Correction of systematic errors in Wide Area Multilateration

Jorge M. Abbud, Gonzalo de Miguel, Juan Besada
GPDS-SSR
Universidad Politécnica de Madrid
Madrid, Spain
jorge.jose.abbud.momma@gmail.com, gonzalo@grpss.ssr.upm.es, besada@grpss.ssr.upm.es

Abstract—**This work presents a method to estimate and correct slow time-dependent position errors due to non perfect ground station synchronization and tropospheric propagation. It uses opportunity traffic emissions, i.e. signals transmitted from the aircrafts within the coverage zone. This method is used to overcome the difficulty of installing reference beacons simultaneously visible by all the base stations in a given Wide Area Multilateration (WAM) system.**

Keywords- Wide Area Multilateration; Air Traffic Control; ADS-B estimation

I. INTRODUCTION

Due to the performance improvements of multilateration systems, their application range has been extended from short-range applications (airport surveillance) to medium-range surveillance, such as surveillance in Terminal Maneuvering Areas (TMA) [1]. This system has been called Wide Area Multilateration system (WAM). Under this positive performance evolution, WAM becomes a firm candidate to replace secondary radars in the surveillance network for Air Traffic Control (ATC) [2].

Multilateration determines the aircraft's position by using the time of arrival (TOA) of the signal travelling from the aircraft itself to a network of fixed receivers (base stations). If the signal is properly coded, it is easy to associate the TOAs relative to one single transmission in the different base stations. This is the typical case in ATC, where RF emissions (ADS-B or TCAS) are used. This way, when the system has the complete set of TOAs in all ground stations, the aircraft position can be determined. The trend in the future ATC surveillance is to use ADS-B as the main source of aircraft positioning. But it is still necessary to have a collaborative backup system in order to enhance surveillance integrity [2]. A promising solution is the use of ADS-B ground stations as WAM base stations. Each base station will send the measured TOA together with the ADS-B information to the ATC control center. Multilateration is performed by processing the TOAs [2].

The accuracy of the multilateration position is determined by the errors in the TOA estimates. From a data processing perspective, these errors can be grouped into three main categories [2]: white noise, synchronization issue among ground stations and propagation effects. The first two are present in any multilateration scenario, although white noise effects in the position determination are not critical for the S/N values usually managed in these systems. On the other hand, although propagation error has not been taken into account for short-range applications, given the distance between base stations in WAM scenarios (up to 20 or 30NM [1]) this source of error has to be considered. This is required in order to preserve the accuracy from suffering degradation along the coverage area (i.e. a low Dilution of Precision).

This characteristic rules out the calibration philosophy to reduce both synchronization and propagation errors, since installing fixed beacons in Line-Of-Sight with all base stations is costly, if not impossible, for large baseline separations. Also, the propagation error has a hard dependence with aircraft altitude. This way, calibrations for on-ground targets are not valid for flying aircrafts.

In order to solve the synchronization issue between stations, GPS-based methods could be used, but these would not reduce propagation errors. Furthermore, a backup synchronization subsystem would be necessary in order to mitigate hypothetical failures of GPS.

Therefore, one solution to this problem is to add a processing subsystem which corrects synchronization and propagation errors simultaneously, by analyzing the signals transmitted by all aircrafts currently present within the WAM coverage (opportunity traffic).

In a recent paper, authors have studied the possibility of performing calibration using opportunity traffic for WAM systems [5]. Figure 1 presents the block diagram of the proposed calibration mechanism. TOAs measured in each station are associated and sent to the central processor which computes the position. This can be either a master station in the multilateration system or a remote station fusing the information of many sensors (this can be the case for ADS-B technology [3]). The first operation is to apply calibration corrections to pseudoranges (for synchronization issues and propagation). Then, target coordinates are determined as if the calibration was perfect, modeling the propagation error as a polynomial depending on distance. The output of this block is delivered as a position determined by WAM system. In order to compensate for slow time variations in the real propagation and calibration conditions, the system has an open-loop

The work has been financed by Spanish Science and Technology Office under projects TEC-2008-06732 y TIN-2008-06742.

control system that modifies the estimated propagation constants, maintaining the system calibrated.

The system selects the targets located at a determined flight level and spread around the coverage area. Then, the system determines their position, as well as the calibration constants, triggering the iterative algorithm from the target coordinates delivered by WAM system and the calibration constants at the output of the averaging filter. Calibration constants are averaged in order to reduce their variance. Once filtered, they are used for the correction of future pseudoranges.

