Space Studies of the Upper Atmospheres of the Earth and Planets including Reference Atmospheres (C)
Active Experiments Related to Space Plasmas (C52)

THE RADIATION IMPEDANCE OF A CURRENT-CARRYING CONDUCTOR IN A JUNO-LIKE JOVIAN ORBIT

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The so-called electrical power generation problem for exploration of the outer planets could be solved deploying an electrodynamic tether. Wave radiation by a conductor carrying a steady current in a polar, highly eccentric, low perijove orbit, as in the planned NASA Juno mission, is considered. The high Jupiter's oblateness produces fast apsidal precession over the meridional plane. In a cold plasma model, radiation occurs in the Alfven and Fast Magnetosonic modes, exhibiting large refraction index. The radiation impedance in both modes is determined for a representative arc in the orbits. Unlike the Earth ionospheric case, the low-dense and highly-magnetized Jovian plasma makes the electron-gyrofrequency to plasma-frequency ratio large [1]; this substantially modifies the power spectrum in either mode.

[1] Sánchez-Torres, A., Sanmartín, J.R., Donoso, J.M., Charro, M., J. Adv. Space Res. (2010), doi :10.1016/j.asr.2009.12.007; Sanmartín, J.R., Martínez-Sánchez, M., J. Geophys. Res. 100, pp. 1677-1686 (1995).

The radiation impedance of a current-carrying conductor in a JUNO-like Jovian orbit

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NASA's Juno mission to Jupiter will need electric power generation. Tethers generate power more efficently than solar panels or (RTG's source).

The baseline Juno mission

- Solar-powered mission.
- high eccentricity polar orbit (avoiding the intense radiation belts).
- perijove ($r_p \sim 1{,}06\,\mathrm{R_J}$), apojove ($r_p \sim 39\,\mathrm{R_J}$).

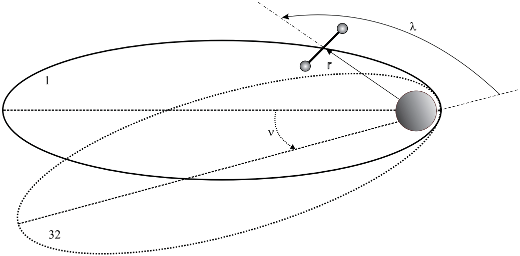
A tether generating power in a Juno-like mission:

- radiates waves as it moves through the Jovian magnetospheric plasma.

- The radiation impedance is required to determine the current in the overall tether circuit.

- The radiated wave signal might be used for Jovian plasma diagnostics.

The large Jovian J_2 zonal harmonic produces substantial apsidal precession



Approximating the orbit as nearly-parabolic $(1 + \cos \lambda = 2r_p/r)$ away from apojove, the orbital velocity modulus reads

$$V_{orb} = \sqrt{\frac{2\mu_J}{r}} \approx 59{,}7\sqrt{\frac{R_J}{r}}\frac{km}{s}$$

The corotating plasma velocity is $V_{pl} = 12{,}6\frac{r}{R_J}\left|\cos(\lambda + \nu)\right| \ \frac{\mathrm{km}}{\mathrm{s}}$

The plasma density (*Divine-Garrett*, **83**) reads ($1{,}0 < r/R_J < 3{,}8$)

$$N_e = 4{,}65 \cdot \exp\left[7{,}68\,\frac{R_J}{r} - \left(\frac{r}{R_J} - 1\right)^2 (\lambda + \nu)^2\right]\ \mathrm{cm}^{-3}$$

The magnetic field neglecting its tilt is

$$B = B_0 \left(\frac{R_J}{r}\right)^3 \sqrt{1 + 3\sin^2(\lambda + \nu)}\,, \qquad B_0 \approx 4{,}23\,\mathrm{Gauss}$$

The Alfven velocity (for S^+ ions) reads

$$V_A = c\,\frac{\Omega_i}{\omega_{pi}} \approx 3{,}85 \times 10^5\,\frac{B\ [\mathrm{Gauss}]}{\sqrt{N_e\ [\mathrm{cm}^{-3}]}}\ \frac{\mathrm{km}}{\mathrm{s}}$$

For a steady current-carrying tether, the Doppler relation gives $\omega = \mathbf{V}_{rel} \cdot \mathbf{k} = (\mathbf{V}_{orb} - \mathbf{V}_{pl}) \cdot \mathbf{k}$, with the x-axis perpendicular to the orbital plane and z along $\mathbf{B}$.

