for satellite communications. Because of its adaptive beam, it is possible to contact several satellites at once, operating at 1.7 GHz . Its complex structure is based on a triangular array configuration, divided into subarrays (cells) of three circular radiant elements, as shown in figures 1 and 2.


Fig. 1 GEODA structure and cell configuration


Fig. 2 GEODA panel

Radiant elements consist of two stacked circular patches. The principal patch is fed in quadrature in two points separated $90^{\circ}$, in order to get circular polarization. The coupled patch, which has smaller size, is not fed, and is used in the aim of improving the bandwidth. Figure 3 shows the radiation pattern of a single radiant element,


Fig. 3. Radiation pattern of a single radiant element
As it is shown, the bandwidth of the radiation pattern of a single radiant element is on the order of $60^{\circ}(\mathrm{deg})$.

Multi-Beam Forming Networks (MBFN) are very effective systems to form multiple beams. Butler matrix is the easiest and smallest lossless network, which can be implemented on printed circuit embedded within a 2D dielectric substrate. Particularly, lossless networks generate orthogonal beams.

Generally, the number of inputs in Butler matrix is a power of two, which do not fit with a triangular subarray. In this case, a three-output network is needed in order to be adapted to the three elements of the subarray.

## II. General MBFM Scheme

Usually, passive multibeam conformation networks are a set of inputs related to the system outputs, which are connected to the radiating elements. Terms input or output depends on the
working way, transmission or reception. In this document, MBFN will be associated to a transmission antenna. The reception network behavior will be the same through network reciprocity.

Regarding the classification, we only distinguish between lossless and lossy networks. In fact, all the networks are lossy but we will consider them lossless when the networks are not dispersive if elements that form the network were ideals.

All ports have to be adapted, both inputs and outputs, and isolated from each. Under these conditions, a MxN network responds to a scattering matrix given by,

$$
[S]=\left[\begin{array}{cccccc}
0 & \cdots & 0 & s_{1, M+1} & \cdots & s_{1, M+N} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & s_{M, M+1} & \cdots & s_{M, M+N} \\
s_{N+1,1} & \cdots & s_{N+1, M} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
s_{M+N, 1} & \cdots & s_{M+N, M} & 0 & \cdots & 0
\end{array}\right]=\left[\begin{array}{cc}
{[0]} & {\left[s_{R}\right]} \\
{\left[s_{T}\right]} & {[0]}
\end{array}\right]
$$

At first, the scattering matrix can be summarized in a pair of matrices, $S_{T}$ and $S_{R}$. If the network is reciprocal, $S_{T}$ and $S_{R}{ }^{t}$ will be the same. Each matrix elements represent mutual coupling between each M input and N output, both amplitude and phase.

The condition for a network to be reciprocal and lossless, on scattering parameters terms, is that the product of the scattering matrix and tis transpose conjugate is the identity matrix. In the case under study,

$$
[S][S]^{H}=\left[\begin{array}{cc}
{\left[S_{R}\right]\left[S_{R}\right]^{H}} & {[0]} \\
{[0]} & {\left[S_{T}\right]\left[S_{T}\right]^{H}}
\end{array}\right]=[I]
$$

This implies that vectors related to the columns of each matrix $S_{T}$ and $S_{R}$ have to be orthonormal to each, hence:

- The number of input and outputs must be the same. Otherwise, matrices would be non-square matrices and one of them would have more vectors than matrix range wherewith it would be impossible to obtain orthogonal beams to each.
- The unit excitations are indicating that the output equals the input power.
- Vectors are orthogonal. When an input port is excited the diagram generated is orthogonal to any other one generated by any other input port.

Once established network parameters, radiation pattern depends on both radiation pattern of the single radiating element and the distribution of each radiating element on the array. We must take into account the polarization also.

## BASIC EQUATIONS

In order to analyse a 'cell', it is assumed patches are located over vertices of an equilateral triangle with 'd' side length, as shown in figure 4.


Fig.4: Subarray distribution
Where $x_{1}=\frac{d}{2 \sqrt{3}}, y_{1}=\frac{d}{2}, x_{2}=\frac{d}{2 \sqrt{3}}, y_{2}=-\frac{d}{2}, \quad x_{3}=-\frac{d}{\sqrt{3}}$, $y_{3}=0$. On the assumption that the steering direction of the first beam is $\theta=\theta_{0}, \phi=0$, and taking into account that the array factor is given by $A F(\theta, \phi)=\sum_{i=1}^{3} A_{i} e^{-j \alpha_{i}} e^{-j k \hat{r}_{i}}$, if we bear in mind the fact that feeding phases must satisfy the condition of adding the contributions of each array element in the steering direction, then, $k_{0} \hat{r}_{1}-\alpha_{1}=k_{0} \hat{r} r_{2}-\alpha_{2}=k_{0} \hat{r} r_{3}-\alpha_{3}$. From here, it is shown that,
$S_{4,1}=S_{5,1}=a e^{j \alpha}$
$S_{6,1}=b e^{j(\alpha-\beta)}$
Where $\beta=\sqrt{3} \frac{\pi d}{\lambda} \sin \left(\theta_{0}\right)$
The other two beams will keep a rotational symmetry steering at the same elevation angle and changing the azimuth angle between $0^{\circ}, 120^{\circ}$ or $240^{\circ}$. Rotating S parameters studied will form three different beams.

Imposing the orthogonal condition $\left[S_{T}\right]\left[S_{T}\right]^{H}=[I]$, then

$$
\sin \left(\theta_{0}\right)=\frac{\lambda}{\sqrt{3} \pi d} \arccos \left(-\frac{a}{2 b}\right)
$$

Steering directions and phase shifts between elements have been calculated for different distances between elements and different amplitude feeding relation, as shown in table 1.

