

REAL OPTIONS: A TOOL FOR MANAGING TECHNICAL RISK IN A MINE PLAN

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ABSTRACT

NPV is a static measure of project value which does not discriminate between levels of internal and external risk in project valuation. Due to current investment project's characteristics, a much more complex model is needed: one that includes the value of flexibility and the different risk levels associated with variables subject to uncertainty (price, costs, exchange rates, grade and tonnage of the deposits, cut off grade, among many others). Few of these variables present any correlation or can be treated uniformly. In this context, Real Option Valuation (ROV) arose more than a decade ago, as a mainly theoretical model with the potential for simultaneous calculation of the risk associated with such variables. This paper reviews the literature regarding the application of Real Options Valuation in mining, noting the prior focus on external risks, and presents a case study where ROV is applied to quantify risk associated to mine planning.

INTRODUCTION

It's important to state that Real Options Valuation (ROV) is not a substitute of the conventional discounted cash flow method (DCF), but rather a complement that fills the gaps that the DCF cannot address [19]. Real Options valuation uses Net Present Value (NPV) calculated through DCF, integrating it in a more sophisticated structure, capable of capturing explicitly the different options that exist on an investment project. This way, they are able to value the flexibility granted by having the possibility of reacting to change. In other words, ROV helps managers study the opportunities that will be presented in the future, being able to plan strategic investments upfront [5].

Real Options Analysis (ROA) is a better approximation of the way an investor sees a project: accepting that the future is uncertain, and suggesting that a good evaluation requires a thorough work identifying potential responses to the ranges of possible future conditions. This so called "future conditions" include internal (technical) as well as external (market) variability. Current applications of the ROA focus mainly on external variables, such as price and exchange rates, with few cases where these variables are integrated with geological uncertainties and conditional simulations. However, the applicability in the "out of the project" area is fairly limited, and it has been widely studied, though not at all widely implemented.

This paper focuses on analyzing the ROV as a tool for quantifying and managing the risk associated to the variability of mine planning variables.

STATE OF THE ART

The Origins of Real Option Valuation

Real Options Analysis and Valuation (ROA/ROV) materialized from the growing need of more confident models when valuing high risk high investment projects. This method was created by acknowledging the similarities that exist between financial options derivatives and investment projects, and replicates the existing financial valuation methods to real project characteristics; thus the name of "Real Options".

Traditional DCF valuation methods provide reliable results when flexibility options are not available, or when the project's uncertainty is

limited and cash flows are fairly constant, but show great inconsistencies when parameters present variability. On the other hand, a real option recognizes the existing uncertainties of the project and turns them into investment opportunities, developing dynamic strategies to manage risk [5].

An option - financial or real - is a right, but not an obligation, to perform an act for a certain cost, at or within a period of time; for example, to buy an asset at a certain price, or to extract mineral from a mine. As so, they are able to add value as they provide opportunities to take advantage of uncertain situations, gaining from favorable scenarios, and hedging from downside risks [15], [23].

Real options differ from financial options in that they deal with tangible uncertain assets, instead financial underlying assets.

The growing application of ROV to project evaluation and investment decision risk is related to the capacity of this methods to overcome the limitations in the traditional valuation methods (in particular the DCF).

Problems of Traditional Valuation Methods

Conventional DCF project valuation method presents some crucial limitations when dealing with uncertainty associated to capital projects, particularly, mining projects. First, DCF assumes that all the related variables are fixed parameters, not considering their stochastic reality. This can be clearly seen for example, in price's input as a fixed value, when it is widely known that price is subject to constant change. A second limitation of traditional methods is that they assume that investments and other relevant decisions must be made "now or never", without considering the value of strategy and management [10]. A third problem is that DCF method collapses all sources of risk in one only discount rate. As the result, conventional DCF methods tend to undervalue projects by applying heavy punishment over the project's last years [22], and make it difficult for decision takers to really comprehend the project's model, and the variables affecting the valuation.

Samis et al [30] states that the fundamental difference between real option valuation and discounted cash flow is in how each of them determines the accountability of cash flow uncertainty on project's value. Amram and Kulatilaka [5] mention that traditional valuation instruments are of no real use in current investment projects, as they don't take into account that the decision making process of managers might have an effect over the project's outcome.

When variables and external conditions change in an ongoing operation, in reality managers tend to react accordingly, leaving obsolete the project's estimated NPV. ROV includes the value associated to these decisions into the initial model, increasing its reliability. Likewise, if managers don't realize these changes, or if the changes find them unprepared, these variations can strongly reduce the project's value. In this case, ROV helps managers foresee future event, so that change finds them prepared and ready to act accordingly, when certain "red light" values are triggered.

Dimitrakopoulos and Sabour [14] and Sabour and Poulin [28] mention that the main advantage of the ROV is that it's able to

incorporate the value of management decision making and flexibility to act according to changes or revise past decisions with time, based on new information (external as well as internal). This method provides a transparent guideline for analyzing the timing of strategic and operational decisions, as it deals with the different sources of uncertainty individually, accounting for all possible scenarios of future outcomes.

Option Valuation

A key concept of options is that they are an asymmetric derivative. This means that to have the option, one must pay a premium upfront (i.e. buy the option). Flexibility is favorable but it's not free, so the important thing to establish is how much I'm willing to pay today for an option, in order to increase future action flexibility, and thus reduce the associated risk.

There are three known methodologies of option valuation resolution: (i) partial differential equations, (ii) binomial models, that include decision trees, and (iii) Simulation models, where the most known and used one is the Monte Carlo simulation.

