Eigenstructure Assignment Based Controllers Applied to Flexible Spacecraft

Crespo, J.*,Aranda, J.**

** Senior Professor at the UPM (Universidad Politécnica de Madrid), Madrid, Spain. javier.crespo@upm.es*

*** Senior Professor at the UNED (Universidad Nacional de Educación a Distancia), Madrid, Spain. jaranda@uned.dia.es*

Abstract

The objective of this paper is to evaluate the behaviour of a controller designed using a parametric Eigenstructure Assignment method and to evaluate its suitability for use in flexible spacecraft. The challenge of this objective lies in obtaining a suitable controller that is specifically designated to alleviate the deflections and vibrations suffered by external appendages in flexible spacecraft while performing attitude manoeuvres. One of the main problems in these vehicles is the mechanical cross-coupling that exists between the rigid and flexible parts of the spacecraft. Spacecraft with fine attitude pointing requirements need precise control of the mechanical coupling to avoid undesired attitude misalignment. In designing an attitude controller, it is necessary to consider the possible vibration of the solar panels and how it may influence the performance of the rest of the vehicle. The nonlinear mathematical model of a flexible spacecraft is considered a close approximation to the real system. During the process of controller evaluation, the design process has also been taken into account as a factor in assessing the robustness of the system.

Keywords: Eigenstructure Assignment; eigenvalue; eigenvector; robustness; flexible spacecraft.

1. Introduction

One interest of researchers and engineers resides in the method of obtaining a high level of mitigation in the interaction between rigid and deformation modes in flexible spacecraft. The controllers used in these vehicles may influence this interaction, which provides an acceptable level of decoupling between system modes. Solving these aspects through the use of specific controllers will facilitate the achievement of decoupled motions between the two vehicle modes. Therefore, any attitude motion around any spacecraft axis will have only minor repercussions on appendage deformation in the spacecraft. Conversely, any deformation of flexible appendages of the spacecraft will have a minor repercussion on its attitude movement.

One of the major tasks of the Attitude and Control Subsystem in any three axis controlled spacecraft is the selection of the controller design method. In this work, we focus on the design and implementation of a controller based on the so-called Eigenstructure Assignment (EA) method.

The method of EA has been attractive to researchers and control engineers in recent decades. The incipient work presented in [1] may be considered the first effort towards developing this technique. This manuscript describes the assignment of eigenvalues and controllability determination applied to multiple-input/multiple-output (MIMO) systems with state feedback. As the technique has progressed, eigenvectors have played a role in influencing the design of the controllers [2]. The relationship between the eigenvalue and the eigenvector in obtaining output feedback controllers has been studied in [3]. Furthermore, it is worth mentioning the method used in [4] to calculate controllers using both state and output feedback. The problem of a lack of toolboxes related to the EA method has been solved by another research group, as described in [5] and [6]. This group has developed suitable toolboxes that can be used in controllers designed using EA methods.

The EA method has been widely applied to the aerospace segment, as illustrated in the following examples. In the field of aeronautics, we wish to acknowledge the work carried out by the GARTEUR group, which has implemented various methods to control a transport aircraft [7] and [8]. Additionally, technical applications of the EA method to helicopter controller design can be found in [9], where the main objective is to achieve acceptable short-term attitude command response and appropriate mode decoupling for the vehicle. The technique developed in [10] involves the design of a controller using an EA method in which the objective is to obtain a robust system by taking into consideration the variation of the system parameters and the dynamics of a helicopter. Other complex aircraft applications involve the control of tailless aircraft using the EA method as described in [11].

The application of the EA technique has also received a great deal of attention from the space sector. The method described in [12] explains the applicability of the EA technique in spacecraft launchers to assess the cross-coupling of the system and how this is solved using a suitable controller. Other applications, such as those described in [13], involve the design of an adaptive controller by combining a gain scheduling approach with the EA method. Recent studies have described the design of a controller for a transport aircraft controlled by FBW [14].

The lack of robustness of the controllers designed via the EA method has been a matter of concern for control researchers. This has led to the development of several techniques, as summarised in [15] and [16]. Recent work focusing on increasing the robustness of the EA technique by improving the orthogonality of the system eigenstructure has appeared in control literature [17]. Last, some researchers have compared the possibilities of the EA technique with other methods, such as LQG [18].

Most attitude control problems in flexible spacecraft are related to the passive damping control of the structural modes. The flexible modes in these vehicles are often stabilised using collocated sensors and roll-off or notch filters in the control channel. The main contributions of this paper are focused on designing a static controller $\boldsymbol{K}$ through the application of the EA method and on illustrating the process of obtaining suitable decoupling between the structural and orbital modes of the spacecraft. We considered on our mathematical model several elastic modes that researchers usually not include in their models. This consideration allows us to obtain a more realistic spacecraft model.

2. Basic Aspects of the Eigenstructure Assignment method

One of the main requirements for developing and designing a controller using the EA method is detailed knowledge of the system modes. Application of the EA method enables the design of both dynamic and static feedback controllers applied to state and output feedback respectively. Another requirement is related to the behaviour of the system in a closed loop. In applying different techniques to obtain the final control law $\boldsymbol{K}$, the EA method provides at least two important advantages. The first advantage is related to the decoupling of the modes; i.e., it minimises the interaction between the two modes. The second advantage is related to eigenvector orthogonality, where a higher level of orthogonality in eigenvectors corresponds to greater system robustness. Ideal solutions that cover all of the aforementioned aspects are of primary interest in this application. While precise decoupling between the orbital and flexible modes is required, the designed system must also be robust.

We consider the following linear system with n states, m inputs and p outputs:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \qquad (1)$$

where $\boldsymbol{x}$ is the vector of states, $\boldsymbol{y}$ is the vector of measurements and $\boldsymbol{u}$ is the control vector. The state matrix is $\boldsymbol{A} \in \mathfrak{R}^{nxn}$, the control matrix is $\boldsymbol{B} \in \mathfrak{R}^{nxm}$, the matrix of observable states is $\boldsymbol{C} \in \mathfrak{R}^{pxn}$, and the matrix that relates inputs and outputs is $\boldsymbol{D} \in \mathfrak{R}^{pxn}$.

The eigenstructure is represented by the eigenvalues and eigenvectors. The eigenvalues represent the system modes and the eigenvectors provide information about the existing system modes. The linear algebra associated with the system is expressed as follows:

$$\begin{aligned} \boldsymbol{A}\boldsymbol{v}_i &= \lambda_i \boldsymbol{v}_i \\ \boldsymbol{w}_i^T \boldsymbol{A} &= \lambda_i \boldsymbol{w}_i^T \end{aligned} \qquad (2)$$

where the eigenvalues are given by λ_i, the right eigenvector is given by a set of $\boldsymbol{v_i}$ and the left eigenvector is given by a set of $\boldsymbol{w_i^T}$. The set of eigenvectors and eigenvalues is expressed in matrix form as follows:

$$\begin{aligned}\Lambda &= [\lambda_1, ..., \lambda_n] \\ \boldsymbol{V} &= [v_1, ..., v_n] \\ \boldsymbol{W}^T &= [w_1, ..., w_n]\end{aligned} \tag{3}$$

The time response of the system in equation (1) is expressed as follows:

$$\boldsymbol{y}(t) = \boldsymbol{C}\sum_{i=1}^{n} \boldsymbol{v}_i e^{\lambda_i t} \boldsymbol{w}_i^T \boldsymbol{x}(0) + \sum_{i=1}^{n} \boldsymbol{v}_i \boldsymbol{w}_i^T \int_0^t \boldsymbol{C} e^{\lambda_i (t-\tau)} \boldsymbol{Bu}(\tau) d\tau \tag{4}$$

