MULTIPLE OBJECT TRACKING USING AN AUTOMATIC VARIABLE-DIMENSION PARTICLE FILTER

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ABSTRACT

Object tracking through particle filtering has been widely addressed in recent years. However, most works assume a constant number of objects or utilize an external detector that monitors the entry or exit of objects in the scene. In this work, a novel tracking method based on particle filtering that is able to automatically track a variable number of objects is presented. As opposed to classical prior data assignment approaches, adaptation of tracks to the measurements is managed globally. Additionally, the designed particle filter is able to generate hypotheses on the presence of new objects in the scene, and to confirm or dismiss them by gradually adapting to the global observation. The method is especially suited for environments where traditional object detectors render noisy measurements and frequent artifacts, such as that given by a camera mounted on a vehicle, where it is proven to yield excellent results.

Index Terms— Tracking, particle filter, mixture model, likelihood, vehicle detection.

1. INTRODUCTION

This work addresses tracking of multiple objects using sequential data measurements. In practice, the acquisition of measurements has an inherent degree of uncertainty due to measurement noise, clutter (false positives), inaccuracies of the model, etc. Hence, a probabilistic framework that reflects this uncertainty is needed. The Bayesian approach, especially the particle filter (PF) [1], is extensively used for this purpose in tracking applications.

Bayesian tracking applied to multiple objects has been performed in the literature both using individual particle filters for each target and defining a joint state space comprising all the objects. The latter is usually preferred when there is some degree of interaction between objects, and is used in many relevant works in the field [2][3]. Nevertheless, joint multi-object filters are unsuitable for tracking a large number of objects in the absence of a reasonable motion model, as the computational complexity increases exponentially with the dimension of the state vector. On the other hand, the use of an individual particle filter for each object complicates the modeling of object interactions [2].

Recently, some works have been proposed that combine particle filtering with other techniques in order to optimize computation in such a way that the joint multi-object tracking is affordable. For instance, in [4] a level set-based active contour method is utilized to build a decision boundary in the state space using shape information and pose invariance of the tracked object, so that particles out of the boundary are removed. In [5] a kernel-based Bayesian filtering framework is proposed which represents likelihood and posterior densities using Gaussian mixture models, enabling more efficient sampling in high dimensional spaces. MCMC-based particle filters that model the interaction of objects using MRF have also been proposed [2][3].

However, most approaches in the literature impose a constant number of objects, or alternatively assume that the entry of new objects is externally managed [6] or manually initialized [7]. In addition, a random walk is assumed to model the motion of objects. Thus, the uncertainty in the location of objects is high and a large amount of particles is needed to sample the state space.

In order to overcome these limitations, in this work a multiple object tracker based on a particle filter is presented, which is not only able to automatically handle a variable number of objects but which also manages object entries and exits intrinsically (i.e., within the framework of the proposed particle filter), without the need of an external control module. The filter involves a joint multidimensional state space with a dynamically changing dimension, which adapts to the number of objects in the scene. Additionally, as opposed to classical random walk assumption, a first-order linear predictor is used for the motion model, which adapts better to the coherent motion of objects. This allows for the use of smaller sample sets and hence for a reduction of the computational complexity. The method uses noisy sequential data measurements, but in contrast to most approaches in the literature, does not require prior data assignment. Instead, the proposed algorithm entails a mixture likelihood model that considers all possible dependencies between objects and measurements. It is especially suited for scenarios that feature small/medium number of objects with noisy measurements and spurious artifacts, such as the traffic scenario given by a camera mounted on a vehicle. The proposed tracker has been successfully applied for vehicle tracking in this kind of traffic sequences.