Partial Differential equations include analytical, numerical and finite difference resolutions. The core of this method's resolution is that it equals the change in the option's value to the change in value of the reference financial portfolio [5]. The most known analytical valuation is the Black & Scholes formula (1973), for European call options. However, these models are not really applicable for valuing real options, as the conditions needed for the formula to actually apply are at most, ideal.

Binomial models are widely used, as they are flexible and easy to understand. Besides, their diagrammed evolution is similar to a cash flow, so the decisions taken upon them tend to be very transparent. However, in binomial trees the level of complexity grows exponentially with the number of uncertainties being considered, and so, their range of use is limited.

In contrast, Monte Carlo simulation enables the analysis of an endless number of variables, as the software developed for this purpose is vast and quite sophisticated. It can simulate European as well as American options, and it allows the valuation of complex problems without the need of over simplified unrealistic assumptions.

Work and applications of ROV

In 1985, Brennan and Schwartz [9] introduced for the first time the concept of real options to a natural resource investment project (in this case, to a copper mine). In 1998, Tulcanaza and Zenteno [34] stated the importance of acknowledging the stochastic quality of crucial variables, such as price and market risk, but focused solely on incorporating price uncertainty into the model. A decade after, the range of application have become vast: from pharmaceutical developments [5], to land lease valuation, oil projects, exploratory campaigns [6], and mine design, operation and scheduling. However, even though the application range in sectors is high, the variables analysis has been limited exclusively to market conditions and external uncertainties.

For example: Samis et al [29] compared the cash flows accountability for the DCF and the ROV methods, taking advantage of the estimated copper forward contracts to include price risk associated to copper price, in order to choose from two possible investment projects. However, the presented procedure fails to recognize the existence of project risk other than price uncertainty. Sabour and Poulin [3] use an elaborate least-square Monte Carlo simulation model to evaluate the operational flexibility of a polymetallic mine, considering as much as 7 different ore types. Mayer and Kazakidis [24] studied a production capacity option, an anticipated shutting down option, and a mining sequence option, all conditional to metal price uncertainties. Sabour and Wood [28] contrast the DCF method with sensibility analysis to the ROV to determine the optimal life of a copper-gold mine, considering the costs associated to mine closure in its valuation. BaoJing and XueSheng [7] compared the option's discrete resolution method of binomial tree, with the continuous Black & Scholes model to evaluate an expansion option, subject to price uncertainty. Sabour and Poulin [1] developed a least-square Monte Carlo simulation to study

the options of abandoning, delaying or expanding a copper mine, dependant of the copper price scenario.

From Options ON Projects, to Options IN Projects

As it was just established, until very recently, all real option applications where focused on market risks, modeling price uncertainties as geometric Brownian motions, with mean reversion for base metals (as it was stated more than two decades ago by Brennan and Schwartz [9], and updated by Schwartz in 1997). The applicability of real options to external market uncertainties has proven to be very useful, however, its characteristics and applications have been widely studied, and thus, further work on this topic line, with the goal of being innovative, is relatively limited.

It's important to state that the applicability of real options are not limited to market changes (price, rates, etc.), but also have the potential to take into account internal, tactical uncertainties that affect the project. Wang and de Neufville [35] define the concept of options "on" projects, as the options that analyze variables that act upon the project (external conditions), and options "in" projects, as options that have the potential to actually change the design of the technical system. In other words, options that work upon the engineering variables, technology uncertainties and technical risks of the system. Kazakidis and Scoble [18] state that flexibility needs to be built into the project to not only act as insurance against adverse scenarios, but also to enable managers to take advantage of opportunities that may develop during the life cycle of the operation.

The previous examples represent options "on" projects. However, real option's applications have matured into "mixed" views, focused on market uncertainties, but also acknowledging, as an "in" project's starting point, the mine's geological uncertainty. Geological uncertainties were first included in the valuations by Dimitrakopoulos, Farely and Godoy [16] in year 2002, by simulating multiple equally probable orebodies based on drillhole's data via conditional simulation (a well known and widely implemented method).

Other examples of this mixed implementation are: Dimitrakopoulos, Martines and Ramazan [15] where price and geological uncertainties are modeled together in order to manage risk associated to grade uncertainty. The authors decide from the simulation results by defining a "minimum acceptable return" on investment. Here, the option is valued as the range between the downside risk insurance, and the upside potential advantage. Musingwini, Minnitt and Woodhall [26] refers to a "flexibility index (FI)" defined by Kazakidis and Scoble [18], that represents the option's impact over the system, and applies it to a production flexibility option by managing ore availability, i.e. the amount of free face required not only to meet production, but to acquire "technical flexibility" (FI>1). All this applied to a South African underground reef mine, located in the Bushveld Complex.

Dimitrakopoulos and Sabour [14] studied an operational flexibility option including price and geological uncertainties, and use ROV and DCF for valuing an actual mine that in real life has already been extracted. Their study shows that the ROV design is about 15% higher than the DCF. Sabour, Dimitrakopoulos and Kumral [2] also refers to price, rates and geological uncertainties, and develop a process to rank the simulated mine designs developed by Monte Carlo simulation and Whittle optimizations, in order to select the most favorable result. Akbari et al [4] include the same uncertain variables in its model, but acknowledges that reserves are not only uncertain due to lack of exploration, but because they are dependant of price (due to cut off changes). Here, the author uses a binomial tree to simulate the metal price, and defines the optimal starting point of the mine, and the ultimate pit limit (dependant of "today's" price. Li and Knights [21] innovate and apply real options analysis into short term mine planning by managing haulage routes to two different dumps, dependant of fuel price, which is also modeled as a mean reverting process of a geometric Brownian motion.