This expression shows the existing relation between the time response of the system and its eigenvalues, the right and left eigenvectors, system inputs and initial conditions. In addition, the homogeneous component is given by the following equation:

$$\boldsymbol{y}(t) = \boldsymbol{C}\sum_{i=1}^{n} \boldsymbol{v}_i e^{\lambda_i t} \boldsymbol{w}_i^T \boldsymbol{x}(0) \tag{5}$$

This term denotes the transient response of the system, which is characterised by the eigenvalues, the right eigenvectors and the product of the left eigenvectors and the initial condition $\boldsymbol{x}(0)$. In expression (5) the term designated as $\alpha_i = \boldsymbol{w}_i^T \boldsymbol{x}(0)$ is a scalar, and the relation given by $\beta_i(t) = \alpha_i e^{\lambda_i t}$ represents the mathematical system modes. The mathematical modes have corresponding system modes, which for flexible spacecraft are called orbital and flexible modes. The real system shows physical coupling between the system modes. In expression (5), the system coupling is given by the value of the product of $\boldsymbol{Cv}_i$.

The control problem in this paper is focused on obtaining a static controller given by $\boldsymbol{K}_{(\mathrm{mxp})}$. Assuming that whole states are available, the control law can be expressed as follows:

$$\boldsymbol{u} = -\boldsymbol{Ky} = -\boldsymbol{KCx} \tag{6}$$

Several techniques have been developed to design suitable controllers according to a desired eigenstructure for the closed loop system. In general, the techniques used are focused on parametric and low sensitivity eigenstructure assignment. The first method considers the value of the eigenvalues in order to establish the best method to solve the problem. The second method addresses the problem through a recursive approach, taking the robustness of the system as the main objective. For the purpose of this paper, it is interesting to consider a parametric eigenstructure assignment and to perform some robustness testing to validate the controller design versus the requirements. From equations (1) and (6) the closed loop system, considering $\boldsymbol{D} = 0$, is given by the following equation:

$$\begin{aligned}\dot{\boldsymbol{x}} &= (\boldsymbol{A} + \boldsymbol{BKC})\boldsymbol{x} \\ \boldsymbol{y} &= \boldsymbol{Cx}\end{aligned} \tag{7}$$

To assign the closed loop eigenstructure, the method defines a desired eigenstructure as a set of desired eigenvalues Λ_d and desired eigenvectors $\boldsymbol{V}_d$, which is given by the following equation:

$$\Lambda_d = \left[\lambda_{d1}.......\lambda_{dp}\right]$$
$$\boldsymbol{V}_d = \left[v_{d1}......v_{dp}\right] \qquad (8)$$

The output feedback problem is to find a real matrix ***K*** such that the eigenvalues of $\left(\boldsymbol{A}+\boldsymbol{BKC}\right)$ include the desired eigenvalues λ_{di} as a subset of the system eigenvalues and the eigenvectors are as close to the desired eigenvectors v_{di} as possible.

For any pair of desired closed loop eigenvalues λ_i and their associated eigenvectors v_i, equation (7) can be expressed as follows:

$$\left(\boldsymbol{A}+\boldsymbol{BKC}\right)v_i = \lambda_i v_i \qquad (9)$$

where the system eigenvectors are $v_i = \left(\lambda_i \boldsymbol{I}-\boldsymbol{A}\right)^{-1}\boldsymbol{BKC}v_i$. The allowable subspace can be defined by the columns of the matrix $\left(\lambda_i \boldsymbol{I}-\boldsymbol{A}\right)^{-1}\boldsymbol{B}$. The best achievable eigenvector may then be obtained by projection of the desired eigenvector onto the allowable subspace. From equation (9), the state matrix for the closed loop system may be expressed as follows:

$$\left[\boldsymbol{A}-\lambda_i\boldsymbol{I} \quad \boldsymbol{B}\right]\begin{bmatrix} v_i \\ \boldsymbol{KC}v_i \end{bmatrix} = 0 \qquad (10)$$

This may be represented for a non trivial solution as a null space given by the following equation:

$$\begin{bmatrix} v_i \\ \boldsymbol{KC}v_i \end{bmatrix} \in \left\{\aleph : \left[\boldsymbol{A}-\lambda_i\boldsymbol{I} \quad \boldsymbol{B}\right]\right\} \qquad (11)$$

The null space is known as the achievable vector space. Thus, all achievable eigenvectors that correspond to the desired closed loop eigenvalues must lie in the subspace spanned by the columns of $\left[\boldsymbol{A}-\lambda_i\boldsymbol{I} \quad \boldsymbol{B}\right]$. Therefore, the desired eigenvectors can be achieved exactly if they belong to this subspace and if there exists a feedback matrix ***K***.

From equation (10): $\bar{\boldsymbol{w}}_i = \boldsymbol{KC}v_i$ (12)

This vector, called the right parameter vector [6], is related to the computation of the controller as follows:

$$\boldsymbol{K} = \bar{\boldsymbol{W}}\left(\boldsymbol{CV}\right)^{-1} \qquad (13)$$

3. Mathematical Model for Flexible Vehicles

We have developed a mathematical model based on the Newton-Euler dynamic equations. This model considers all possible perturbations affecting the movement of the spacecraft along with the inertia moments for rigid body, flexible panels and reaction wheels. The mathematical model has been developed using three reference frames. The first is the Earth-Centred Inertial (ECI) reference frame. The second is an orbit reference frame, located in the mass centre of the spacecraft. In this frame, the **z**-axis points to the earth centre, the **x**-axis is tangential to the orbit, and the **y**-axis is perpendicular to the orbit plane. The attitude manoeuvres are related to the rotations around **x**, **y** and **z** axes, called roll, pitch and yaw, respectively. These rotations are identified as attitude modes for the purpose of the controller design through the EA method. The final frame is the body reference frame located at the centre of mass of the satellite, coincident with the principal axis of inertia.

Potential and kinetic energies are considered in the derivation of the mathematical model. Additionally, the environmental perturbations are included in the model, along with the necessary forces applied by actuators, by application of Hamilton's principle to the system Lagrangian:

$$\begin{aligned} &\boldsymbol{L} = \boldsymbol{E}_L - \boldsymbol{V}_L \\ &\frac{d}{dt}\left(\frac{\partial \boldsymbol{L}}{\partial \dot{\boldsymbol{u}}}\right) - \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{u}} = \boldsymbol{Q}_{nc} \end{aligned} \tag{14}$$

$\boldsymbol{E}_L$ and $\boldsymbol{V}_L$ are the kinetic and potential system energies, respectively, and $\boldsymbol{Q}_{nc}$ are the non conservative forces applied to the system. While obtaining potential and kinetic energies, the influence of the rigid and flexible part of the spacecraft must be considered.

The elastic movements of the solar panels with respect to the body frame have been modelled by the assumed modes method. The elastic deformation of the panels is modelled as a function of time and several generalised coordinates. The displacements of the solar panels must satisfy the geometric boundary conditions imposed on the system to avoid structural component failure. The solar panels can be considered geometrical rectangular plates, and their modes of vibration can be those of clamped-free and free-free beams.