The position determination is done using the proposed mechanism in [6]. It uses an iterative algorithm with the linearized multilateration equation [7]. This algorithm needs an initial position which is determined using the closed form algorithm of [8]. The system can be implemented in TOA form (determining time of emission, cT_e) or in TDOA form (eliminating cT_e). If cT_e is not necessary, the TDOA form is more accurate due to the complete ignorance about the time of emission in WAM. This is the approach assumed in this work. The same method applies to the determination of calibration constants plus position.

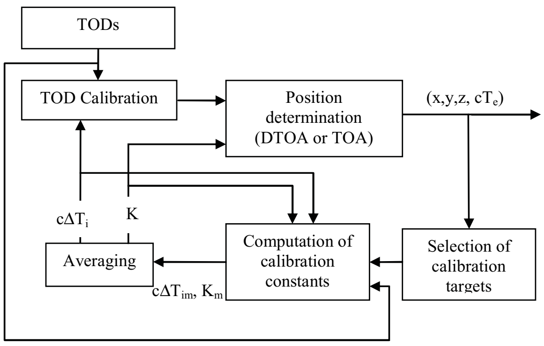

Figure 1. Block diagram of calibration mechanism using opportunity traffic.

This approach uses a linear model for propagation error and a constant error to represent the synchronization error of each base station. Linear propagation models are appropriate for medium distances (around 75-100 km). For longer distances, calibrations based on linear models do not have the required performance. This paper will extend the method proposed in [5] by including second-order propagation models. The inclusion of a polynomial model to correct the synchronization error in each base station will be studied as well.

The paper is structured as follows: section 2 states the problem of position determination with WAM including propagation and clock synchronization errors. Section 3 compares the accuracy of the system using opportunity traffic when propagation effects are modelled through first or second-order model (with respect to distance). Finally, section 4 analyzes the accuracy degradation due to the clock drifts, followed by a mitigation method proposal.

II. Characterisation of Slowly-Variant Errors in WAM Systems

The aircraft position is determined by means of the time difference of arrival (TDOA) of the signal at the different base stations. As a first step, a method based on hyperbolic location as described in [9] can be used. Additionally, in the presence of error, a gradient method starting from the solution of the previous treatment will be used in order to refine the location [7].

Each base station in the scenario measures the TOA of the signal received from the target aircraft. The TOA of the signal traveling from the j-th aircraft located in (x^j, y^j, z^j) to the i-th base station located in (x_i, y_i, z_i) can be represented by the following expression:

$$\tau_i^j = TOA_i = \frac{1}{c}\sqrt{\left(x_i - x^j\right)^2 + \left(y_i - y^j\right)^2 + \left(z_i - z^j\right)^2} + \frac{1}{c}\Delta P_i^j + T_e + \Delta T_i + n_i \tag{1}$$

Equation (1) can be rewritten as:

$$\tau_i^j = TOA_i = \frac{1}{c}R_{ij} + \frac{1}{c}\Delta P_i^j + T_e + \Delta T_i + n_i \tag{2}$$

where R_{ij} stands for the Euclidean distance between the i-th station and j-th aircraft, ΔP_i^j represents the propagation error, T_e represents the signal emission time, ΔT_i is the synchronism error, and n_i the white noise random error.

In order to eliminate the signal emission time uncertainty, the aircraft position will be assessed based on the Time Difference of Arrival (TDOA). This means that all available TOAs for a single emission are referenced to the TOA on one of the base stations. Thus, the TOA equation system is now replaced by a TDOA equation-system, with one less unknown, as well as one less equation:

$$\tau_{i,m}^j = \tau_i^j - \tau_m^j = TDOA_i^j = \frac{1}{c}\left(R_{ij} - R_{mj}\right) + \frac{1}{c}\left(\Delta P_i^j - \Delta P_m^j\right) + \left(\Delta T_i - \Delta T_m\right) + \left(n_i - n_m\right) \tag{3}$$

where the reference base station is the m-th station.

Let us now characterize the propagation error ΔP_i^j. It is the uniform vertical gradient of atmospheric refractive index that 'bends' the signal propagation trajectory and changes the velocity of light, delaying its arrival to the base station. Figure 2 represents this propagation error with respect to the slant range for aircrafts flying at altitudes between 6000m and 14000m AMSL, using the expressions defined in [10]. The base station is considered to be at sea level.