TABLE I. STEERING DIRECTION AND PHASE SHIFTS

|  |  | $\mathbf{d} / \boldsymbol{\lambda}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 7}$ | $\mathbf{0 . 8}$ |
| $\mathrm{a} / \mathbf{b}$ | $\boldsymbol{\beta}$ | $\boldsymbol{\theta}_{\mathbf{0}}$ | $\boldsymbol{\theta}_{\boldsymbol{0}}$ | $\boldsymbol{\theta}_{\boldsymbol{0}}$ | $\boldsymbol{\theta}_{\boldsymbol{0}}$ |
| $\mathbf{0 . 5}$ | 104.5 | 42.1 | 34.0 | 28.6 | 24.8 |
| $\mathbf{0 . 7}$ | 110.5 | 45.1 | 36.2 | 30.4 | 26.3 |
| $\mathbf{1 . 0}$ | 120.0 | 50.3 | 39.9 | 33.4 | 28.8 |
| $\mathbf{1 . 3}$ | 130.5 | 56.9 | 44.3 | 36.7 | 31.6 |
| $\mathbf{1 . 5}$ | 138.6 | 62.8 | 47.8 | 39.4 | 33.8 |

Should be noted that when the feeding amplitude is the same for the three elements then it is needed a $120^{\circ}$ feeding phase shifts.

## Network Designs

Different MBFM schemes for the radiant elements will be described. Designs will be based on hybrid couplers and fixed phase shifters.

## A. Butler Network with odd number of inputs

As Shelton studied in [3], one of the possible Butler matrices may consist of a combination of three coupler circuits and two fixed phase shifters. Figure 2 shows a scheme when uniform amplitude is applied. Two equilibrated coupler and another non equilibrated can be seen. Hereby, if a uniform output power is desired, a coupler non equilibrated ( $2 / 3$ direct and $1 / 3$ coupled) is needed.


Fig.5: Scheme of a three-port network

## B. Three-Port Symmetric Network

A rotation symmetric scheme can be set in order to get a perfectly symmetrical behavior between inputs and outputs. Figure 3 shows a scheme of the network with non equilibrated $90^{\circ}$ couplers.


Fig.5: Scheme of a three-port symmetric network

In particular, if we use -3 dB hybrid couplers ( $\mathrm{c}=0.5$ ), $20.7^{\circ}$ or $159.3^{\circ}$ and $45^{\circ}$ or $135^{\circ}$ phase shifts are obtained, hence, amplitude and phase relations are:

| Case a) $0.707_{0^{\circ}}$ | $0.707_{0^{\circ}}$ | $1_{110.7^{\circ}}$ |  |
| :--- | ---: | ---: | ---: |
| Case b) | $1_{0^{\circ}}$ | $1_{0^{\circ}}$ | $0.707_{110.7^{\circ}}$ |



Fig.7: Radiation pattern of the network with -3 dB couplers. Feedings related to case a) and b). Curves of -1 and -3 dB .

## C. Scattering Networks with non Orthogonal Pattern

When the number of beams is higher than the number of elements of the array, it is needed to work with scattering networks that have no orthogonal restriction between output vectors.

If we need to get four beams using only a triangular array of three radiating elements, one of which have a normal steering direction to the array and the other three are equidistant with respect to the principal steering direction which have and angle $\theta_{0}$ respect to the normal direction. In order to get this network we can design a $4 \times 4$ Butler matrix loading one of the outputs with the adaptive load as shown in Figure 5.


Fig.8: 4x4 Butler matrix

## Beam-Forming Network Implementation

A prototype of a Butler matrix with odd number, shown in figure 5, has been built in microstrip technology. Both, fixed phase shifters and hybrid coupled has been designed in microstrip transmission lines. Figure 9 shows the $3 \times 3$ beamforming network board.


Fig.9: $3 \times 3$ odd Butler matrix

## Beam-Forming Network Measurements

Firstly, adaptation and coupling ports are measured. In the figures bellow it is shown the scattering parameters at the output when the beam-forming network is fed by port 1 (a), port 2 (b) and port 3 (c),


Fig.10: Scattering parameters

As expected, at 1.7 GHz amplitude power is the same for all the three output ports. Also, a $120^{\circ}$ phase shift is found when is needed.

This network has also been connected to a subarray of three elements and it has been measured. A spherical compact range has been used to acquire a full field acquisition for the three input ports. The radiation pattern obtained is shown in figure 11 (azimuth) and 12 (elevation).


Fig.11: Azimuth radiation pattern


Fig.12: Elevation radiation pattern
As it is seen in figures 11 and 12 a multi-beam forming network has been designed. Steering directions for the three beams are $0^{\circ}, 120^{\circ}$ and $240^{\circ}$ azimuth for each of them and $23^{\circ}$ elevation for the three of them. Thus, radiation patterns obtained are the expected and therefore the implemented network is working properly.

## References

[1] Sierra Pérez, M, A. Torres, J.L. Masa, M. Gómez, I. Montesinos, "GEODA: Adaptive Antenna Array for Satellite signal Reception Antennas and Propagation" in Proceedings of The Second European Conference on Antennas and Propagation, EuCAP 2007. Edinburgh, Scotland, November 2007, 1-4.
[2] J. Butler and all "Beamforming matrix simplifies design of electronically scanned antennas", Electronic Design, Vol. 9, pp. 170173, April 1961.
[3] Shelton, J. P., "Multibeam, hexagonal, triangular-grid, planar arrays," Antennas and Propagation Society International Symposium, vol.3, 9097, 1965.
[4] Moody-The Systematic Design of the Butler Matrix IEEE trans. On Antennas and Propagation Nov. 1964.
[5] R. J. Mailloux. Phase Array Antenna Handbook. Artech House 1994.