The elastic displacements at any point on the solar panels are discretised by a series of admissible functions $w_i(x,t)$ and $\upsilon_i(y,t)$, and their associated time-dependent generalised coordinates are represented by $q_i(t)$ for longitudinal bending and $r_i(t)$ for torsional deformations about the longitudinal axis:

$$\begin{aligned} w_i(x,t) &= \sum_i \phi_i(x) q_i(t) \\ \upsilon_i(y,t) &= \sum_i \psi_i(y) r_i(t) \end{aligned} \tag{15}$$

The graphical representations of the solar panel bending depicted in Figure 1 show the bending at any point. From this figure, we can confirm that the solar panel may bend in two dimensions according to the functions described in equation (15). Figure 2 shows the bending of a single solar panel.

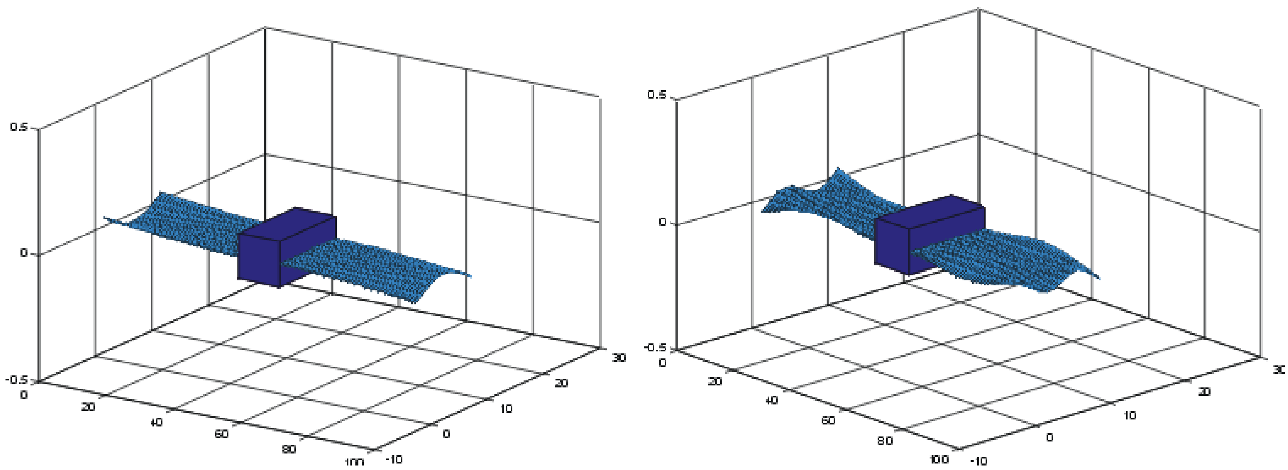
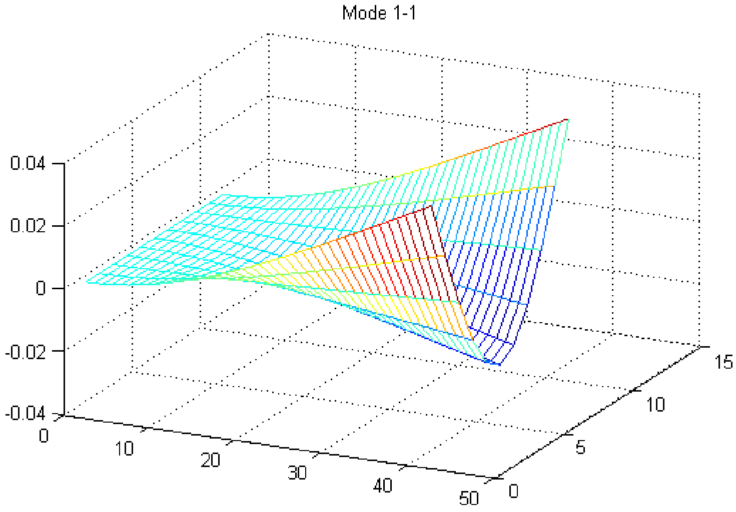
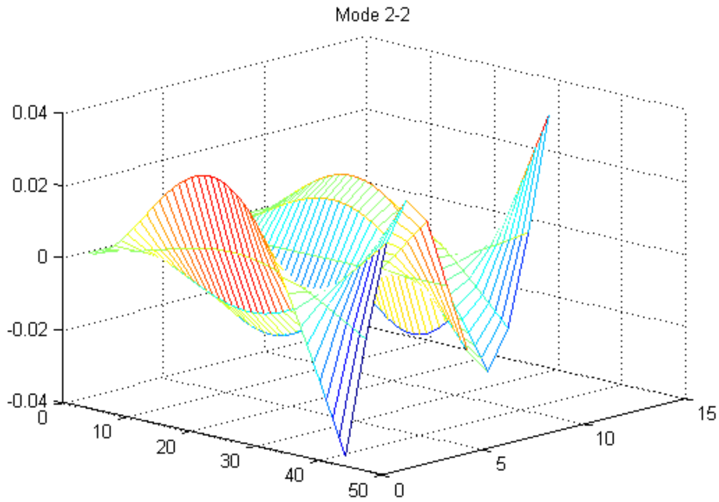


Figure 1: Graphical representation of solar panel bending on the spacecraft model.



(a)



(b)

Figure 2: Elastic displacement of a single solar panel. Vibration modes: (a) mode 1-1, (b) mode 2-2.

Analysing the energies needed to develop the mathematical model, the components of the kinetic energy can be described as those related to the rigid component of the spacecraft motion, together with the flexion and torsional effects of solar panel bending, as follows:

$$\boldsymbol{E}_L = \boldsymbol{E}_{c\_Rigid} + \boldsymbol{E}_{c\_Flexion} + \boldsymbol{E}_{c\_Torsion} \tag{16}$$

This term expressed in matrix form represents the kinetic energy for the entire system, including the rigid body contribution and elastic displacements of solar panels, as follows:

$$\boldsymbol{E}_L = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix} \begin{bmatrix} I_X & 0 & 0 \\ 0 & I_Y & 0 \\ 0 & 0 & I_Z \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} + \sum_{i=1}^{2} \frac{1}{2} \begin{bmatrix} \dot{\phi} & \dot{q}_i \end{bmatrix} \begin{bmatrix} I_{Xi} & Q_{Xi} \\ Q_{Xi} & m_i \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{q}_i \end{bmatrix} + \sum_{i=1}^{2} \frac{1}{2} \begin{bmatrix} \dot{\theta} & \dot{r}_i \end{bmatrix} \begin{bmatrix} I_{Yi} & Q_{Yi} \\ Q_{Yi} & J_{Yi} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{r}_i \end{bmatrix} \tag{17}$$

$$\boldsymbol{E}_L = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} & \dot{q}_1 & \dot{r}_1 & \dot{q}_2 & \dot{r}_2 \end{bmatrix} \begin{bmatrix} I_X + I_{X1} + I_{X2} & 0 & 0 & Q_{X1} & 0 & Q_{X2} & 0 \\ 0 & I_Y + I_{Y1} + I_{Y2} & 0 & 0 & Q_{Y1} & 0 & Q_{Y2} \\ 0 & 0 & I_Z & 0 & 0 & 0 & 0 \\ Q_{X1} & 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & Q_{Y1} & 0 & 0 & J_{Y1} & 0 & 0 \\ Q_{X2} & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & Q_{Y2} & 0 & 0 & 0 & 0 & J_{Y2} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{q}_1 \\ \dot{r}_1 \\ \dot{q}_2 \\ \dot{r}_2 \end{bmatrix}$$

The total potential energy of the spacecraft is the contribution of the gravitational effect [19] and the elastic potential energy. The gravitational effect of the system is given by the following equation:

$$\boldsymbol{V}_G = -\mu \int_m \frac{dm}{R} \qquad (18)$$

The elastic motions for flexion of solar panel are expressed as follows:

$$\boldsymbol{E}_{D\_Flexion} = \frac{1}{2}\int_0^L EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx \qquad (19)$$