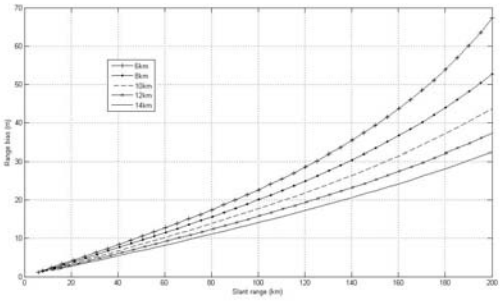

Figure 2. Systematic error (range bias) due to radio wave propagation for a standard atmosphere.

Two relevant characteristics can be observed concerning this propagation error. First, a second degree polynomial seems to be a good fit for modeling the range bias with respect to the distance. Actually, even a linear approximation can prove to be sufficient for short distances [5]. Second, the coefficients of the polynomial depend on the aircraft altitude. The last observation forces to estimate different propagation models as a function of aircraft height. The system divides the height in different layers (with 1-2 km of thickness). For each altitude layer, the system performs an independent propagation calibration based on the aircrafts inside it.

Taking into account the previous observations, equation (2) can be approximated substituting propagation error by a first-order or second-order model versus Euclidean range:

$$\tau_i^j = TOA_i \approx \frac{1}{c}(1+K)R_{ij} + T_e + \Delta T_i + n_i \tag{4}$$

$$\tau_i^j = TOA_i \approx \frac{1}{c}\left[(1+K_1)R_{ij} + K_2 R_{ij}^2\right] + T_e + \Delta T_i + n_i \tag{5}$$

The method to estimate parameters for both models will be described in section 3.

Now we shall characterize the clock drift occurring in the base stations. In this study, we consider that since the signal emissions are quasi-periodic, a Time Interval Error (TIE) model of a local clock will be used based on the philosophy of [11]. It consists of a polynomial model projecting ahead on a horizon of N points from the starting point with the k-th degree Taylor expansion:

$$\Delta T_i(n) = \sum_{p=0}^{k} \lambda_p \frac{n^p T^p}{p!} + w_1(n,T) \tag{6}$$

where n is the sample number, T is the time step, $\lambda_{p+1} \equiv \lambda_{p+1}(0), p \in [0,k]$ the initial states of the clock and $w_1(n,T)$ is a clock noise with known properties. For large values of n, the polynomial component dominates over $w_1(n,T)$. Thus, in this study, we shall characterize this local clock with a second-order polynomial without considering $w_1(n,T)$. Expression (6) now becomes:

$$\Delta T_i(nT) = \lambda_{i,0}(0) + \lambda_{i,1}(0)nT + \frac{1}{2}\lambda_{i,2}(0)n^2T^2 \tag{7}$$

This way, TDOA between stations i-th and m-th for a signal transmitted at time $t=nT$ for the j-th aircraft under coverage can now be written in the following ways:

$$\begin{aligned}\tau_{i,m}^j(nT) = TDOA_i^j(nT) = \frac{1}{c}[\ &(1+K_1)(R_{ij}-R_{mj}) \\ &+ K_2\left(R_{ij}^2 - R_{mj}^2\right)\ ] + (\Delta T_i(nT) - \Delta T_m(nT)) + (n_i - n_m)\end{aligned} \tag{8}$$

On the right side, we are now in a position to set a system of TDOA equations in order to determine the aircraft position. On the other hand, the number of unknowns has increased due to the characterization of propagation effects and relative synchronization errors. Therefore, the solution of the system shall not only contain the aircraft coordinates, but also the constants relative to both propagation error and clock drift.

In order to avoid an indeterminate system of non-linear equations, a set of new independent equations must be obtained. For this purpose, the opportunity traffic method will be used.

III. CORRECTION OF THE SYSTEMATIC ERROR DUE TO PROPAGATION

This section focuses solely on the propagation effects and the technique used to solve the equation system without considering clock drifts (synchronization error is considered constant with time).

Since the calibration constants must be determined together with coordinates (three spatial coordinates plus emission time for each aircraft, two propagation constants and one synchronization constant for each base station minus one for the reference station), there is a need to process jointly the TDOA of M ($\geq N/(N$-4)) aircrafts for the linear propagation error model to obtain the sufficient number of equations. It is necessary that the number of base stations, N, is greater than 4, the minimum number of pseudo-ranges required to determine spatial coordinates and emission time. The pseudo-ranges of the extra stations are used as equation to determine the additional unknowns of the calibration models. When using the parabolic propagation error model, the minimum number of aircrafts under coverage shall be $M \geq (N+1)/(N-4)$.