However, studies done solely over internal, technical, "in" the project variables are very limited, although they applicability in this area is wide. One relevant example is Kazakidis and Scoble [18],

where the author presents three option scenarios that appear due to ground-related problems: first, a sequencing option for increasing the flexibility of the production plan. Second the option of hiring extra rehabilitation crew to deal with ground-related problems; and third, a trade-off study between different flexible alternatives in order to optimize the mine plan. These alternatives are: to add a second crusher, to increase hoisting capacity to drill an extra vent raise, or to construct a second unlined orepass system. The decision is taken by using the same flexibility index as in [26], and its capital cost (the option's price). This is a very good example of real option's applications "in" projects, but the study's interest is placed on increasing project flexibility rather than managing its risk.

RISK MANAGEMENT MODEL

One of the goals of this paper is to demonstrate the potential of real options analysis to quantify and manage the risk associated to the project. This is, studying uncertain scenarios of feasible technical solutions, and including all of them, as the project's new flexibility, in the design and valuation.

Risk is a function of all the uncertainties present in the project. These uncertainties can create value and opportunities, but to do so, they must be diligently considered and classified, in order to account for their impact on the project. Some uncertainties may be eliminated by investing in more and better information, like investing in more drill holes in order to have a better idea of the ore body. However, there are some variables that can't be eliminated or diminished as the last one presented: the commodity's price is uncertain, no matter how much effort you put in estimating it with sophisticated models.

Because of this, project uncertainty must be quantified and managed and, in many cases, real option philosophy proves to be a powerful tool to implement over the design to achieve flexibility as the means to manage risk.

Botín et al [8] presents a classification of investment projects' internal risks that consists in four groups, depending on their probability of occurrence, and on the impact of the events. The groups mentioned are:

- A = Fatal Flaws (high impact, high prob.)
- B = Manageable (low impact, high prob.)
- C = Catastrophic (high impact, low prob.)
- D = Bearable (low impact, low prob.)

Using this classification, a risk management flow model has been developed and is shown in **figure 1**. Its goal is to define the actions that must be taken to value these risks, depending of each type. Risks A and C must be taken into account by a change in the basic engineering model, as they must be prevented at all cost. This action is especially focused on type "A" risks, which need a thorough revision of all possible project outcomes, in order to be prevented. At the opposite end are risks type D, where the cost of managing these risks is higher than the maximum possible gain obtained by eliminating them, and thus, they are not considered in the analysis.

Anyhow, the scope of this study focuses only on type B risks, particularly to technical risks that may affect the project at prefeasibility stages, that is, that stay latent along the project cycle, from early exploration, to mine closure. These risks are associated to variables that have an effect over the project's value, but can be controlled through different measures.

Type B risks can be subdivided into two classes: on one hand, variables which impact is limited to one process within the value chain (**figure 2**), but do not have an effect in downstream processes (left flow of **figure 1**), like for example, ore hardness. The higher the hardness, it will be more difficult to comminute, requiring more energy, and thus, higher energy costs that reduce the project's NPV. However, these effects don't really change the output. Here, the model is selected based on the difference of NPV between reality and the alternative without risk.

On the other hand, are the risks associated to variables that do have an impact on downstream processes, like ore grade and dilution:

a decrease in the grade or an increase in dilution has an impact on the amount of metal produced. As shown in **figure 1**, these types of risks may be evaluated and managed by using real options analysis, as different scenarios must be analyzed to define the model. In this case, the project's value corresponds to the value of the different options considered, defining different scenarios and their occurrence, according to the variable's probability distribution model.

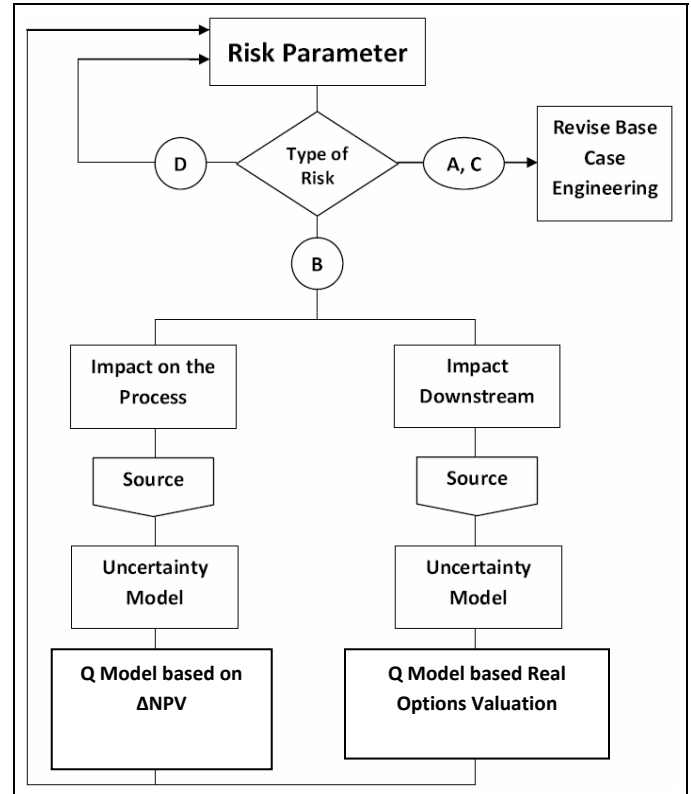


Figure 1. Risk Management Model.