The solar panel torsion is given by the following equation:

$$\boldsymbol{E}_{D\_Torsion} = \frac{1}{2}\int GK \left(\frac{\partial \theta_T}{\partial \xi} \right)^2 d\xi \qquad (20)$$

The total contribution to the potential energy is expressed as follows:

$$\begin{aligned} \boldsymbol{V}_L &= E_{D\_Flexion} + E_{D\_Torsion} = \sum_{i=1}^{2} \frac{1}{2} q_i^2 \int E\overline{I}_X \left(\phi''(\xi) \right)^2 d\xi + \sum_{i=1}^{2} \frac{1}{2} r_i^2 \int GK \left(\psi'(\xi) \right)^2 d\xi \\ \boldsymbol{V}_L &= \sum_{i=1}^{2} \left[\frac{1}{2} K_{Fi} q_i^2 + \frac{1}{2} K_{Ti} r_i^2 \right] \end{aligned} \qquad (21)$$

$$\boldsymbol{V}_L = \begin{bmatrix} \phi & \theta & \psi & q_1 & r_1 & q_2 & r_2 \end{bmatrix} \begin{bmatrix} 4\omega_0^2 (I_Y - I_Z) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3\omega_0^2 (I_X - I_Z) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_0^2 (I_Y - I_X) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{F1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{T1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{F2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{T2} \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ q_1 \\ r_1 \\ q_2 \\ r_2 \end{bmatrix}$$

Applying Hamilton's principle to the Lagrangian $\boldsymbol{L}$ and defining a generalised coordinate vector $\boldsymbol{u}$ using a set of attitude angles together with the corresponding deflections of the solar panels, we can obtain a non linear mathematical model, represented as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{G}\dot{\boldsymbol{u}} + \boldsymbol{K_m u} = \boldsymbol{Q}_{nc} \qquad (22)$$

The elements of the matrices expressed in expanded form for the governing equations of motion are the following:

$$
\boldsymbol{M} = \begin{bmatrix} I_X + I_{X2} + I_{X2} & 0 & 0 & Q_{X1} & 0 & Q_{X2} & 0 \\ 0 & I_Y + I_{Y1} + I_{Y2} & 0 & 0 & Q_{Y1} & 0 & Q_{Y2} \\ 0 & 0 & I_Z & 0 & 0 & 0 & 0 \\ Q_{X1} & 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & Q_{Y1} & 0 & 0 & J_{Y1} & 0 & 0 \\ Q_{X2} & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & Q_{Y2} & 0 & 0 & 0 & 0 & J_{Y2} \end{bmatrix}
$$

$$
\boldsymbol{G} = \begin{bmatrix} 0 & 0 & I_Y - I_Z - I_X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_X + I_Z - I_Y & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\boldsymbol{K_m} = \begin{bmatrix} 4\omega_0^2 \left(I_Y - I_Z\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3\omega_0^2 \left(I_X - I_Z\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_0^2 \left(I_Y - I_X\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{F1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{T1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{F2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{T2} \end{bmatrix}
$$

The ***M*** matrix in expression (22) represents the mass generalised matrix, which includes the inertia moments for rigid body, solar panels and reaction wheel actuators, the generalised moments for solar panel deformations and the generalised mass of the solar panels. The gyroscopic matrix ***G*** includes the results for the calculated moment of inertia. Finally, the stiffness matrix $\boldsymbol{K_m}$ includes the effects of angular movement around the Earth and the damping ratio considerations. The internal damping of the structure is integrated in the mathematical model by adding any suitable value to the mathematical model. The generalised forces $\boldsymbol{Q_{nc}}$ include the non conservative forces applied to the spacecraft with relevant repercussions on attitude movement. It should be noted that the actions from internal actuators, such as the reaction wheels, are considered by their angular moments. Therefore, reaction wheel actions, together with external actions, complete the system model, leading to a complete definition of $\boldsymbol{Q_{nc}}$, as follows:

$$
\boldsymbol{Q_{nc}} = \left[\boldsymbol{T}_x \quad \boldsymbol{T}_y \quad \boldsymbol{T}_Z \quad 0_{(n-m)}\right]^T \tag{23}
$$

The vector ***u*** represents the vector of generalised coordinates, which are selected to match the objective of the controller design, as follows:

$$
\boldsymbol{u} = \left[\phi \quad \theta \quad \psi \quad q_1 \quad r_1 \quad q_2 \quad r_2\right]^T \tag{24}
$$

The mathematical model in equation (22) represents the non linear model of the system; i.e., it represents the real model. Figure 3 shows the process followed to obtain the linear model from the non linear system and how it is used to validate the controller. The nonlinear model is linearised around an equilibrium point, considering in this case a fine pointing of the spacecraft to Earth. This requirement leads to the linear model of the system. The relation of the system matrix described in equation (22) with the linear model is expressed as follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & \boldsymbol{I}_{nxn} \\ -\boldsymbol{M}^{-1}\boldsymbol{K}_{\boldsymbol{m}} & -\boldsymbol{M}^{-1}\boldsymbol{G} \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1}\begin{bmatrix} 0_{(n-m)xm} \\ \boldsymbol{I}_m \end{bmatrix} \end{bmatrix} \qquad \boldsymbol{C} = [\boldsymbol{I}] \qquad \boldsymbol{D} = 0 \tag{25}$$

Therefore, the linear system in equation (1) becomes:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{Ax} + \boldsymbol{Bu} \\ \boldsymbol{y} &= \boldsymbol{Cx} \end{aligned} \tag{26}$$

Assuming that the system is controllable and observable and that all states are accessible, the required control law is given by equation (6). In this case, the output feedback system becomes a state feedback system. A particularity of the system in equation (26) is related to the state matrix ***A.*** This matrix is considered ill-conditioned due to the nature of its elements, particularly those related to solar panel vibration. This characteristic may affect the robustness of the system. A state vector represented by the Euler angles and elastic displacements of the solar panels along with their first derivatives is given as follows:

$$\boldsymbol{x} = \begin{bmatrix} \phi & \theta & \psi & q_1 & r_1 & q_2 & r_2 & \dot{\phi} & \dot{\theta} & \dot{\psi} & \dot{q}_1 & \dot{r}_1 & \dot{q}_2 & \dot{r}_2 \end{bmatrix}^T \tag{27}$$

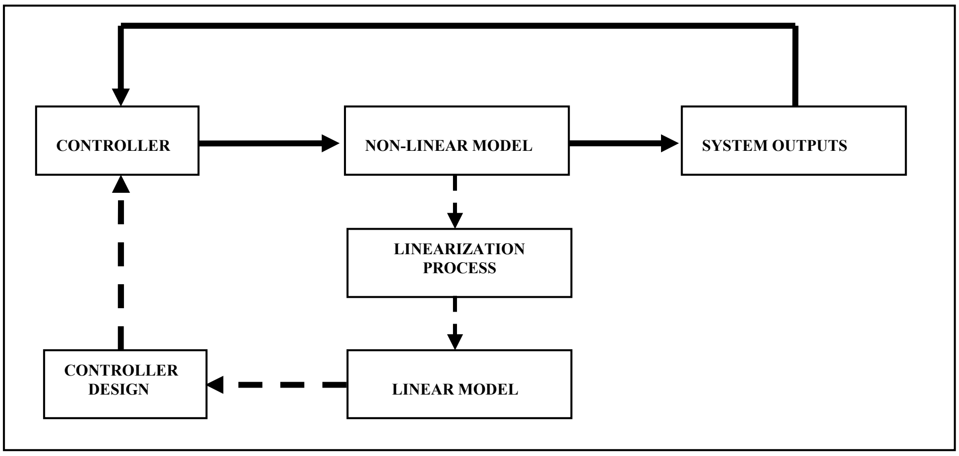


Figure 3: Modelling process.

