Thermodynamically consistent dynamic formulation of discrete thermoviscoelastic elements

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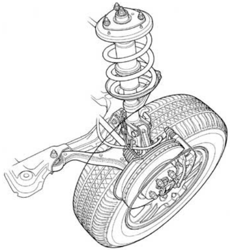
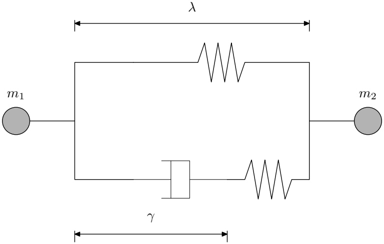
Motivation

- Multibody systems incorporating discrete viscoelastic elements. Example: vehicle shock absorber devices, spatial mechanisms, etc.





- Temperature effects
  - Heat generated by viscous dissipation can flow to the environment and/or change the temperature of the element itself
  - Damping and stiffness may depend on temperature
  - Thermal expansion
  - ...

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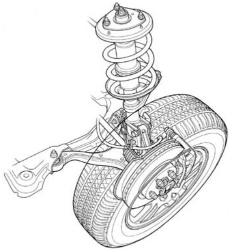

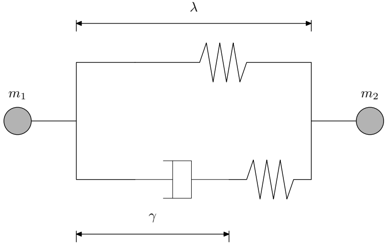

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Formulation

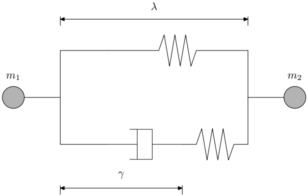
Positions: $\mathbf{q}_1, \mathbf{q}_2$

Length: $\lambda = |\mathbf{q}_1 - \mathbf{q}_2|$

Viscous stretch: γ

Element temperature: θ

Ambient temperature: θ_r
(constant)



- Free energy function [1] :

$$\psi(\lambda, \gamma, \theta) = \psi^{\infty}(\lambda, \theta) + \Gamma(\lambda, \gamma, \theta)$$

- Legendre transform: $\psi(\lambda, \gamma, \theta) \rightarrow e(\lambda, \gamma, s)$

$$\text{with} \quad s = -\frac{\partial\, \psi(\lambda, \gamma, \theta)}{\partial\, \theta} \quad \rightarrow \quad \theta = \hat{\theta}(\lambda, \gamma, s)$$

- Internal energy:

$$e(\lambda, \gamma, s) = \psi(\lambda, \gamma, \hat{\theta}(\lambda, \gamma, s)) + \hat{\theta}(\lambda, \gamma, s)s$$

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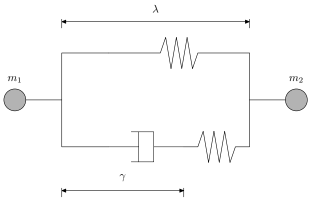

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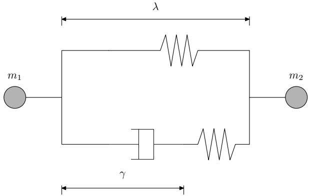

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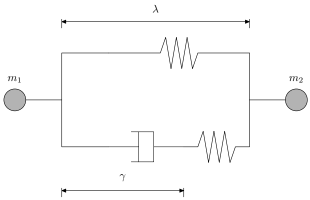

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Formulation

- Evolution equations in entropy form

$$\dot{\mathbf{q}}_1 = \frac{1}{m_1}\mathbf{p}_1$$
$$\dot{\mathbf{q}}_2 = \frac{1}{m_2}\mathbf{p}_2$$
$$\dot{\mathbf{p}}_1 = -f\,\frac{\mathbf{q}_1 - \mathbf{q}_2}{\lambda}$$
$$\dot{\mathbf{p}}_2 = -f\,\frac{\mathbf{q}_2 - \mathbf{q}_1}{\lambda}$$
$$\dot{\gamma} = \frac{1}{\eta}g$$
$$\dot{s} = \frac{1}{\theta}\left(\frac{g^2}{\eta} - h\right)$$

with

$$f = \frac{\partial e}{\partial \lambda}\,, \qquad g = -\frac{\partial e}{\partial \gamma}\,, \qquad \theta = \frac{\partial e}{\partial s}\,, \qquad h = c(\theta - \theta_r)$$

damping of dashpot: $\eta(\theta)$ thermal conductivity: $c > 0$

Formulation

This continuous mathematical model satisfies:

- Non-negative dissipation (Clausius-Plank) condition:

$$\mathcal{D} = f\dot{\lambda} - \dot{\psi} - s\dot{\theta} = g^2/\eta \geq 0$$

- First law of thermodynamics: total energy of a closed system (element + ambient) must be constant
- Second law of thermodynamics: total entropy of a closed system (element + ambient) must not decrease.
- Symmetries: Conservation of linear and angular momentum (under free motion)

We say that a discrete model that satisfies the previous relations is thermodinamically consistent

Standard time integration methods may not be thermodinamically consistent for moderate time steps.

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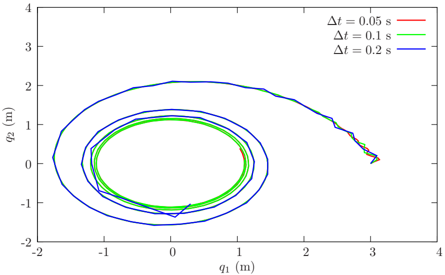
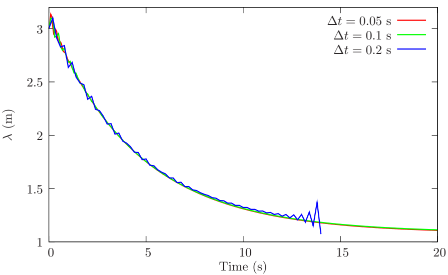
Numerical experiment

Standard integrator (midpoint rule)

Single particle with mass m connected by a thermoviscoelastic element with length 1 m to a fixed point in space.



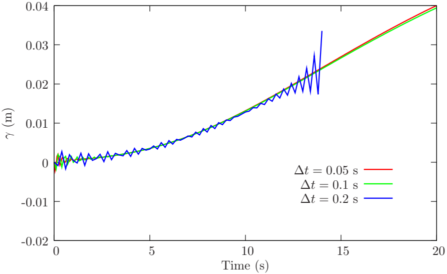
Trajectory



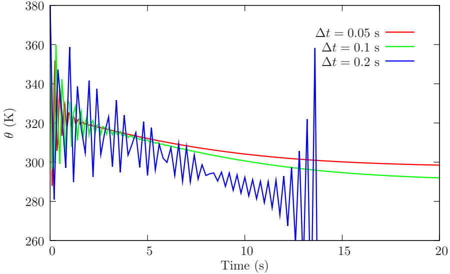
Length (λ) vs. time

Numerical experiment

Standard integrator (midpoint rule)



Viscous stretch (γ) vs. time



Temperature (θ) vs. time

Numerical experiment

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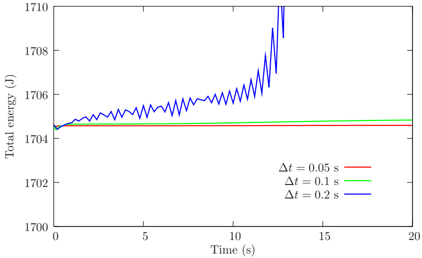
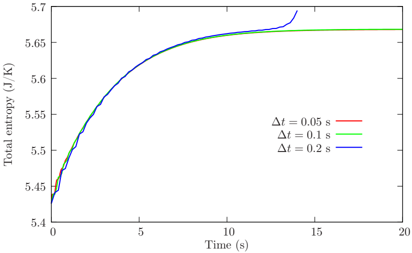


Total energy (E) vs. time



Total entropy (S) vs. time

Objective and methodology

- **Objective**: For discrete thermoelastic elements, define a time integration scheme accurate and thermodinamically consistent

  Expected superior stability and long term accuracy compared standard integrators.