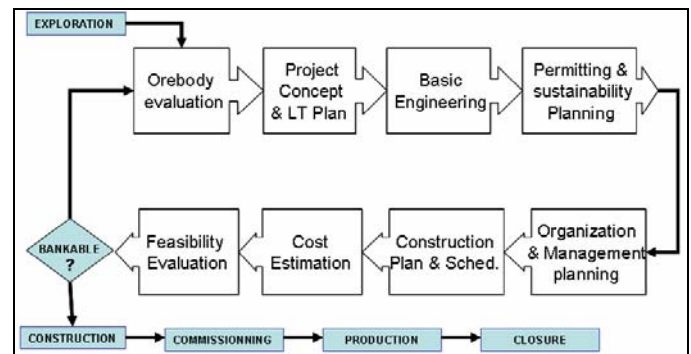


Figure 2. Project's value chain for feasibility decision [8].

It is important to notice, however, that both sides of the flowchart work with uncertainty models as the input value of the variable. This simple measure provides much more accurate results, as each event is simulated taking into account not only the simple mean NPV, but more importantly, the volatility and probability of occurrence. This concept is defined as the expected net present value (ENPV) by Brennan & Schwartz [9].

CASE STUDY

Option Contextualization

Risk in a Mine Planning process is related to the uncertainty in some critical variables such as ore grade, dilution, efficiency and performance, etc. In this case study ROV and Monte Carlo simulation are used to quantify the risk associated to ore dilution. It is worth

noting that this ROV model will be focused solely on internal technical variables, staying independent from external variables (market conditions), thus extending the common “safe ground” applicability of Real Options.

Dilution is a key variable in a mining project, especially when dealing with complex geometry low grade ore bodies. Its value depends on rock quality, equipment used and mining method, may reach values as high as 30%. In a mine plan, grade, ore recovery, dilution, and other variables are input to the planning model to estimate production rates. Moreover, in many feasibility studies, dilution is often assumed constant, and there’s rarely an estimate correction once production begins. Real Options Analysis is a method that has the potential to take its influence into account.

The objective of this Case Study is to apply ROA in the development of risk quantification and management strategies to account for the uncertainty associated to key project variables. Although ROV can be applied to many other internal and external variables, this paper focuses on dilution, a high impact technical variable that, when underestimated, may cause significant production losses.

Case Background

The Case Study being analyzed corresponds to a mining project, with several changes and simplification in the Base Case parameters (**table 1**) and the economic model (**table 2**).

Table 1. Project’s base case assumptions.

Ore Reserves (Mtonnes)	500
Monthly Ore Extraction (t/month)	3 185 841
Metallurgical Recovery	93.0%
Concentrate Grade	25.0%
Concentrate moisture	10.0%
Payable Cu	85.0%
Cu Price -US\$/lb	3.20
Treatment Costs – US\$/t cc	200
Distribution Costs – US\$/t cc	20
Cu Refining charges (US\$/kg Cu)	0.26
Fixed Operating Costs (MUS\$/year)	500
Variable Operating Costs (US\$/t min)	15
Non Operational Costs (US\$’000/year)	3 000
BC Extractable Reserves (Mtonnes)	565
BC Diluted Production (t/month)	3 600 000
BC Dilution	13.0%

Table 2. Project’s base case cash flow for the 3 years development and first 10 years of production.

Period	-3	-2	-1	1	2 - 10
Ore ROM (Mton)				34	43
Grade ROM (%)				0.885	0.885
Concentrate (Mton)				1 138	1 422
OPEX (MUS\$)				371	589
TAX 40% (MUS\$)				80	168
CAPEX (MUS\$)	200	500	1000	0	0
Cash Flow (MUS\$)	-200	-500	-1000	290	421
	VAN (10%)			MUS \$ 510	

The Base Case parameters in **table 1**, were used to develop a simplified annual cash flow statement shown in **table 2**, with a resulting NPV of MUS\$ 510, considering a discount rate of 10%. This base case is the starting point for the future option’s value.

The three first time periods (-3, -2, -1) in **table 2** correspond to preparation and development stages, and so, there are only capital expenses involved. On the first period, a ramp up at 80% production rate is considered, and full plant capacity is applied from the second production period onwards. Although ore reserves allow a longer

project life, a base reference valuation of 10 years, is used for all cases presented.

Simulation Model Description

The economic impact of extra dilution corresponds to a reduction of mill feed grade, therefore, the strategy translates into one, apparently simple objective: to maintain the projects output (tones of concentrate). To do this, it is necessary to implement extra flexibility into the mining and mineral processing systems of the project. Extra flexibility calls for extra capital requirements (CAPEX) to ensure that the operating systems (i.e. mine, mill and operating services), are sized to handle the waste generated by dilution. Extra CAPEX should be allowed for increased mine load-haul and ore transportation capacity, increased mineral processing rates, etc.

Dilution Model

Dilution uncertainty will be represented by a continuous probability distribution that depends on operational and geological conditions (both estimated and relatively uncertain). In this Case Study, dilution uncertainty is modeled by a parametric stochastic model with a most likely value (mode), equal to the Base Case dilution. Monte Carlo simulation is used to integrate the uncertainty associated to dilution into the project evaluation model, where all other variables are kept constant.