4. Modal Analysis

The modal analysis performed on the open loop system reveals the main characteristics of the system in terms of natural frequency and damping. This open loop analysis allows the classification of the spacecraft modes into the following two categories: orbital and deformation. This trade-off is performed with the nonlinear system linearised around the equilibrium point. Any motion around the equilibrium point is considered an attitude misalignment that must be corrected to obtain correct attitude pointing.

The data taken from open loop system analysis are shown in Table 1, including the system eigenvalues for different system modes. These data are obtained for a general spacecraft with a solar panel built with the parameters and dimensions belonging to a spacecraft of minisat type, shown in Table 2. These data show a spacecraft that is unstable because of the external configuration of the solar panels.

<table>
<tr><th colspan="2">Dynamics</th><th>Eigenvalues</th><th>Damping</th><th>Natural Frequency (rad/sec)</th></tr>
<tr><td rowspan="3">Orbital Modes</td><td>Roll</td><td>± 2.91e-002</td><td>1.00e+000</td><td>2.91e-002</td></tr>
<tr><td>Pitch</td><td>0 ± 3.24e-002i</td><td>0.0</td><td>3.24e-002</td></tr>
<tr><td>Yaw</td><td>-6.15e-004 ± 3.48e-002i</td><td>1.76e-002</td><td>3.48e-002</td></tr>
<tr><td rowspan="2">Bending Modes</td><td>Flexion</td><td>0 ± 2.77e+001i</td><td>0.0</td><td>2.77e+001</td></tr>
<tr><td>Torsion</td><td>0 ± 4.00e+002i</td><td>0.0</td><td>4.00e+002</td></tr>
</table>

Table 1: Open loop system data.

Parameter	Notation	Value
Solar panel length	*L*	2 m
Appendage Stiffness	*EI*	1.3 e+09
Hub dimensions	*a*	0.6 x 0.6 x 0.6 m
Moments of Inertia	*Ix,Iy,Iz*	14.11, 12.072, 12.60 Kg.m^2

Table 2: Parameters of the simulation model.

The stability of the spacecraft depends primarily on the moment of inertia. This is conditioned by the spacecraft configuration, i.e., the position of the solar panels with respect to the main rigid body of the spacecraft. According to the data in Table 1, the pitch and yaw eigenvalues in this system are stable, whereas the roll eigenvalues are located on the right semi-plane of the complex plane.

5. Requirements for the Controller Design

The problem of attitude manoeuvres is submitted to the design of a regulator where attitude angles and attitude rates lead to a known reference. In rigid spacecraft, this implies a movement of the spacecraft structure in a coordinated manner. However, in flexible spacecraft, the attitude motion to obtain a known position reference may cause the excitation of vibration modes belonging to the solar panels. The alleviation of vibration modes to avoid potential material stress is the main purpose of attitude manoeuvres. This can be obtained via a controller that enables decoupling of the orbital spacecraft motion from the elastic deflections of the solar panels.

The EA method implies the choice of the closed loop eigenvalues and eigenvectors in order to determine the performance of the closed loop system. The following three performance requirements must be identified to design the controller: settling time, overshoot and response time. These system characteristics must be contained in the closed loop desired eigenvalues. Table 3 shows the decoupling criteria followed to obtain real decoupling. The data included in this table affect directly the desired closed loop eigenvectors [20], [21].

Orbital Manoeuvres	Required Decoupling		
Roll	Yaw	Flexion	Torsion
Pitch	-	-	Torsion
Yaw	Roll	Flexion	Torsion

Table 3: Decoupling criteria for system modes.

The decoupling pursued in this paper involves the separation of yaw and pitch manoeuvres from flexion and torsion deformations in solar panels. According to the spacecraft configuration, roll manoeuvres cause flexion deformation in panels because of the inherent coupling that exists between the roll and yaw axes.

With respect to the parametric behaviour of the closed loop system, the criteria followed are based on restricting the overshoot by damping in the system. This requirement affects the closed-loop eigenvalue and the system response time, which under these circumstances is not very fast. The proposed design procedure uses the following steps:

- Identification of the open loop modes and determination of the system stability.
- Identification of the coupling between different system modes.
- Determination of the desired closed loop eigenstructure, eigenvalues and eigenvectors.
- Simulation of the closed loop system with the controller ***K***.

To obtain the desired eigenstructure, it may be necessary to iterate between the last two steps of the procedure to obtain the most suitable controller.

6. Simulations

Based on the mathematical model presented previously, several approximations to a final controller have been performed. One of the major concerns is the behaviour of the bending modes (flexion and torsion) when the spacecraft is performing any attitude manoeuvre. The coupling between flexion and orbital modes may be determined by system modal analysis applied to the closed loop system. Although the potential coupling between the system modes may be understood by splitting the MIMO system into three single-input/single-output (SISO) channels, this solution does not represent the inherent coupling of the real system. The bending modes appear at frequencies of approximately 27 rad/sec and 400 rad/sec.

The EA method of designing the controller uses with two eigenstructures. One is the called the desired eigenstructure, which corresponds to the behaviour of the closed-loop system after the controller has been set up in the loop. The other is called the obtained eigenstructure, which is the actual eigenstructure obtained after application of the EA method. The desired eigenstructure is characterised by the required eigenvalues and eigenvectors. At times when it is not possible to obtain the desired eigenstructure directly, a trial-and-error process must be used to obtain suitable results for the controller. Table 4 lists the desired and obtained eigenvalues and their relationship with system modes. The required desired eigenvalues are selected to give an appropriate damping and time response to the orbital modes, while the deformation modes are required with the same open loop eigenvalue. This is based on the low probability of exciting these modes in performing attitude maneuvers and it is considered as a design strategy on eigenstructure assignment. A close match is observed between the desired and obtained eigenvalues after the iterative process.

Dynamic Modes	Desired Eigenvalues	Obtained Eigenvalues	Damping	Natural Frequency (rad/sec)
Roll	-6.2e-002 ± 3.5e-002i	-6.1457e-002 ± 3.4821e-002i	8.70e-001	7.06e-02
Pitch	-8.7e-002 ± 3.2e-002i	-8.6736e-002 ± 3.2408e-002i	9.37e-001	9.26e-02
Yaw	-2.9e+000 ± 2i	-2.9122e+000 ± 2.0000e+000i	8.24e-001	3.53
Flexion	0 ± 2.7746e+001i	0 ± 2.7746e+001i	0	2.77e+001
Torsion	0 ± 4.0000e+002i	0 ± 4.0000e+002	0	4.00e+001

Table 4: Desired eigenvalues and the values obtained after the iterative process.

The pole-zero map depicted in Figure 4 shows the position of the eigenvalues in the complex plane for both open and closed loop systems. It is interesting to note that some eigenvalues move to new positions according to the desired eigenstructure, whereas other eigenvalues stay in the same position, as in the open loop system. Specifically, the flexion and torsion eigenvalues are located in the same position as in the open loop system. This result does not imply any problem in stability because of the difficulty in exciting such frequencies once the system has been decoupled.