2. MULTIPLE OBJECT TRACKING

The Bayesian approach provides an ideal framework for dynamic state estimation, as it allows to recursively update the state of the system with the new measurements. Namely, the aim of the Bayesian algorithms is to recursively construct the posterior pdf of the state sequence $\{\mathbf{x}_k, k \in \mathbb{N}\}$ given the measurements $\mathbf{z}_{1:k}$. Within the Bayesian framework, particle filtering has been extensively used in many recent works in the field of tracking. The underlying idea of particle filters is to approximate the posterior probability density function with a set of discrete representations

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(i.e., particles), and their associated importance weights, as [1]:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) \approx \sum_{s=1}^{n_s} w_k^{(s)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(s)})$$

where $w_k^{(s)}$ is the weight associated to particle $\mathbf{x}_k^{(s)}$. Among the multiple variations of the importance sampling algorithm, in this work the SIR (Sampling Importance Resampling) filter is used. As shown in [1], in this particular case the weights are given by $w_k^{(s)} \propto p(\mathbf{z}_k|\mathbf{x}_k^{(s)})$. Additionally, the particles are resampled at every time index as explained in [1].

2.1. Motion and observation models

In the designed particle filter, the measurements at time k are given by the observed positions of the objects. Hence, $\mathbf{z}_k$ is a vector containing the measured positions of all objects in the image, i.e., $\mathbf{z}_k = \{z_{x,k}^j, z_{y,k}^j\}_{j=1}^M$, where $z_{x,k}^j$ and $z_{y,k}^j$ are the x- and y-coordinates of measurement j at time k. The operation of the PF relies on a good choice of both the motion model and the observation model, which links the state to the measurements. In this work, as opposed to classical random walk approaches, the motion model is assumed to be linear with constant velocity. This model approximates better the real motion of objects in many applications, such as vehicle or pedestrian tracking. In reality, the linear motion of the object is usually distorted by the perspective effect due to the position of the camera. Hence, we propose to work on a transformed domain obtained through a plane rectification (usually known as IPM [8]) that gives a bird's-eye view of the scene, in which the movement of the objects is linear. The state vector, $\mathbf{x}_k$, is composed of the position, (x_k^i, y_k^i), and velocity, $(\dot{x}_k^i, \dot{y}_k^i)$, of all objects in the transformed domain, thus if there are N objects in the scene at time k, it is given by $\mathbf{x}_k = \{x_k^i, y_k^i, \dot{x}_k^i, \dot{y}_k^i\}_{i=1}^N$.

On the other hand, a mixture model [9] is used to characterize the observation process. Since the state vector contains all the objects in the scene, it is expected that a measurement is found for each of the objects near the position indicated by the corresponding element of the state vector. Hence, if the observation comprises M measurements, the pdf of each measurement, $p(\mathbf{z}_k^j|\mathbf{x}_k)$, is represented as a mixture model,

$$p(\mathbf{z}_k^j|\mathbf{x}_k) = \sum_{i=1}^{\mathrm{N}} \alpha_i p_i(\mathbf{z}_k^j|\mathbf{x}_k^i) + \alpha_u \mathcal{U}(\mathbf{z}_k^j) \quad (1)$$

where $p_i(\mathbf{z}_k^j|\mathbf{x}_k^i) = \mathcal{N}(\mathbf{z}_k^j; \mu_k^i, \sigma_k^i)$ for $i = 1, \ldots, \mathrm{N}$, with $\mu_k^i = (x_k^i, y_k^i)$. The standard deviation σ_k^i depends on the measurement noise. That is, we model the probability density of each measurement via N component densities mixed together with mixing coefficients α_i. These component densities are bivariate Gaussian distributions centered around the positions of the existing objects given by the state vector. Observe that measurements that are close to any of the modes representing the positions of the objects in the state vector, and measurements that are in the neighborhood of several of these modes will render high density values. However, some measurements may be produced by noise rather than by one of the objects, therefore we include an additional component in (1) given by a uniform distribution, $\mathcal{U}(\cdot)$, with coefficient α_u, to model scattered noise. Formally, to be a proper density function, it must be $\int_W \mathcal{U}(\cdot) = 1$, where W is the domain of the uniform pdf, i.e., an image of width w and height h; thus, $\mathcal{U}(\cdot) = 1/wh$.