In most mining projects, the variability of dilution is characterized by a strong skewness towards higher dilution and therefore, a continuous gamma distribution model has been used to replicate the typical behavior of dilution. This probability density function is defined in equation 1.

$$f(x) = \frac{\left(\frac{x - \mu}{\beta}\right)^{\gamma-1} \cdot \exp\left(-\frac{x - \mu}{\beta}\right)}{\beta \cdot \Gamma(\gamma)} \tag{1}$$

In the previous equation, γ represents the shape parameter, which establishes how near is the mode from the minimum value; μ is the location parameter, that indicates the minimum value of the distribution; β is the scale parameter, and $\Gamma(\gamma)$ is the gamma function. In the current Case Study, the parameters of a gamma model were determined to fit a simple triangular distribution, of minimum value 12, maximum 21, and mode 13. The resulting model is shown in equation 2.

$$f(x) = \frac{\left(\frac{x - 12}{1}\right)^{2-1} \cdot \exp\left(-\frac{x - 12}{1}\right)}{1 \cdot \Gamma(2)} \tag{2}$$

With the corresponding distribution parameters:

Mean: $\beta \cdot \gamma + \mu = 1 \cdot 2 + 12 = 14$

Mode: $\beta + \mu = 1 + 12 = 13$

Standard Deviation: $\beta + \sqrt{\gamma} = 1 + \sqrt{2} = 1.4142$

This dilution distribution is an input to our valuation model. The objective here is to quantify the economic impact of a high dilution value. Dilution values higher than 13% (base case dilution) would increase the amount of waste through the production system and hence, reduce the capacity to mine and treat valuable ore and consequently, reduce copper production and project’s value. **Figure 3(b)** shows that the probability of this happening is almost 75%.

The integration of dilution as a probability function into the calculation of NPV results in a 8% reduction in the project NPV for every 1% extra dilution, and that the ENPV corrected by probability, defined in Brennan and Schwartz [9], has an actual value of MUS\$469, almost a 10% lower than the NPV calculated by the DCF. This relation can be seen in **figure 4**.

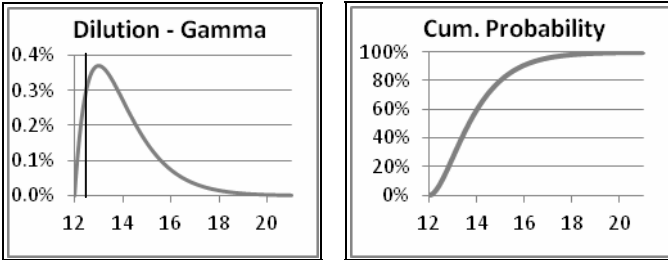


Figure 3. (a) Dilution as a Gamma distribution (b) Dilution's cumulative probability.

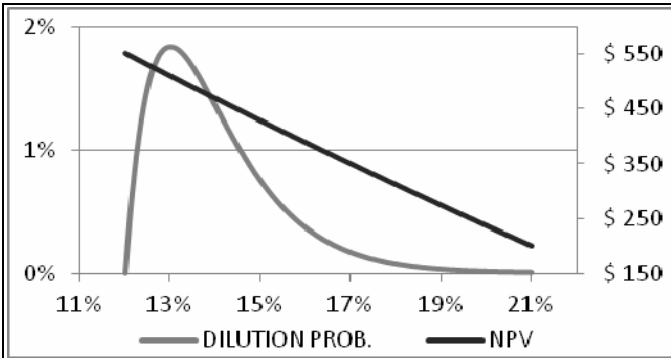


Figure 4. Base Case NPV for different dilutions.

To manage this risk, an increase in plant capacity is proposed to provide flexibility to the systems, enabling it to process the extra waste rock resulting from higher dilution, so that copper production rates may be sustained when dilution is higher than the expected 13% (Base case). In other words, a production system capable of maintaining copper production rates at maximum dilution value should be considered as a "dilution risk-free option". The determination of the "maximum dilution value" should be tied to a confidence level in the dilution model and in this case study, a value of "3-σ" (mean plus 3*standard deviation), equals to 17.3% dilution, with 96.3% probability of not being exceeded was used as the "risk free" value, as shown in table 3.

Table 3. Cumulative probability for different dilution limits.

Dilution	Cum. Prob.
d	13.0%
d + σ	14.4%
d + 2σ	15.8%
d + 3σ	17.3%

The cost of the real option is estimated as the extra capital expenditures required providing the project with the extra plant capacity and flexibility required by the "risk free" value of dilution. To calculate the value of the real option, it is required to evaluate: i) the extra capacity required to compensate the negative effect of increased dilution over the project's NPV (figure 4), ii) The variation of the project's NPV derived from the option.

Extra Capital Cost Model

To calculate the extra cost associated to a higher production rate, the William's model is used. This model is based in the cost relationship that exists between two plants or between two equipments of different capacity, power or volume, but of similar characteristics, and establishes the following relation:

$$C_A = C_B \left(\frac{Q_A}{Q_B} \right)^m \quad (3)$$

Where:

C_A ; C_B = Capital expenditures of plants A and B

Q_A ; Q_B = Production capacity of plants A and B
 m = Williams' exponent

In this case the Williams model is applied to evaluate the expansion of a mining and processing system, compared to its original capacity. In other words, plant A corresponds to the system with increased production rate, and plant B corresponds to the system with the Base Case capacity.