A larger amount of jointly-processed aircrafts yield a stronger degree of over-determination, providing therefore higher stability in the estimates. The drawback is that larger sparse matrices and larger amount of data shall be handled.

The non-linear equation system allowing the simultaneous determination of calibration constants as well as the location of the aircrafts is as follows (noise terms are not included for the sake of clarity):

$$\tau^1_{2,1}(0) = TDOA^1_2(0) \approx \frac{1}{c}\left[(1+K_1)(R_{21}-R_{11}) + K_2\left(R_{21}{}^2 - R_{11}{}^2\right)\right] + \Delta\lambda_{21}(0)$$

$$\cdots$$

$$\tau^1_{N,1}(0) = TDOA^1_N(0) \approx \frac{1}{c}\left[(1+K_1)(R_{N1}-R_{11}) + K_2\left(R_{N1}{}^2 - R_{11}{}^2\right)\right] + \Delta\lambda_{N1}(0)$$

$$\cdots$$

$$\tau^j_{2,1}(0) = TDOA^j_2(0) \approx \frac{1}{c}\left[(1+K_1)(R_{2j}-R_{1j}) + K_2\left(R_{2j}{}^2 - R_{1j}{}^2\right)\right] + \Delta\lambda_{21}(0)$$

$$\cdots$$

$$\tau^j_{N,1}(0) = TDOA^j_N(0) \approx \frac{1}{c}\left[(1+K_1)(R_{Nj}-R_{1j}) + K_2\left(R_{Nj}{}^2 - R_{1j}{}^2\right)\right] + \Delta\lambda_{N1}(0) \tag{9}$$

where $\Delta\lambda_{21}(0)=\lambda_2(0)-\lambda_1(0)$, denoted $\Delta\lambda_{21}$ within the remaining part of this paper represent the difference between synchronization errors in two base stations. Note that it is sufficient to determine the difference between synchronization errors since the measured magnitude is the TDOA. So, in a scenario involving N stations and M aircrafts, the amount of TDOA equations is $M(N-1)$.

One way to solve this system is by using a gradient method [7][8], setting the initial value around the intersection of the hyperboloids. The initial condition is determined as indicated in figure 1. Iteration using the linearized system shall be performed until the convergence criteria based on the accuracy requirements have been met.

The vector composed by the unknowns is as follows:

$$\bar{\mathbf{x}} = \left[x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_M, y_M, z_M, c\Delta\lambda_{2,1}, \ldots, c\Delta\lambda_{N,1}, K_1, K_2\right]^T \tag{10}$$

(x_j, y_j, z_j) being the position of the j-th aircraft, $c\Delta\lambda_{i,1}$ being the synchronization errors between the clocks of stations 1 and i, and K_1, K_2 the coefficients relative to degrees 1 and 2 modeling the propagation error effect.

The system in (9) can be solved using the following iteration:

$$\bar{\mathbf{x}}^k = \bar{\mathbf{x}}^{k-1} + \boldsymbol{\delta}\bar{\mathbf{x}}^k \tag{11}$$

The initial value $\bar{x}^0$ must be set around the true solution. This is done using previous estimations of calibration constants and the position determined for each aircraft by the WAM system before calibration process.

For each iteration process, an estimation of the TDOAs is performed using the estimates of vector $\bar{\mathbf{x}}^{k-1}$. The error between the measured values and its estimates is defined as follows:

$$\varepsilon^k = \bar{\tau} - \mathbf{f}\left(\mathbf{x}^{k-1}\right) \tag{12}$$

where $\bar{f}$ is the set of TDOA estimates. As an example, $f^j_{i,1}(\mathbf{x}^k)$ is defined in expression (13) below:

$$f^j_{i,1}\left(\mathbf{x}^k\right) = \frac{1}{c}\left[\left(1+K^k_1\right)\left(R^k_{i,j} - R^k_{1,j}\right) + K^k_2\left(R^k_{i,j}{}^2 - R^k_{1,j}{}^2\right)\right] + \Delta\lambda^k_{i,1} \tag{13}$$