- **Methodology**: Geometric or structure preserving integrator. (*warning: non Hamiltonian system !*)

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Methodology

Hamiltonian systems

$\longrightarrow$ Hamilton equations ($\dot{z} = \mathbf{J}\nabla H$)

$\longrightarrow$ Energy-Momentum methods

Non-Hamiltonian systems

$\longrightarrow$ GENERIC [2] ($\dot{z} = \mathbf{L}\nabla E + \mathbf{M}\nabla S$)

$\longrightarrow$ Energy-Entropy-Momentum methods [3]

First time applied to a dissipative system with internal variables.

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Energy-Entropy-Momentum (EEM)

- Discrete derivative operator. [4] in a partitioned case, for a smooth $f : \mathbb{R}^N \to \mathbb{R}$:

$$\mathsf{D} f(\boldsymbol{x}, \boldsymbol{y}) \cdot \boldsymbol{u} = \sum_{i=1}^{N} \mathsf{D}^i f(x_i, y_i) \cdot u_i \quad , \quad \text{for } \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^N, \boldsymbol{u} \in \mathbb{R}^N$$

verifying:

Directionality $\quad \mathsf{D} f(\boldsymbol{x}, \boldsymbol{y}) \cdot (\boldsymbol{y} - \boldsymbol{x}) = f(\boldsymbol{y}) - f(\boldsymbol{x})$

Consistency $\quad \mathsf{D} f(\boldsymbol{x}, \boldsymbol{y}) = Df\left(\frac{\boldsymbol{x} + \boldsymbol{y}}{2}\right) + \mathcal{O}(||\boldsymbol{y} - \boldsymbol{x}||)$

Operator D denotes the standard derivative.

[4] González, O. *Design and analysis of conserving integrators for nonlinear hamiltonian systems with symmetry*. PhD Thesis, Stanford Univ., Dep. of Mechanical Engineering (1996)

EEM scheme

$$\dot{\mathbf{q}}_1 = \frac{1}{m_1}\mathbf{p}_1 \qquad\longrightarrow\qquad \frac{\mathbf{q}_{1,n+1} - \mathbf{q}_{1,n}}{\Delta t} = \frac{1}{m_1}\mathbf{p}_{1,n+1/2}$$

$$\dot{\mathbf{q}}_2 = \frac{1}{m_2}\mathbf{p}_2 \qquad\longrightarrow\qquad \frac{\mathbf{q}_{2,n+1} - \mathbf{q}_{2,n}}{\Delta t} = \frac{1}{m_2}\mathbf{p}_{2,n+1/2}$$

$$\dot{\mathbf{p}}_1 = -f\frac{\mathbf{q}_1 - \mathbf{q}_2}{\lambda} \qquad\longrightarrow\qquad \frac{\mathbf{p}_{1,n+1} - \mathbf{p}_{1,n}}{\Delta t} = -f^*\frac{(\mathbf{q}_1 - \mathbf{q}_2)_{n+1/2}}{\lambda_{n+1/2}}$$

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$$\dot{\gamma} = \frac{1}{\eta}g \qquad\longrightarrow\qquad \frac{\gamma_{n+1} - \gamma_n}{\Delta t} = \frac{1}{\eta^*}g^*$$

$$\dot{s} = \frac{1}{\theta}\left(\frac{g^2}{\eta} - h\right) \qquad\longrightarrow\qquad \frac{s_{n+1} - s_n}{\Delta t} = \frac{1}{\theta^*}\left(\frac{{g^*}^2}{\eta^*} - h^*\right)$$

with

$$f = \partial e/\partial\lambda \qquad\longrightarrow\qquad f^* = \mathsf{D}^{\lambda}e(\lambda_n, \lambda_{n+1})$$

$$g = -\partial e/\partial\gamma \qquad\longrightarrow\qquad g^* = -\mathsf{D}^{\gamma}e(\gamma_n, \gamma_{n+1})$$

$$\theta = \partial e/\partial s \qquad\longrightarrow\qquad \theta^* = \mathsf{D}^{s}e(s_n, s_{n+1})$$

and $\eta = \eta(\theta)\ ,\ \ h = c(\theta - \theta_r) \longrightarrow \eta^* = \eta(\theta^*)\ , \qquad h^* = c(\theta^* - \theta_r)$