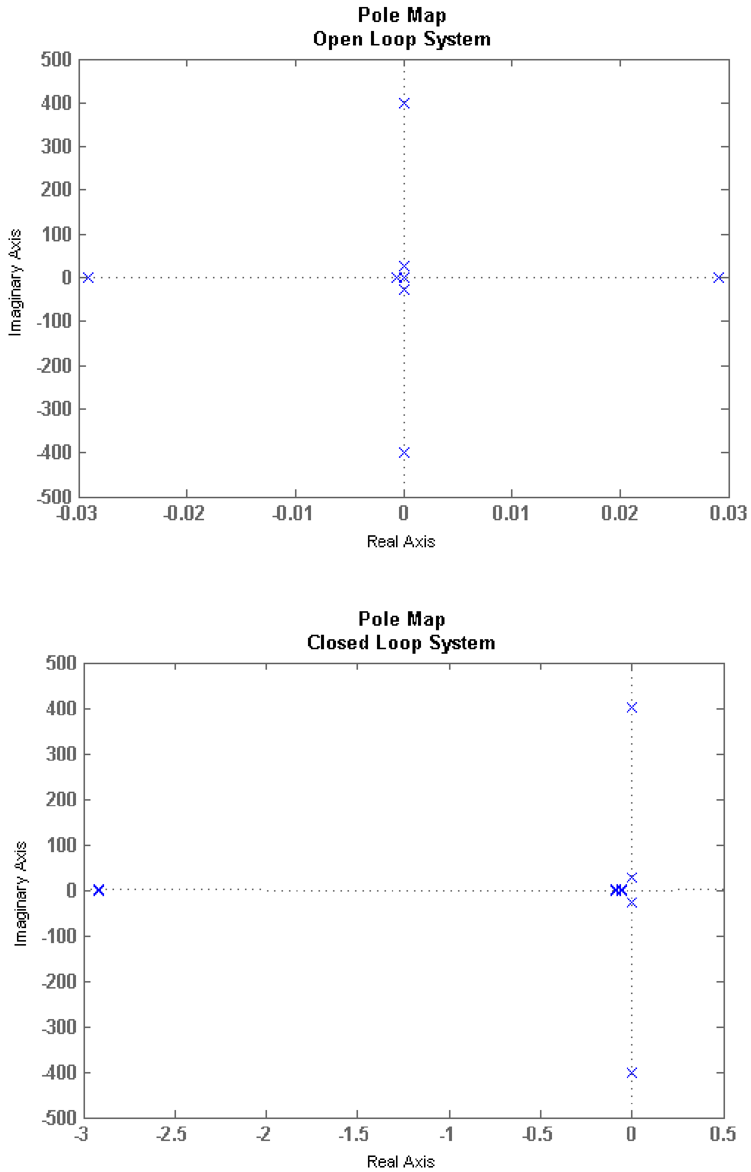
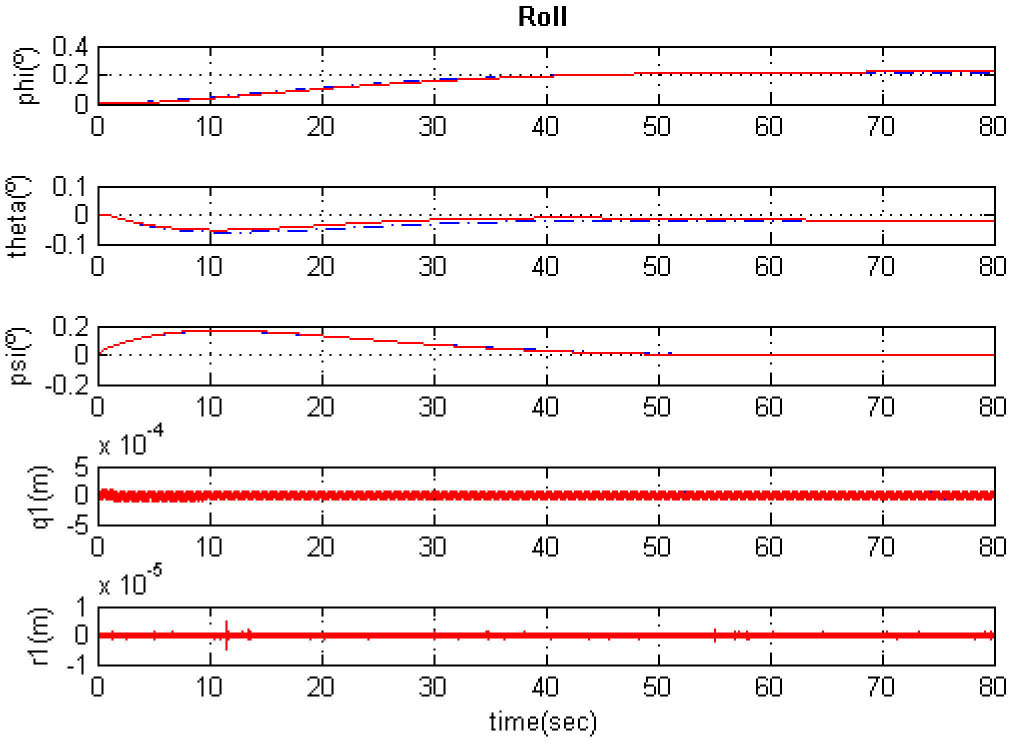


Figure 4: Open and Closed System Eigenvalues.

The obtained eigenstructure is completed by the closed loop eigenvectors. In this type of system, the eigenvectors provide information about the coupling of the system modes and represent the vibration modes in modal coordinates. Other aspects, such as the orthogonality of the eigenvectors, are related to the system robustness [17].

In order to verify whether the main requirement has been imposed on the system, i.e., whether the system decoupling has been obtained as desired, a step simulation has been performed for both linear and non-linear systems. Figure 5 shows the results of these tests. The graph shows that the behaviour of linear and non-linear systems is similar for different correction manoeuvres. Figure 5 also shows that the decoupling between the orbital and bending modes has been obtained. Thus, a roll manoeuvre that is decoupled from the flexion mode in the real system will not cause solar panel bending. This situation is also applicable to solar panel bending in the transverse motion. A relative decoupling of the orbital modes has also been obtained with the desired eigenstructure. Therefore, a motion around any axis of the spacecraft induces only light movement in the remaining axes. This situation is not of significant concern due to the time response obtained for the closed loop system.