The differential increment is obtained by solving the first derivative terms of the linearized version of system (9). Thus, expression (12) can also be written as:

$$\boldsymbol{\varepsilon}^k = \mathbf{A}^k \boldsymbol{\delta}\mathbf{x}^k \tag{14}$$

where $\mathbf{A}^k$ is the gradient matrix of system (9) at the *k*-th iteration. Matrix **A** is obtained using the following expressions:

$$\mathbf{A} = \begin{bmatrix} \nabla_{2,1} & \bar{\mathbf{1}} & \bar{\mathbf{0}} & \cdots & \bar{\mathbf{0}} & \bar{\mathbf{R}}_{2,1} & \bar{\mathbf{R}}^2_{2,1} \\ \nabla_{3,1} & \bar{\mathbf{0}} & \bar{\mathbf{1}} & \cdots & \bar{\mathbf{0}} & \bar{\mathbf{R}}_{3,1} & \bar{\mathbf{R}}^2_{3,1} \\ \nabla_{4,1} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \cdots & \bar{\mathbf{0}} & \bar{\mathbf{R}}_{4,1} & \bar{\mathbf{R}}^2_{4,1} \\ \vdots & \vdots & \vdots & \cdots & \bar{\mathbf{0}} & \vdots & \vdots \\ \nabla_{\mathbf{N_S},1} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \cdots & \bar{\mathbf{1}} & \bar{\mathbf{R}}_{\mathbf{N_S},1} & \bar{\mathbf{R}}^2_{\mathbf{N_S},1} \end{bmatrix} \tag{15}$$

where:

$$\nabla_{\mathbf{i,1}} = \begin{bmatrix} \bar{\boldsymbol{\Delta}}^1_{\mathbf{i,1}} & \bar{\mathbf{0}}_{\mathbf{i,1}} & \cdots & \bar{\mathbf{0}}_{\mathbf{i,1}} \\ \bar{\mathbf{0}}_{\mathbf{i,1}} & \bar{\boldsymbol{\Delta}}^2_{\mathbf{i,1}} & \cdots & \bar{\mathbf{0}}_{\mathbf{i,1}} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{\mathbf{0}}_{\mathbf{i,1}} & \bar{\mathbf{0}}_{\mathbf{i,1}} & \cdots & \bar{\boldsymbol{\Delta}}^{\mathbf{N_{A/C}}}_{\mathbf{i,1}} \end{bmatrix}$$

$$\bar{\mathbf{0}}_{\mathbf{i,1}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\mathbf{1}} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T \quad ; \quad \bar{\mathbf{0}} = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T$$

$$\bar{\mathbf{R}}_{\mathbf{i,1}} = \left[R_{i,1} - R_{1,1},\ \ R_{i,2} - R_{1,2},\ \ \cdots R_{i,j} - R_{1,j},\ \ \ldots\ \ R_{i,\mathbf{N_{A/C}}} - R_{1,\mathbf{N_{A/C}}}\right]^T$$

$$\bar{\mathbf{R}}^2_{\mathbf{i,1}} = \left[R^2_{i,1} - R^2_{1,1},\ \ R^2_{i,2} - R^2_{1,2},\ \ \ldots\ R^2_{i,j} - R^2_{1,j},\ \ \ldots\ \ R^2_{i,\mathbf{N_{A/C}}} - R^2_{1,\mathbf{N_{A/C}}}\right]^T$$

$$\bar{\boldsymbol{\Delta}}^{\mathbf{j}}_{\mathbf{i,1}} = \nabla f^j_{i,1} = \begin{pmatrix} \left(1+K_1\right)\left(\dfrac{x_j - x^b_i}{R_{ij}} - \dfrac{x_j - x^b_1}{R_{1j}}\right) + 2K_2\left(x^b_1 - x^b_i\right) \\ \left(1+K_1\right)\left(\dfrac{y_j - y^b_i}{R_{ij}} - \dfrac{y_j - y^b_1}{R_{1j}}\right) + 2K_2\left(y^b_1 - y^b_i\right) \\ \left(1+K_1\right)\left(\dfrac{z_j - z^b_i}{R_{ij}} - \dfrac{z_j - z^b_1}{R_{1j}}\right) + 2K_2\left(z^b_1 - z^b_i\right) \end{pmatrix}^T$$

Since system (14) is usually over-determined, it must be solved using the minimum mean square error (MMSE) expression:

$$\boldsymbol{\delta}\mathbf{x}^{k}=\left(\left(\mathbf{A}^{k}\right)^{T}\cdot\left(\mathbf{S}^{k}\right)^{-1}\cdot\mathbf{A}^{k}\right)^{-1}\cdot\left(\mathbf{A}^{k}\right)^{T}\cdot\left(\mathbf{S}^{k}\right)^{-1}\cdot\boldsymbol{\varepsilon}^{k} \tag{16}$$

where $\mathbf{S}^k$ is the TDOA covariance matrix and can be estimated from the S/N in each receiver.