EEM Scheme

A closer look at the partitioned discrete derivative:

$$\mathsf{D}^{\lambda} e(\lambda_n, \lambda_{n+1}) = \frac{e(\lambda_{n+1}, \gamma_{n+1}, s_{n+1}) - e(\lambda_n, \gamma_{n+1}, s_{n+1})}{2(\lambda_{n+1} - \lambda_n)} + \frac{e(\lambda_{n+1}, \gamma_n, s_n) - e(\lambda_n, \gamma_n, s_n)}{2(\lambda_{n+1} - \lambda_n)}$$

$$\mathsf{D}^{\gamma} e(\gamma_n, \gamma_{n+1}) = ...$$

$$\mathsf{D}^{s} e(s_n, s_{n+1}) = ...$$

EEM Scheme

It can be shown that this scheme satisfies:

- Non-negative viscous dissipation: $\mathcal{D}_n = g_n^2/\eta_n \geq 0 \; , \; \forall n$
- First law of thermodynamics in discrete form: $E_{n+1} - E_n = 0$

  $$\text{with} \quad E = \frac{1}{2m_1}\,\mathbf{p}_1^2 + \frac{1}{2m_2}\,\mathbf{p}_2^2 + e(\lambda(\mathbf{q}_1, \mathbf{q}_2), \gamma, s) + \sigma\theta_r$$

- Second law of thermodynamics in discrete form: $S_{n+1} - S_n \geq 0$

  $$\text{with} \quad S = s + \sigma$$

- Symmetries: linear and angular momentum (under free motion): $\mathbf{L}_{n+1} - \mathbf{L}_n = \mathbf{0} \; , \quad \mathbf{J}_{n+1} - \mathbf{J}_n = \mathbf{0}$

  $$\text{with} \quad \mathbf{L} = \mathbf{p}_1 + \mathbf{p}_2 \quad \text{and} \quad \mathbf{J} = \mathbf{q}_1 \times \mathbf{p}_1 + \mathbf{q}_2 \times \mathbf{p}_2$$

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with $$S = s + \sigma$$

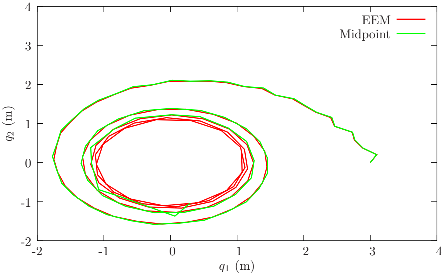
- Symmetries: linear and angular momentum (under free motion):
$\mathbf{L}_{n+1} - \mathbf{L}_n = \mathbf{0} \ , \quad \mathbf{J}_{n+1} - \mathbf{J}_n = \mathbf{0}$

with $\mathbf{L} = \mathbf{p}_1 + \mathbf{p}_2$ and $\mathbf{J} = \mathbf{q}_1 \times \mathbf{p}_1 + \mathbf{q}_2 \times \mathbf{p}_2$