In addition, all measurements are drawn independently in the measurement generation process, therefore the joint measurement density function, $p(\mathbf{z}_k|\mathbf{x}_k)$, is modeled as

$$p(\mathbf{z}_k|\mathbf{x}_k) = \prod_{j=1}^{\mathrm{M}} [\sum_{i=1}^{\mathrm{N}} \alpha_{ij} p_i(\mathbf{z}_k^j|\mathbf{x}_k^i) + \alpha_u \mathcal{U}(\mathbf{z}_k^j)] \quad (2)$$

As regards the mixing coefficients, they are defined as

$$\alpha_{ij} = (1-\alpha_u) p_i(\mathbf{z}_k^j|\mathbf{x}_k^i) / \sum_{t=1}^{\mathrm{N}} p_i(\mathbf{z}_k^j|\mathbf{x}_k^t) \quad \begin{cases} i = 1, \ldots, \mathrm{N} \\ j = 1, \ldots, \mathrm{M} \end{cases} \quad (3)$$

Namely, the coefficients α_{ij} of the Gaussian components are weighted proportionally to the likelihood of the object to which they are associated. That is, a high weight is given to the model $p_i(\mathbf{z}_k^j|\mathbf{x}_k^i)$, which indicates the likelihood that object i generated measurement j, when this likelihood is larger than that of other objects. The factor $(1 - \alpha_u)$ in equation (3) guarantees that $\sum_{i=1}^{\mathrm{N}} \alpha_{ij} + \alpha_u = 1 \quad \forall j$, as imposed by the mixture model. As mentioned above, the components p_i of the mixture model in (1) are bivariate Gaussian distributions centered around $\mathbf{x}_k^i$, $p_i(\mathbf{z}_k^j|\mathbf{x}_k^i) = \frac{1}{2\pi\sigma_1\sigma_2}\exp(-\frac{1}{2}[\frac{(z_{x,k}^j - x_k^i)^2}{\sigma_1^2} + \frac{(z_{y,k}^j - y_k^i)^2}{\sigma_2^2}])$. Therefore, the coefficients α_{ij} are given by

$$\alpha_{ij} = (1-\alpha_u) \frac{\exp(-\frac{1}{2}[\frac{(z_{x,k}^j - x_k^i)^2}{\sigma_1^2} + \frac{(z_{y,k}^j - y_k^i)^2}{\sigma_2^2}])}{\sum_{t=1}^{\mathrm{N}} \exp(-\frac{1}{2}[\frac{(z_{x,k}^j - x_k^t)^2}{\sigma_1^2} + \frac{(z_{y,k}^j - y_k^t)^2}{\sigma_2^2}])}$$

On the other hand, the coefficient of the uniform component α_u, parameterizes the importance of the scattered noise within the mixture model. This coefficient will adopt different values depending on the reliability of the observation model, as shall be discussed in Section 3.1.

The power of the mixture model lies in that, given one measurement, the likelihood of the state vector (i.e., the probability that such state vector generated that measurement) is composed of all the the individual probabilities of each object having generated the measurement, rather than just the likelihood of one of the objects (namely, the closest to the measurement in typical approaches). Hence, although a larger weight is assigned to the likelihood of the object closest to the measurement, the probability of the measurement being generated by another object or by clutter is also considered.

In summary, as opposed to classical approaches (e.g., Kalman filtering, particle filtering after data association stage), with the proposed approach all objects may contribute to the probability of observation of each measurement given the state vector, hence allowing the observation model to gather all possible dependencies between each measurement and all the set of objects. Besides, the uniform distribution component of the mixture model captures possible uncertainties in the measurement generation process.

2.2. Inference of results

At every stage of the object tracking, results must be rendered for the positions of the objects in the image. These are inferred from the set of particles active at that point in time, after the resampling stage. Among all possible average values, the conditional mean estimate of the state is chosen, which is given by

$$\bar{\mathbf{x}}_k = \frac{1}{n_s} \sum_{s=1}^{n_s} \mathbf{x}_k^{(s)}$$

where $\mathbf{x}_k^{(s)}$ is the state vector of the s-th particle.