In order to assess the performance of the opportunity traffic method, a hypothetical WAM scenario is simulated. The system here considered is composed by six stations, located in the corners of a square (side: 100 Km) and two in the middle of two vertical sides. The station altitudes are arbitrary, but near sea level (this implies a poor performance in aircraft altitude determination). Six aircrafts are considered for the calibration process, all of them outside the square delimited by base stations 1 to 4 with a height of 10 Km ((x Km,y Km): (-150,60) (100,90) (80,-40) (-120,-70) (20,80) (-30,-110)).

Figures 3 to 6 display the mean and standard deviation of the WAM location error for pseudo-range error standard deviation values ranging between 1 and 10m. The results have been obtained using Monte Carlo experiments, averaging sufficient independent solutions to turn the simulation variance negligible. Results displayed in figures 3 and 4 have been obtained using the linear model for propagation error, whereas figures 5 and 6 show the results obtained using the parabolic model.

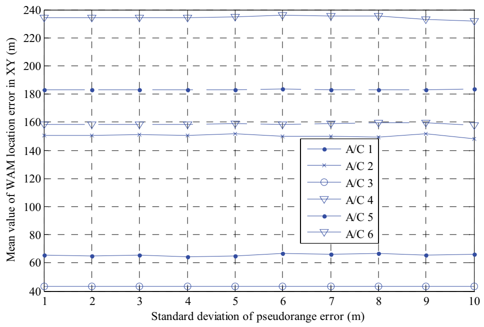

Figure 3. Mean of WAM position error using the order 1 estimation of the propagation error.

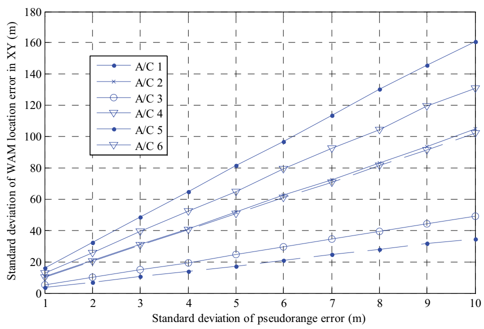

Figure 4. Standard deviation of WAM position error using the order 1 estimation of the propagation error.

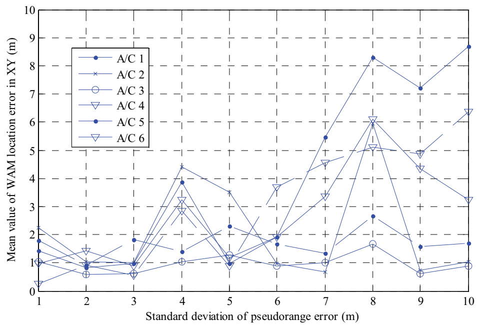

Figure 5. Mean of WAM position error using the order 2 estimation of the propagation error.

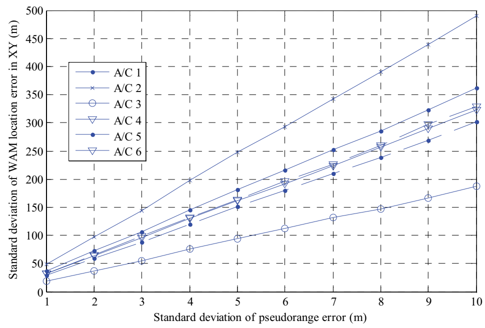

Figure 6. Standard deviation of WAM position error using the order 2 estimation of the propagation error.

Concerning the performance in the X-Y plane, results show that the parabolic model sacrifices the variance of the location error on behalf of its mean value.

IV. CLOCK DRIFT EFFECT ON POSITION ACCURACY

This section covers the assessment of the dual correction opportunity traffic method, since it considers the propagation effects, as well as the drifts suffered by the clocks placed on each base station. Expression (6) suggests that the TOA measurement induced by this drift varies over time. Figure 7 shows how the errors increase rapidly in a biased way. Therefore, the TDOA estimates generated by the algorithm must consider the behavior of each local clock (more precisely, the difference between drifts).