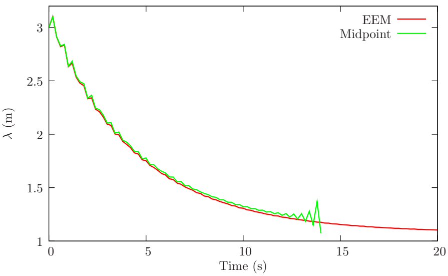
Numerical experiment

EEM vs. midpoint

$\Delta t = 0{,}2$ s



Trajectory

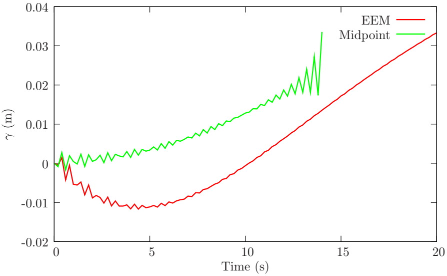


Length (λ) vs. time

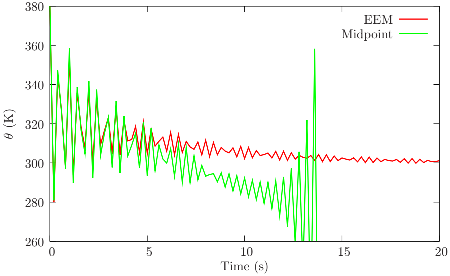
Numerical experiment

EEM vs. midpoint

$\Delta t = 0{,}2$ s



Viscous stretch (γ) vs. time

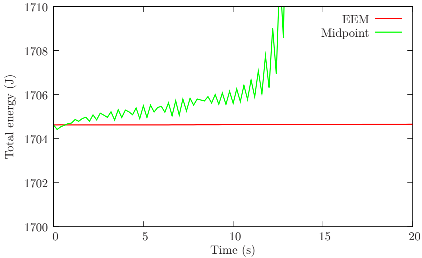


Temperature (θ) vs. time

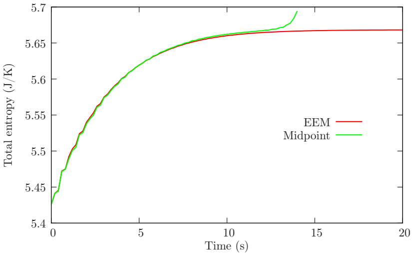
Numerical experiment

EEM vs. midpoint

$\Delta t = 0{,}2$ s

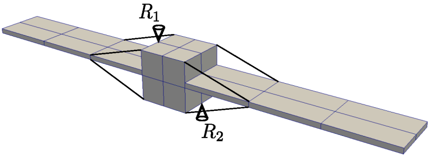


Total energy (E) vs. time

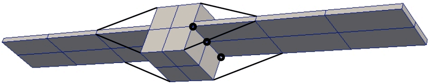


Total entropy (S) vs. time

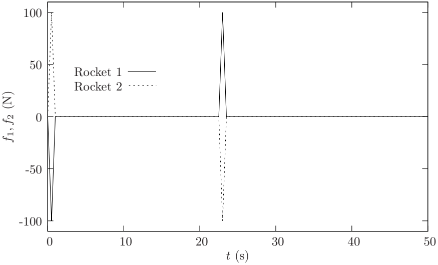
Application: satellite maneuver



(a) Upper view, showing the rockets R_1 and R_2



(b) Lower view, showing the hinges of the right solar panel with the satellite body.



Rocket forces vs. time

[movie]