Williams' exponent depends on the investment's volatility and the presence of economies of scale, and varies by equipment, plant's processes or industry [25]. Mining process plants are commonly evaluated with an exponent of $m = 0.8$; however, as the plants being compared in this case are "the same one" the risk and variability is minor, and so, an exponent of $m = 0.7$ is used.

As a cost estimating method, Williams' model is only valid for an "order of magnitude estimate" but when used to estimate CAPEX for the same system at two different production rates, it provides the necessary accuracy. Also to be noticed that this option is intended for implementing flexibility in the system through a small capacity expansions, never higher than 10%. As such, rounding up the production rate, expansion costs must be calculated for capacities from 3.6Mt/mo to 4.0Mt/mo.

For this case, the Williams formula may be expressed in a simplified form:

$$C_A = C_B \cdot (1 + r)^m$$

Where r = plant's expansion weight factor

The determination of the weight factor r is a considerable problem, since it depends of dilution, which is represented by a probability function.

Option Valuation

Once the dilution distribution and the extra expansion costs available, the next step is to simulate the expected NPV for different monthly production rates, i.e. from 3.60Mton (base case), to 4.00 Mton (the 10% round up expansion). It is important to notice that there are actually two options present: (1) to maximize the project's NPV, producing at the plant's full capacity and (2) to maintain the ore feed constant. These two cases provide different results, and the decision to choose one over the other depends solely on the strategic plans of the mine management.

These production options can be considered as a "catalogue of possible responses" to dilution uncertainty in the operation plan, as presented by Cardin et al [10]. The results obtained for some of the simulations are presented in figure 5. In this figure it is also included the base case's NPV limit (the vertical line), the "d+3*s" confidentiality limit (the horizontal straight line) and the NPV's cumulative probability of a constant undiluted ore feed strategy, all three in segmented lines. This last curve intersects the base case curve and the NPV's base case limit in the same point, at 75% probability. This point represents the base case's context: its risk, ore production and value; and it shows that under these conditions, there's a 75% chance that the project's value will actually be lower than estimated. The goal is to lower this risk to 4%.

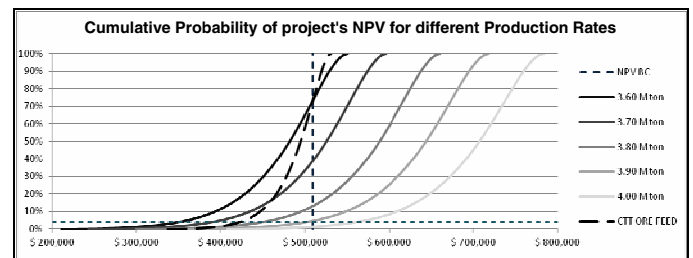


Figure 5. Simulation results for the cumulative probability of the project's NPV for different production rates

The zoom image presented in **figure 6** shows a zone of interest from **figure 5** which corresponds to the section where the mentioned curves intersect with the simulated production rates, at the required confidence level. It can be noticed (**figure 6**) that the two relevant productivities are: 3.74Mton/month for a minimum ore feed, and 3.90Mton/month for a minimum project NPV, both with a 96% confidence. It's important to differentiate between these two cases: the first one assumes that the system will operate to maintain copper contained in the mill feed at "base case" values. The second case assumes that the system will operate at its maximum capacity taking advantage of the extra flexibility to increase project's throughput. This other option ensures with a 96% of confidence that the project's value will be at least the base case's NPV (MUS\$510).

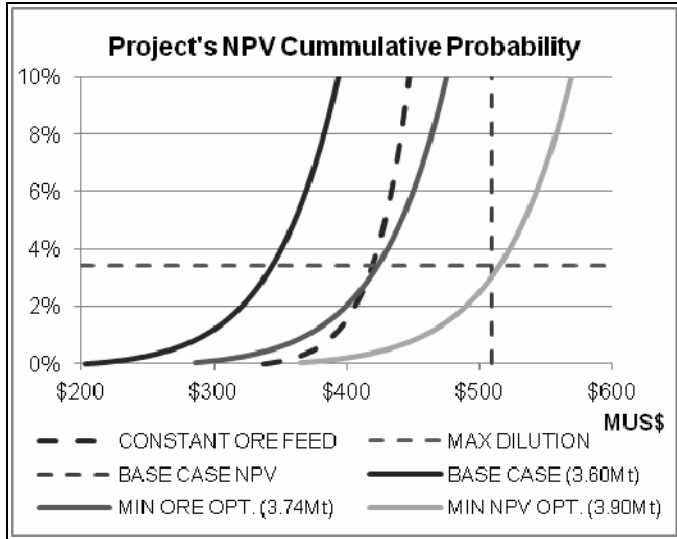


Figure 6. Detail of interest zone of figure 5.

These options leave us with two possible production rates and according to equation (4), the corresponding capital expenses for each option are shown in **table 4**.

Table 4. Capital expenditure for each option (MUS\$).

Period	3.60 Mt/mo	3.74 Mt/mo	3.90 Mt/mo
(-3)	200	205	210
(-2)	500	515	530
(-1)	1,000	1,030	1060

Capital expenditures vary with the flexibility being integrated into the project. However, as shown in **figure 7**, this cost is buffered by the income received by the extra mineral being processed. In this figure, the two upper curves correspond to fulfilling the plant's capacity, even if the amount of ore being processed is higher than the base case. On the other hand, the two inferior curves correspond to the options of investing in a higher flexibility operation, to produce exactly the amount of ore considered in the base case, even if the plant has spare capacity.