Consequently, the iterative algorithm presented in section 3 is to be used once again, in order to refine the target location. Since two new unknowns shall be taken into account for each equation of the TDOA system, necessary expansions are to be made in expressions (9) to (16) in order to accommodate the TDOA measurements gathered at different instants.

Besides, the scenario described in section 3 has been upgraded, as local oscillators are modeled with a second-order polynomial along the time axis. The experiments have been carried out considering clock parameters typically used in base stations (e.g. atomic clocks, with the following coefficients for (6): 10^{-10}, $3\cdot10^{-11}$, 10^{-13}).

Figure 8 show how the two-step approach based on the opportunity traffic method is capable of bounding the mean error with a small increase in standard deviation.

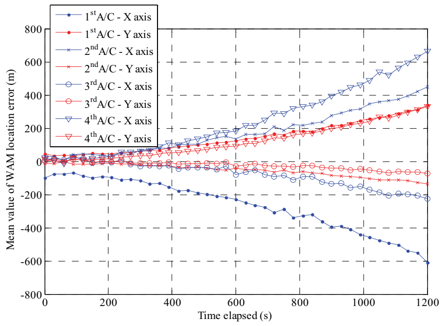

Figure 7. Mean of WAM position error obtained without considering clock drift.

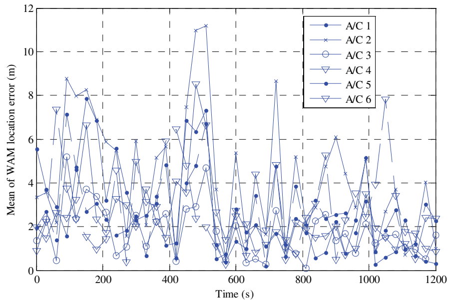

Figure 8. Mean value of horizontal position error with the two-step approach.

V. Conclusion

This paper presents a collaborative backup system that enhances ATC surveillance integrity. Its cost is relatively low since this system reuses the ADS-B ground stations.

This system is able to mitigate the impact of the propagation effects, as well as the impact of the clock drift effects for a limited period of time without using calibration stations. Cases in point are scenarios leading to temporary GPS unavailability, such as spoofing, insufficient number of acquired satellites or even a failure in the GPS receiver aboard the aircraft.

References

[1] W.H.L.Neven, T.J. Quilter, R. Weedon, R. A. Hogendoorn, “Wide Area Multilateration”, Report on EATMP TRS 131/04. Version 1.1 - National Lucht en Ruimtevaartlaboratorium, August 2005.

[2] “SESAR Definition Phase – Deliverable 3: the ATM target concept”, SESAR Consortium, September, 2007.

[3] “EUROCONTROL standard document for surveillance data exchange. Part 12: Category 21, ADS-B messages”, EUROCONTROL, ed 1.1 September 2008.

[4] M. J. Leeson, “Error Analysis for a Wide Area Multilateration System”,QinetiQ/C&IS/ADC/520896/7/19, 2006.

[5] G. de Miguel, J. Besada, J. García, “Correction of propagation errors in Wide Area Multilateration systems”, European Radar Conference 2009 (EuRad 2009), Rome (Italy), September 2009.

[6] G. Galati, M. Leonardi, P. Magarò, V. Paciucci, “Wide Area Surveillance using SSR Mode S Multilateration: advantages and limitations”, European Radar Conference 2005, EURAD 2005.

[7] W. H. Foy, “Position-Location Solutions by Taylor-Series Estimation”, IEEE Trans. on Aer. and Elec. Syst., Vol. AES-12, No. 2, pp. 187-194, March 1976.

[8] G. Strang, K. Borre, “Linear Algebra, Geodesy, and GPS”, Wellesley-Cambridge Press, 1997.

[9] Y.T.Chan, and K.C.Ho, “A simple and efficient estimator for hyperbolic location”, IEEE Transactions on Signal Processing, Vol. 42, No. 8, pp. 1905-1915, August 1994.

[10] D. K. Barton, “Radar System Analysis and Modeling”, Artech House, 2005.

[11] Y. S. Shmaliy, “An unbiased FIR filter for TIE model of a local clock in applications to GPS-based timekeeping”, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, UFFC-53, pp. 862-870, 2006.