Application: satellite maneuver

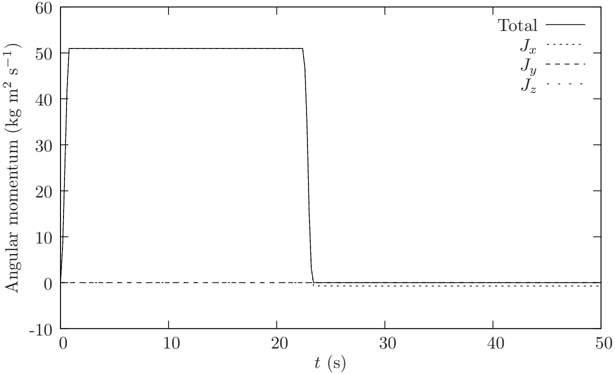


Figura: Angular momentum vs. time. EEM, $\Delta t = 0{,}2$ s

Application: satellite maneuver

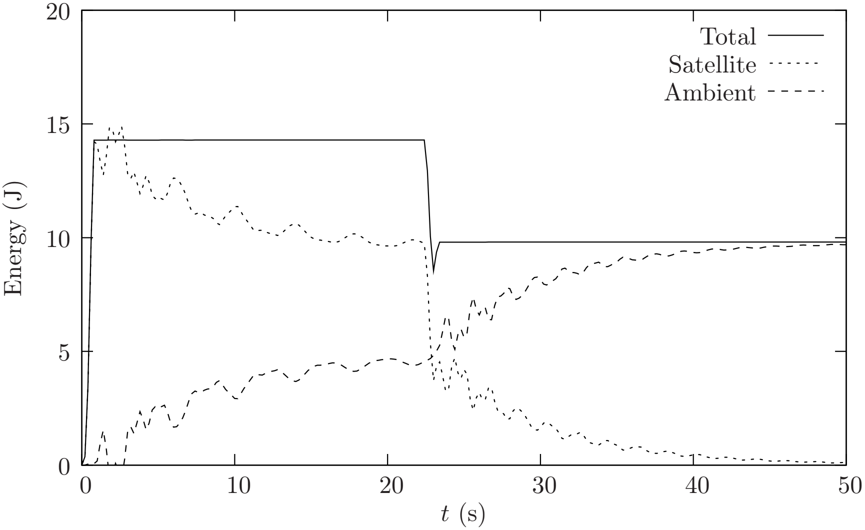


Figura: Energy vs. time. EEM, $\Delta t = 0{,}2$ s

Application: satellite maneuver

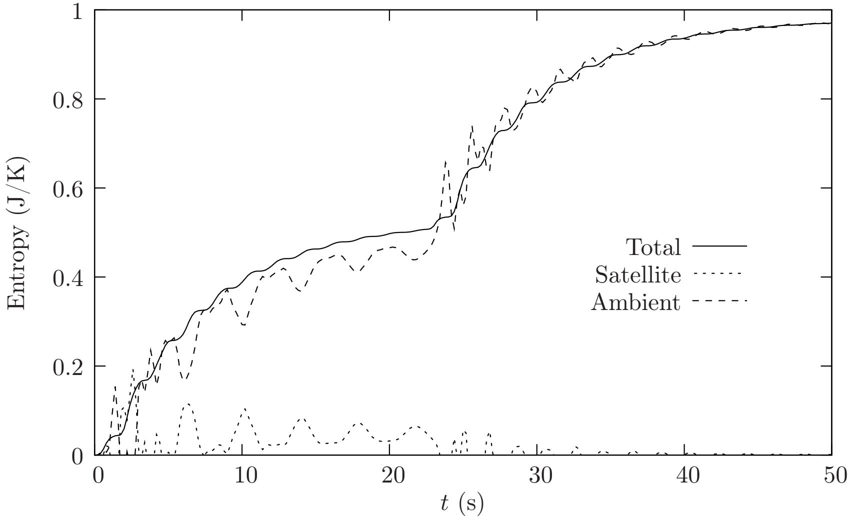


Figura: Entropy vs. time. EEM, $\Delta t = 0{,}2$ s

Application: satellite maneuver

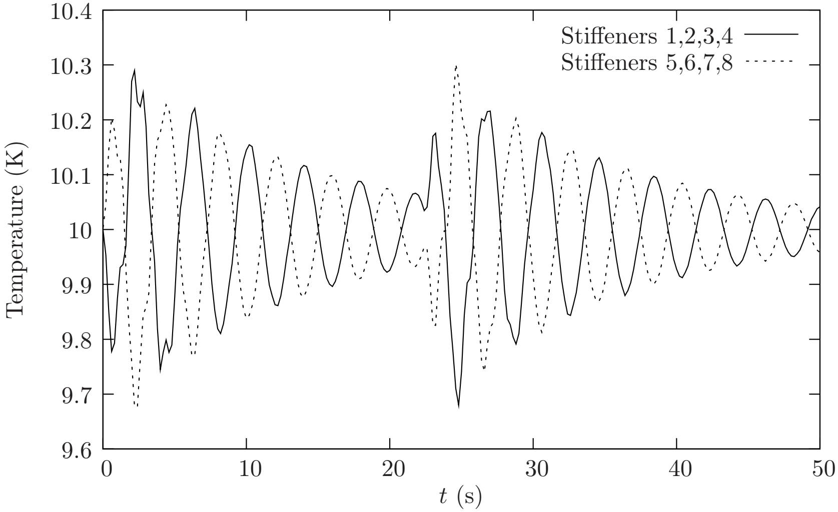


Figura: Stiffeners' temperature vs. time. EEM, $\Delta t = 0{,}2$ s

Conclusions

- Presented novel procedure to represent the nonlinear dynamics of a discrete viscoelastic element with temperature effects
- Large displacements, non-linear elastic behaviour and temperature-dependent mechanical properties
- Energy-entropy-momentum method: complies with first and second law of thermodynamics in a discrete form and preserves symmetries.
- First time applied to a dissipative system with internal variables.
- Numerical experiments suggest that proposed method possesses superior stability compared to typical implicit methods
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Thermodynamically consistent dynamic formulation of discrete thermoviscoelastic elements

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