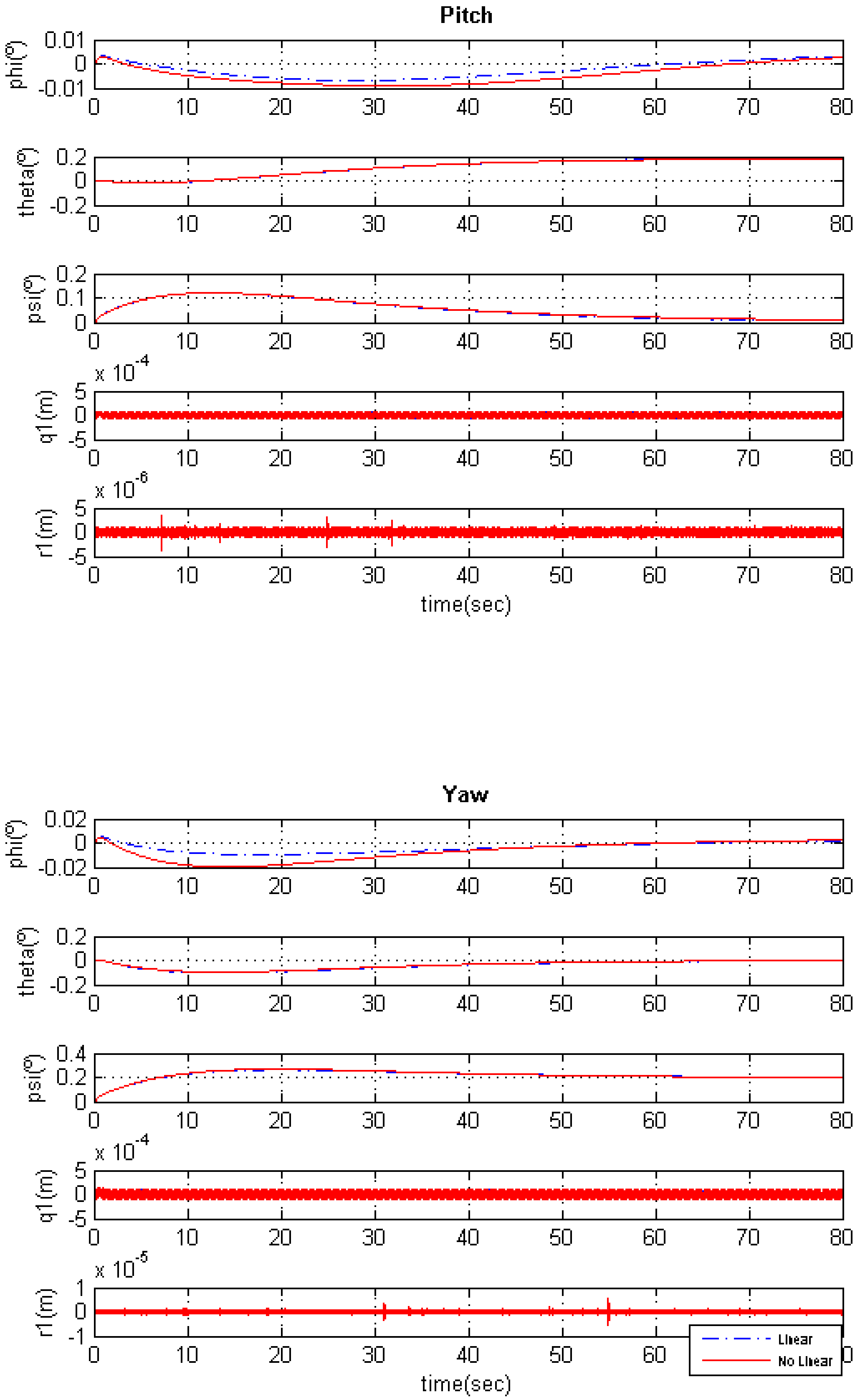


Figure 5: Linear and non-linear step response.

7. Robustness Analysis by LFT

Any change in the dynamics or internal parameters of the spacecraft may represent a lack of robustness. It is also necessary to consider the action of non-modelled dynamics and its uncertainty on the parameters of the system. All of these factors affect the robustness and tolerance of the system to both internal and external disturbances. A linear fractional transformation (LFT) has been used to model the system with the aforementioned disturbances. Figure 6 shows the LFT model in which Δ represents the system uncertainty. In this LFT representation, the controller ***K*** has been obtained using the EA process. It must be considered that the EA method applied to the MIMO system can be made potentially robust by defining suitable locations for the closed loop eigenvalues and selecting the eigenvector to ensure robustness.

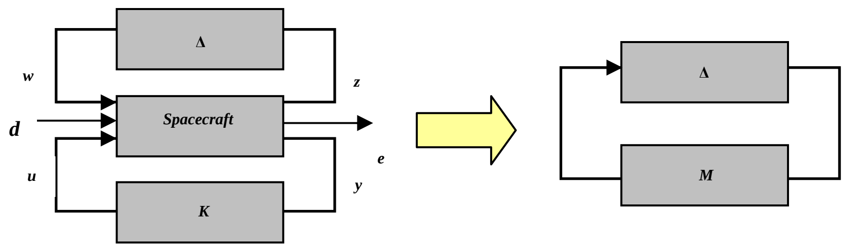
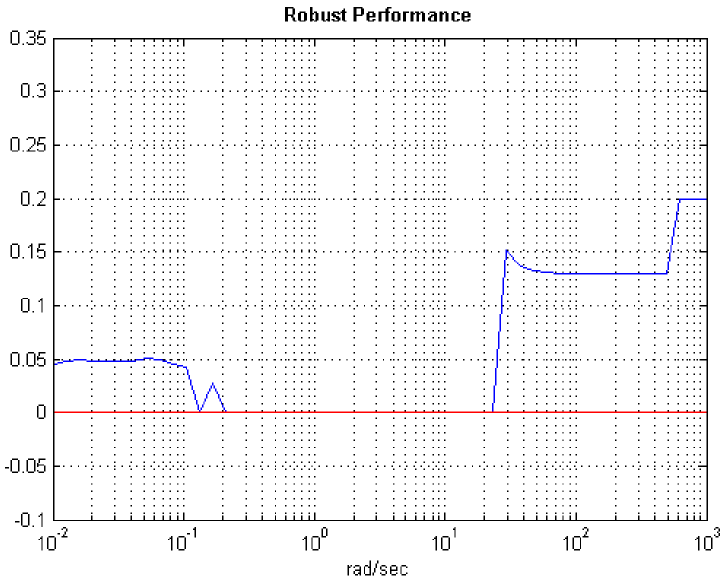


Figure 6: Linear fractional model.

The nominal performance, robust stability and robust performance are obtained by weighting functions, which allow the estimation of the upper and lower limits for all frequencies in the system. Figure 7 depicts the system behaviour with respect to robustness performance and stability. Both response behaviours perform below the critical value of one, which demonstrates that the system shows robust performance for all of the frequencies considered. The frequency response corresponding to the robustness stability shows a peak measuring approximately 0.9 located around the flexion frequency. This response is appropriate for the nature of the system, and the frequency belongs to the first vibration mode (flexion mode).



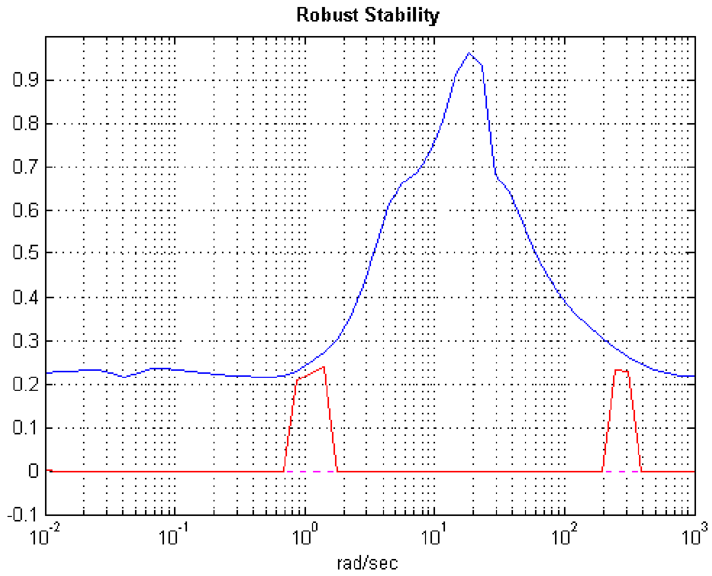


Figure 7: Robustness performance and Robustness Stability.