3. AUTOMATIC MANAGEMENT OF ENTRIES AND EXITS

In contrast to most methods in the literature, which make use of an external detector, we propose to use the intrinsic power of the particle filter to also manage the entry and exit of objects, hence avoiding the need of an external module. As regards object entries, the system enters a transitory period whenever a measurement that is not adequately represented by the estimated pdf arises. In this period, aside of the pdf predicted from previous instants, a subset of particles is devoted to hypothesize a new pdf with an extended state vector (that is, to hypothesize a new object). During the transitory period, if the incoming measurements support the hypothesis, particles associated to the newly hypothesized pdf will propagate extensively implying the appearance of a new object, and only few (if any) particles with the original dimension will remain. Then, the transitory period ends and the new pdf will be selected. Conversely, the previous pdf will be propagated if the measurements do not support the extension of the state vector.

More formally, if at time k one of the elements of the state vector is not adequately represented by the current set of objects, the transitory period is triggered. An arbitrary function can be selected to model the adequation of objects to measurements (in this case, a Euclidean distance criterion over the transformed domain is used). At the start of the transitory period, n_r particles from the total pool of n_s ($n_r << n_s$) particles hypothesize the existence of a new object. The state vector of these particles is denoted by an apostrophe and initialized to $\mathbf{x}'_k = \{\bar{\mathbf{x}}_k, \mathbf{x}_k^{\mathrm{N}+1}\}$, where $\mathbf{x}_k^{\mathrm{N}+1}$ is given by the new measurement $\mathbf{z}_k^j$. Therefore, two pdfs, p and p', are hypothesized at time k; the former of dimension N, the latter of dimension N+1:

$$p = p(\mathbf{x}_k|\mathbf{z}_{1:k}) \approx \sum_{s=1}^{n_s-n_r} w_k^{(s)}\delta(\mathbf{x}_k - \mathbf{x}_k^{(s)})$$
$$p' = p(\mathbf{x}'_k|\mathbf{z}_{1:k}) \approx \sum_{s=1}^{n_r} w_k^{(s)}\delta(\mathbf{x}_k - \mathbf{x}'^{(s)}_k)$$

All particles are propagated with the motion model specified in Section (2.1). The likelihood will be larger for those particles whose state matches the measurements. Being so, if there are measurements for the hypothesized new object in the subsequent frames, the (N+1)-dimension particles (hereafter denoted simply (N+1)-particles) will prevail in the resampling process due to their larger likelihood, and therefore they will propagate progressively. Finally most particles will belong to this class, and the function p' will be said to model better the posterior density. Conversely, if the measurements do not support the hypothesized dimension increase, (N+1)-particles will not propagate. In this case, the posterior density is modeled by p.

The tracker is also able to manage object exits. In effect, the tracker correlates the positions of objects at different times, thus it can also detect when an object abandons the region of interest. When this happens, the state vector of all particles is reduced accordingly.

3.1. Selection of coefficient α_u

The component α_u determines the weight of the uniform noise component in the mixture model. In essence, it parameterizes the reliability of the measurement generation process. The selection of this coefficient thus determines the adaptation of the state vector to the observations: if α_u is low, the particles that better match the measurements will propagate rapidly; conversely, if α_u is high, little confidence is given to the measurements, and the particles will only slowly adapt to these. Hence, the selection of α_u is tightly related to the multidimensionality of the state vector; indeed, during transitory periods there is a degree of uncertainty in the reliability of the measurement vector, as a measurement exits that could either be produced by noise or due to the entry of an object in the scene. It is wise then to increase the noise in the observation model via a larger value of α_u during transitory periods. By doing so, the particles adapt progressively (but slowly) to the measurements, in such a way that only if the measurements are coherent for a sufficiently large period of time, one of the dimensions prevails. Then the dimension of all the particles is homogenized and the particle filter returns to normal operation.