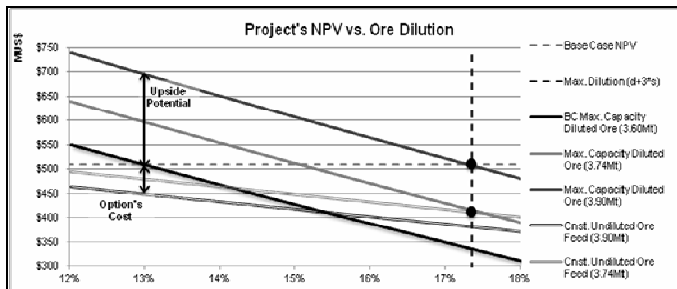


Figure 7. Project's NPV dependant of ore dilution for all studied options.

Analysis of the Results

The option of introducing flexibility to maintain a constant ore feed represents more clearly the structure of real option, where there's an option cost (the extra investment of building a bigger plant, presented in **figure 7**), and the flexibility presented as extra capacity available that can be occupied once unknown variables (in this case dilution) become known, to achieve the expected throughput. It's worth noting that the "option cost" presented in **figure 7** is not only the price of flexibility, but the cost of the risk associated to ore dilution.

However, the case of constant undiluted ore feed to the plant is not quite realistic, as if a plant is built for a given production capacity, it is operationally optimal to keep it working at its maximum, if ore is available. Taking advantage of this extra capacity available due to the introduced flexibility, may be considered as the upside potential of the option. Hence, only the two upper curves from **figure 7** are considered. A summary of the results is presented in **table 5**.

Table 5. Summary of the study's valuation results.

Option (Mt/mo)	Option's Cost PV	Upside Potential	ENPV MUS\$	Conf. Of BC's NPV
3.60	MUS\$ -	MUS\$ -	\$ 470	26.5%
3.74	MUS\$ 39.5	MUS\$ 86.2	\$ 554	80.8%
3.90	MUS\$ 79.0	MUS\$185.2	\$ 651	96.9%

Here is shown, the option's cost present value, the upside potential income due to the advantage of a higher capacity plant, the expected net present value (corrected by dilution's probability), and the confidentiality of achieving a higher NPV than the one considered in the base case, for each option.

As shown before, the two options have the same cost structure, and, as any option, this structure presents two different expenses: first, a premium acquisition cost, that's paid up front (first column of the previous table), that corresponds to an increase in the base CAPEX as the cost of dilution, and second, an exercise cost, that is accounted only if the option is applied. In this case study, this last expense is accounted for in the OPEX, as it represents the extra costs associated to a larger operation (all variable costs).

CONCLUSIONS

As shown in the previous case study, a real options analysis was successfully executed to measure the impact and manage the risk associated to dilution uncertainty in a mining project. Results show that the evaluated options presented almost an 18% improvement over the project's base case ENPV in the "constant undiluted ore feed" option, with a 96% confidentiality that ore feed will be the planned one. And more than a 38% improvement over the "minimum NPV" option, also with 96% confidentiality.

Further work can be done by extending the analysis to multiple concatenated variables, such as equipment performance, recovery, grade variability due to geological or sampling uncertainties. A grade equivalent could be obtained by multivariate simulation using least-square Monte Carlo simulation, "correcting" the grade by both uncertainties (dilution and ore grade). Even though the origins of dilution and grade uncertainties are different, the impact associated to their variability is the same: lower value in the run-of-mine ore, and so, the decision to manage the risk is the same, and should be taken in the same stage of the system (mine planning), and thus, it can be evaluated using real option analysis.

Even though ROV's implementation is not as direct as applying the DCF's net present value formula, the increasing complexity and marginal economics of future mining projects should drive a shift in perception towards the benefits of applying ROV to evaluate risk and take advantage of the upside potential of production systems.

REFERENCES

1. Abdel Sabour, S. a. & Poulin, R., 2010. [10] Mine expansion decisions under uncertainty. *International Journal of Mining, Reclamation and Environment*, 24(4), pp.340-349.