8. Conclusions

In this paper, we have addressed the problem of controlling a flexible spacecraft with solar panels as the external appendages. As described above, the main objectives of the controller are to stabilize the open-loop system and to obtain suitable decoupling between spacecraft. Finally, the influence of these aspects on system robustness has also been considered. The application of the EA method for controller design resulted in a stable system with optimum decoupling between system modes. The EA method, however, did not provide intrinsic robustness to the system. This factor needs to be carefully analysed further. A μ-analysis has been performed in this study and showed acceptable system behaviour throughout the entire considered frequency range. The EA method is classified as a modal process in the design of controllers. Thus, it is very important to have a precise mathematical model of the system to obtain suitable results. The suitability of any EA method depends largely on the application and a thorough knowledge of the system. This has been achieved through a precise selection of the closed loop eigenvalues and by the results obtained for the closed loop eigenvectors. It is important to select an appropriate group of eigenvalues to overcome pole placement problems. The role of eigenvectors in these types of applications results in adequate closed loop system performance.

Further work in applications using the EA method is needed to address the problem of orthogonality of eigenvectors because this characteristic of the systems is directly related to system robustness.

References

[1] W. M. Wonham, "On Pole Assignment in Multiinput, Controllable Linear Systems", IEEE Transactions on AC, Vol. AC-12, 1967.

[2] B. C., Moore, "On the Flexibility Offered by State Feedback in Multivariable Systems Beyond Closed-Loop Eigenvalue Assignments", IEEE Transactions on AC, Vol. AC-21, 1976.

[3] S., Srinathkumar, "Eigenvalue/Eigenvector Assignment Using Output Feedback", IEEE Transactions on AC, Vol. AC-23, No 1, 1978.

[4] B., Porter, and J.J.D'Azzo, "Algorithm for Closed-Loop Eigenstructure Assignment by State Feedback in Multivariable Linear Systems", International Journal of Control, Vol. 27, No.6, 1978.

[5] J.F. Magni,"Robust Modal Control with a Toolbox for Use with MATLAB", ONERA-Toulouse. Kluwer Academic/Plenum Publishers, 2002.

[6] G.P. Liu, R.J. Patton, "Eigenstructure Assignment Toolbox for Use with MATLAB", Pacilantic International Ltd. Oxford, 1999

[7] L. Faleiro and R. Pratt, “RCAM Design Challenge Presentation Document: An Eigenstructure Assignment Approach”, GARTEUR/TP-088-18, April 1997.

[8] J.M. De la Cruz, P. Ruipérez, J. Aranda, “RCAM Design Challenge Presentation Document: An Eigenstructure Assigment Approach”. GARTEUR/TP-088-22. Abril 1997.

[9] T Clarke, J Ensor and S J Grifon. “Desirable eigenstructure for good short-term helicopter handling qualities: the attitude command response case”. Proc. Instn Mech. Engrs Vol. 217 Part G: J. Aerospace Engineering, 2003

[10] J Ensor and T Clarke. “Analysis and design using the allowable eigenvector subspace.” Proc Instn Mech Engrs Vol 216 Part G: J Aerospace Engineering, 2002.

[11] Nieto-Wire, C., Sobel, K. “Eigenstructure Assignment for a Tailless Aircraft”. AIAA paper 2007-6417, Aug. 2007.

[12] Oliva, A. P., Leite Filho, W. C.”Eigenstructure versus Optimal Control for Decoupling”. Control Engineering Practice, 2002, pp. 1059-1079.

[13] Ali Reza Mehrabian1_ and Jafar Roshanian. “Skid-to-turn missile autopilot design using scheduled eigenstructure assignment technique.” Proc. IMechE Vol. 220 Part G: J. Aerospace Engineering, 2006.

[14] F. Holzapfel,M. Heller,M. Weingartner,G. Sachs and O. da Costa. “Development of control laws for the simulation of a new transport aircraft.” Proc. IMechE Vol. 223 Part G: J. Aerospace Engineering. 2009

[15] Juang, J, Lim, K. and Junkins, J. L: “Robust eigensystem assignment for flexible structures”. Journal of Guidance, Vol 12. no. 3, May – June 1989, pp. 381-387.

[16] Faleiro, L. F., Magni, J. F., De la Cruz, J. M., Scala, S.: “Robust Flight Control”, Cap. 3. ISBN 978-3-540-76151-8, pp. 22-32.

[17] M. A. Rastgaar, M. Ahmadian, S. C. Southward. “Orthogonal Eigenstructure Control with Non-collocated Actuators and Sensors.” Journal of Vibration and Control, 15, 2009.

[18] Huan-Liang Tsai, Jium-Ming Lin, Tsang Chiang and Cheng-Yu Cheng, “Optimal Satellite Attitude Control System Design by Combination of Eigenstructure Assignment and LEQG/LTR Methods”, Proceedings of the 5th World Congress on Intelligent Control and Automation, June 2004, Hangzhou, P.R. China.

[19] Meirovitch, L. and Quinn, R. D: “Equations of motion for maneuvering flexible spacecraft”. Journal of Guidance, Vol 10, no. 5, Sept – Oct 1987.

[20] J. Aranda and J. Crespo. “Eigenstructure Assignment Based Controllers in Flexible Spacecrafts”. Brussels, June 2007, 2ND EUROPEAN CONFERENCE FOR AEROSPACE SCIENCES (EUCASS).

[21] J. Aranda and J. Crespo. “Modal Analysis Applied To Spacecraft Attitude Control”. AIAA Guidance, Navigation and Control Conference and Exhibit 20 - 23 August 2007, Hilton Head, South Carolina. AIAA 2007-6444.

TABLES

ILLUSTRATIONS

Figure 6: Linear fractional model.

Figure 7: Robust performance and Robust Stability.

LIST OF NOTATIONS

$\boldsymbol{A}$= State Matrix

$\boldsymbol{B}$= Control Matrix

$\boldsymbol{C}$= Output Matrix

$\boldsymbol{D}$= Input-Output Matrix

$\boldsymbol{E_L}$ = Kinetic Energy

G = Stiffness Modulus

$\boldsymbol{G}$ = Gyroscopic Matrix

I_X, I_Y, I_Z = Moments of Inertia

J_{Xi}, J_{Yi} = Generalised moment of inertia

$\boldsymbol{K_m}$ = Stiffness Matrix

$\boldsymbol{K}$ = Gain Controller

K = Poisson's ratio

$\boldsymbol{L}$ = Lagrangian

$\boldsymbol{M}$ = Mass Matrix

m_i = Generalised Mass for Solar Panels

$\boldsymbol{Q_{nc}}$ = Nonconservative generalised forces

Q_{Xi}, Q_{Yi} = First-order generalised moment.

q_i = Longitudinal generalised coordinate

r_i = Transversal generalised coordinate

$\boldsymbol{u}$ = Generalised Coordinates Vector

$\vec{\boldsymbol{u}}$ = Control Signal

$\boldsymbol{V}$ = Right Eigenvector Matrix

$\boldsymbol{W}$ = Left Eigenvector Matrix

$\overline{\boldsymbol{W}}$ = Right Parameter Matrix

$\boldsymbol{V_L}$ = Potential Energy

$w_i(x,y,t)$ = Elastic displacement

$\bar{\boldsymbol{w}}_i$ = Right Parameter Vector

ω_0 = Spacecraft angular speed

$\boldsymbol{x}$ = State Vector

$\boldsymbol{y}$ = Output Vector

Λ = Eigenvalue Matrix

λ_{di}= Desired eigenvalues

v_{di}= Desired eigenvectors

$\boldsymbol{v_i}$ = Right Eigenvectors

$\boldsymbol{w_i}$ = Left Eigenvectors

ϕ,θ,ψ = Roll, pitch and yaw angles

$\phi_i(x)$= Longitudinal shape factor

$\Psi_i(x)$= Transversal shape factor