The value of α_u for transitory periods is fixed as a function of the ratio, F, between the likelihood of a (N+1)-particle and that of a N-particle, assuming that M measurements, $\{\mathbf{z}_k^j\}_{j=1}^{\mathrm{M}}$, exist ($\mathrm{M} > \mathrm{N}$): N of them ($\mathbf{z}_k^j$, $1 < j \leq \mathrm{N}$) are close (ideally coincident) to the N existing objects, one of them ($\mathbf{z}_k^{\mathrm{N}+1}$) is close (ideally coincident) to the object hypothesized in the transitory period for the (N+1)-particles, and the remaining measurements ($\mathbf{z}_k^j$, $j > \mathrm{N}+1$) are far from the objects. This will be the scenario when a new object enters the scene, for which there are consistent measurements in time. Then, for (N+1)-particles, it is

$$p(\mathbf{z}_k^j|\mathbf{x}_k) \simeq \begin{cases} (1-\alpha_u)/(2\pi\sigma_1\sigma_2) + \alpha_u/(wh) & \text{if } j \leq \mathrm{N}+1 \\ \alpha_u/(wh) & \text{otherwise} \end{cases}$$

Therefore, according to equation (2), the likelihood equals

$$p(\mathbf{z}_k|\mathbf{x}_k) = [(1-\alpha_u)/(2\pi\sigma_1\sigma_2)+\alpha_u/(wh)]^{\mathrm{N}+1}(\alpha_u/wh)^{\mathrm{M}-\mathrm{N}-1}$$

In contrast, for N-particles, the state vector does not hypothesize a new object. Thus, the measurement $\mathbf{z}_k^{\mathrm{N}+1}$ is not near any of the objects in the state vector, and we have

$$p(\mathbf{z}_k^j|\mathbf{x}_k) \simeq \begin{cases} (1-\alpha_u)/(2\pi\sigma_1\sigma_2) + \alpha_u/(wh) & \text{if } j \leq \mathrm{N} \\ \alpha_u/(wh) & \text{otherwise} \end{cases}$$
$$p(\mathbf{z}_k|\mathbf{x}_k) = [(1-\alpha_u)/(2\pi\sigma_1\sigma_2) + \alpha_u/(wh)]^{\mathrm{N}}(\alpha_u/wh)^{\mathrm{M}-\mathrm{N}}$$

The ratio F between likelihoods is then

$$\mathrm{F} = \frac{p(\mathbf{z}_k|\mathbf{x}_k)\mid_{(\mathrm{N}+1)-particle}}{p(\mathbf{z}_k|\mathbf{x}_k)\mid_{\mathrm{N}-particle}} = \frac{(1-\alpha_u)/(2\pi\sigma_1\sigma_2) + \alpha_u/(wh)}{\alpha_u/(wh)}$$

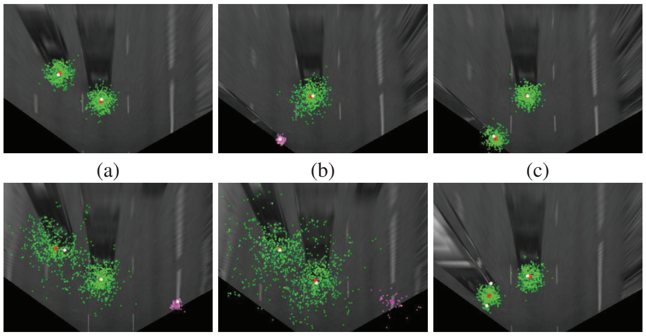
which after some calculation yields

$$\alpha_u = 1/[1 + (2\pi\sigma_1\sigma_2)(\mathrm{F}-1)/wh)] \quad (4)$$

The parameter F represents the ratio of propagation between both types of particles in the transitory period when the measurements support the hypothesized dimension increase. As discussed previously, during the transitory period the ratio F shall be kept low (e.g., $1 < \mathrm{F} < 10$) in order for the (N+1)-particles not to propagate too quickly. Finally, recall that during normal operation the measurements are assumed to be highly reliable, and consequently particles must propagate according to their adequation to the measurements. In this case, α_u may adopt a wide range of values (always smaller than for the transitory period), provided that it ensures only a marginal contribution of the scattered noise term in the observation model.