2. Abdel Sabour, S. a., Dimitrakopoulos, R.G. & Kumral, M., 2008. [01] Mine design selection under uncertainty. *Mining Technology: IMM Transactions section A*, 117(2), pp.53-64.
3. Abdel Sabour, Sabry a. & Poulin, Richard, 2006. [17] Valuing Real Capital Investments Using The Least-Squares Monte Carlo Method. *The Engineering Economist*, 51(2), pp.141-160.
4. Akbari, A.D., Osanloo, M. & Shirazi, M.A., 2009. [02] Reserve estimation of an open pit mine under price uncertainty by real option approach. *Mining Science and Technology (China)*, 19(6), pp.709-717.
5. Amram, M. & Kulatilaka, N., 1999. [36] *Real options: managing strategic investment in an uncertain world*.
6. Armstrong, M. et al., 2004. [33] Incorporating technical uncertainty in real option valuation of oil projects. *Journal of Petroleum Science and Engineering*, 44(1-2), pp.67-82.
7. Baojing, S. & XueSheng, F., 2010. [08] The Option Value Analysis and Application of Uncertainty of Mining Investment Projects. In *2nd IEEE International Conference on Information Management and Engineering (ICIME)*. pp. 411-415.
8. Botín, J.A., Guzmán, R.R. & Smith, M.L., 2011. [28] A Methodological model to assist the optimization and risk management of mining investment decisions. *The Society for Mining, Metallurgy and Exploration Annual Meeting*, 11(124), pp.1-6.
9. Brennan, M.J. & Schwartz, E.S., 1985. [23] Evaluating Natural Resource Investments. *Journal of Business*, 58(2), pp.135-157.
10. Cardin, M.-A., de Neufville, R. & Kasakidis, V., 2008. [19] A Process to Improve Expected Value on Mining Operations. *Mining Technology*, pp.1-15.
11. Cortazar, G, Gravet, M. & Urzua, J., 2008. [25] The valuation of multidimensional American real options using the LSM simulation method. *Computers & Operations Research*, 35(1), pp.113-129.
12. Cortazar, Gonzalo & Schwartz, E.S., 1993. [26] A Compound Option Model of Production and Intermediate Inventories. *The Journal of Business*, 66(4), pp.517-540.
13. Cortazar, Gonzalo, Schwartz, E.S. & Casassus, J., 2003. [20] Chapter 9 Optimal exploration investments under price and geological – technical uncertainty: a real options model. In *Real R&D Options*. pp. 149-165.
14. Dimitrakopoulos, R & Abdelsabour, S., 2007. [09] Evaluating mine plans under uncertainty: Can the real options make a difference? *Resources Policy*, 32(3), pp.116-125.
15. Dimitrakopoulos, R.G., Martinez, L. & Ramazan, S., 2007. [05] A Maximum Upside /Minimum Downside Approach to the Traditional Optimization of Open Pit Mine Design. *Journal of Mining Science*, 43(1), pp.73-82.
16. Dimitrakopoulos, Roussos, Farrelly, C.T. & Godoy, M., 2002. [32] Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. In *Transactions - Institution of Mining and Metallurgy*. pp. 82-88.
17. Hull, J.C., 1997. [24] *Options, Futures, and Other Derivatives* Seventh Ed. D. Clinton, ed., Pearson Education, Inc.
18. Kasakidis, V.N. & Scoble, M., 2003. [13] Planning for flexibility in underground mine production systems. *Mining Engineering*, 55(8), pp.33-38.
19. Kodukula, P. & Papudesu, C., 2006. [35] *Project valuation using real options: a practitioner's guide*.
20. Lee, E. & Strang, D., 2003. [04] Valuation techniques used in the mining industry Part three: Real option valuation approach. *Mining Engineering*, 55(12), pp.8, 9.
21. Li, S.-xing & Knights, P., 2009. [06] Integration of real options into short-term mine planning and production scheduling. *Mining Science and Technology (China)*, 19(5), pp.674-678.
22. Martinez, L., 2009. [31] Why accounting for uncertainty and risk can improve final decision making in strategic open pit mine evaluation. In *Project Evaluation Conference*. p. 113.
23. Martinez, Luis & McKibben, J., 2010. [30] Unverstanding Real Options in Mine Project Valuation: A Simple Perspective. In *MININ 2010*. pp. 223-234.
24. Mayer, Z. & Kazakidis, V., 2007. [07] Decision Making in Flexible Mine Production System Design Using Real Options. *Journal of Construction Engineering and Management*, (February), pp.169-180.
25. Mular, A.L., 2002. [34] Major Mineral Processing Equipment Costs and Preliminary Capital Cost Estimations. In *Mineral Processing Plant Design, Practice and Control. Proceedings*. pp. 310-325.
26. Musingwini, C., Minnitt, R.C.A. & Woodhall, M., 2007. [12] Technical operating flexibility in the analysis of mine layouts and schedules. *The Journal of the Southern African Institute of Mining and Metallurgy*, 107(February), pp.129-136.
27. Northcote, A.E.A., 2007. [27] Managing the Study Risk. In *Project Evaluation Conference*. pp. 223-227.
28. Sabour, S.A.A. & Wood, G., 2009. [14] Modelling financial risk in open pit mine projects: implications for strategic decision-making. *The Journal of the Southern African Institute of Mining and metallurgy*, 109(March), pp.169-175.
29. Samis, M et al., 2006. [11] Valuing uncertain asset cash flows when there are no options: A real options approach. *Resources Policy*, 30(4), pp.285-298.
30. Samis, Michael, Laughton, David & Davis, Graham, 2005. [03] Valuing resource extraction projects using real options. *CIM Bulletin*, 98(May), p.82.
31. Sepúlveda, G.F., Branch, J.W. & Jaramillo, P., 2010. [21] An Approximation to the Open Pit mine Planning Approach Based on Decisions under Uncertainty. *Boletín de Ciencias de la Tierra*, 28(November), pp.7-14.
32. Snowden, D.V., Glacken, I. & Noppe, M., 2002. [15] Dealing With Demands of Technical Variability and Uncertainty Along the Mine Value Chain Nodes of uncertainty. In *Value Tracking Symposium*. pp. 1-8.
33. Tan, W., Liu, H. & Zhang, W., 2009. [22] Real Option Model Application and Sensitivity Analysis in Mineral Resources Investment Project. In *International Conference on Information Management, Innovation Management and Industrial Engineering*. IEEE, pp. 200-203.
34. Tulcanaza, E. & Zenteno, L., 1998. [29] Options for Hedging Mine Planning Scenarios To Meet Contingencies. *Engineering and Mining Journal*, 199(5, 6, 7), pp.56-60, 36-38, 46-49.
35. Wang, T. & de Neufville, R., [18] Identification of Real Options “in” Projects. *Proc. of INCOSE 16, Orlando*, pp.1-10.