4. EXPERIMENTS AND RESULTS

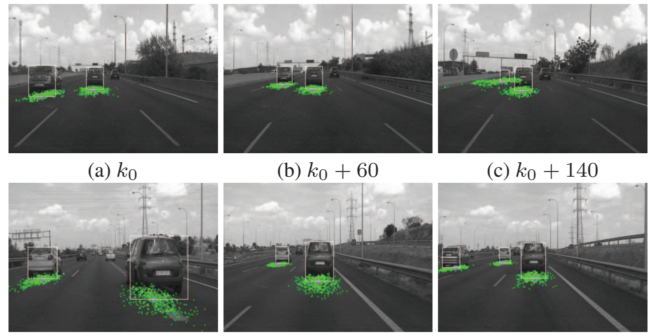
The method presented in this paper has been applied for vehicle tracking in the traffic scenario given by a vehicle-mounted camera. In this environment, vehicle detectors are especially prone to errors due to the ego motion of the camera. The proposed PF is built upon the measurements given by an external object detector. The state vector is composed of the positions (defined as the mid-lower point) and velocities of the vehicles in the transformed domain given by the IPM. Tests show that the method is able to provide accurate tracking of the time-uncorrelated object measurements given by the external detector, to efficiently manage object entries and to dismiss false positives due to errors delivered by the detector.



(a) (b) (c)

(d) (e) (f)

Fig. 1. Distribution of particles (green) in the transformed domain. Measurements are painted in white and inferred estimates in red.



(a) k_0 (b) $k_0 + 60$ (c) $k_0 + 140$

(d) k_0 (e) $k_0 + 160$ (f) $k_0 + 215$

Fig. 2. Tracking for two example sequences, in the upper and lower row, respectively.

An example of the behavior of the proposed PF is shown in Fig. 1. Observe that images represent the bird's-eye view given by the IPM. In the normal operation, particles are tightly gathered around the measured positions of the vehicles (see Fig. 1a). Most importantly, the proposed method is able to detect and track new objects, and to discard artifacts. As shown in Fig. 1b, when an object enters the scene, the PF initially hypothesizes a new object with a small set of particles (painted in pink). The hypothesis matches the incoming measurements, thus particles containing it propagate quickly (see Fig. 1c), and the object is confirmed and tracked. Analogously, when the detector delivers an erroneous measurement, the PF initially hypothesizes a new object with a small set of particles (see Fig. 1d). However, after some frames (Fig. 1e) these particles fade away, since they are not supported by subsequent measurements, and the tracker discards the artifact. Remarkably, the tracker is able to overcome the false positives generated around existing objects. In effect, in complex environments (such as the addressed vehicle tracking) detectors are prone to deliver several measurements for the same object. This is the case for the left vehicle in Fig. 1f. Traditional rule-based approaches would assign the nearest measurement to the vehicle and probably create a new track for the other measurement. In contrast, in our method all measurements contribute to the likelihood model and therefore the object is correctly estimated to be in the intermediate region, and false detections are avoided.

The positions of the vehicles are inferred as the average of the particles, transformed back to the original domain, as shown in the examples in Fig. 2. In order to also characterize vehicle dimensions, bounding boxes are drawn for each vehicle around the position given by the PF using a gradient-based strategy as in [8]. Fig. 2a-c shows tracking of two vehicles for normal operation. In Fig. 2d-f, a more complex scenario is also proven to be satisfactorily solved: first, the vehicle appearing in the right-hand side is detected (Fig. 2d), then tracking of the two vehicles in the image is maintained (Fig. 2e), and finally a new vehicle is detected in the left-hand side (Fig. 2f).

5. CONCLUSIONS

In this work a novel multiple object tracking strategy based on a variable dimension particle filter has been presented. The proposed method provides a complete object tracking framework including object entry and exit management. The tracker is able to overcome the limitations of classical object detectors, which typically involve noisy measurements and spurious artifacts. New object appearance is managed intrinsically by the proposed PF, which adapts its dimension to the number of objects in the scene. The strength of the method lies on the designed mixture observation model, which gathers all possible dependencies between measurements and objects. The method is especially applicable for tracking applications with a small/medium number of objects, such as vehicle tracking. The method has been proven to perform well for complex traffic environments with a vehicle-mounted camera.

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