It follows that the refraction index is very large.

$$n \equiv \frac{ck}{\omega} = \frac{ck}{\mathbf{V}_{rel} \cdot \mathbf{k}} \gg 1$$

$\Rightarrow$ Both Alfven and Fast magnetosonic waves are radiated in the cold plasma model. The equation for the Fourier transform of electric field is

$$-\frac{\mathbf{k} \wedge (\mathbf{k} \wedge \mathbf{E})}{k^2} - \frac{\overline{\overline{\varepsilon}}_c \cdot \mathbf{E}}{n^2} = \frac{4\pi i \mathbf{j}_s}{\omega n^2}, \quad \overline{\overline{\varepsilon}}_c(\omega) = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \quad (*)$$

with the Astrom dispersion relation reading

$$\begin{aligned} \mathcal{D}(k,\theta,\omega) \quad \equiv \quad & \left(\varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta - \frac{\varepsilon_3\varepsilon_1}{n^2}\right)\left(1 - \frac{\varepsilon_1}{n^2}\right) \\ & + \left(\sin^2\theta - \frac{\varepsilon_3}{n^2}\right)\frac{\varepsilon_2^2}{n^2} = 0, \quad \theta = \left(\widehat{\mathbf{B},\mathbf{k}}\right) \end{aligned}$$

With the Jovian plasma condition $\Omega_e \gg \omega_{pe}$ FM resonances ($k \to \infty$) occur at $\omega = \omega_{pe}$ and $\omega_{LH} = \sqrt{\omega_{pi}^2 + \Omega_i^2}$ for $\theta = 0$ and $\pi/2$ respectively

For fast magnetosonic waves the matrix elements of the dielectric tensor are further simplified

$$\varepsilon_1 \approx 1 - \frac{\omega_{LH}^2}{\omega^2}\ , \quad \varepsilon_2 \approx -\frac{\omega_{pe}^2}{\omega\Omega_e}\ , \quad \varepsilon_3 = -\frac{\omega_{pe}^2 - \omega^2}{\omega^2}$$

and all three ratios $|\varepsilon_j/n^2|$, $j = 1, \ldots, 3$ are small as in LEO $\Rightarrow$

$$\mathcal{D}_{FM} = \varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta$$

For Alfven waves $(\omega < \Omega_i)$, the ratios ε_1/n^2, ε_2/n^2 are small and $|\varepsilon_3/n^2| \leq \mathcal{O}(1) \Rightarrow \mathcal{D} \approx \mathcal{D}_A(k, \theta, \omega) = \varepsilon_1 \left(\sin^2\theta - \frac{\varepsilon_3}{n^2}\right) + \varepsilon_3 \cos^2\theta\,.$

In both modes, $\mathbf{k}$ is nearly perpendicular to $\mathbf{B}$, $\cos\theta \equiv k_z/k \ll 1$.

The electric field $\mathbf{E}$, in the Fourier transform equation $(*)$, is decomposed as $\mathbf{E} = \mathbf{E}_t - i\mathbf{k}\phi$. Using $\mathbf{k}\cdot\mathbf{E}_t = 0$ and $(*)$ determines both $|\mathbf{E}_t| \ll |\mathbf{E}_l|$ and the electric potential, reading for either mode

$$\phi \simeq \frac{4\pi}{\omega}\mathbf{k}\cdot\mathbf{j}_s\, F\,, \qquad F \equiv \begin{cases} \left[1-\varepsilon_3/n^2\right]/k^2\mathcal{D}_A & A \\ 1/k^2\mathcal{D}_{FM} & FM \end{cases}$$

The radiated power (for $|\mathbf{E}_t| \ll |\mathbf{E}_l|$)

$$\dot{W}_{rad} = -\int \mathbf{j}_s\cdot\mathbf{E}d\mathbf{r} = -\int \phi\nabla\cdot\mathbf{j}_s d\mathbf{r}$$

depends on the source current-density $\mathbf{j}_s$.

Introduce a normalized Fourier transform of the current-density divergence

$$g(\mathbf{k}) \equiv -i\int d\mathbf{r}\nabla\cdot j_s(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}/2\pi I_s,$$

and consider a tether of lenght L spinning to keep it taut with $\varphi(t)$ the angle between tether and y-axis in the orbital plane.

The angle-averaged impedance then reads

$$\dot{W}/I_s^2 = Z \equiv \int_0^{2\pi} \frac{d\varphi}{2\pi} \int \frac{2i\,|g(\mathbf{k})|^2\, d\mathbf{k}}{\omega}\ F$$

The current divergence $\nabla \cdot \mathbf{j}_s(\mathbf{r})$ is assumed as occurring on spherical surfaces at end-connectors of dimension R small compared with L

$$\nabla \cdot \mathbf{j}_s(\mathbf{r}) = \frac{I_s}{4\pi R^2}\left[\delta\left(\left|\frac{L}{2}\mathbf{u}_t + \mathbf{r}\right| - R\right) - \delta\left(\left|-\frac{L}{2}\mathbf{u}_t + \mathbf{r}\right| - R\right)\right]$$

$$g(\mathbf{k}_\perp) \simeq \frac{1}{\pi}\sin\left(\frac{L}{2}k_y \sin\varphi\right)\frac{\sin(k_\perp R)}{k_\perp R}$$

with $\mathbf{k}\cdot\mathbf{u}_t \simeq k_y \sin\varphi$ and $\mathbf{u}_t$ the unit vector along the tether for each rotation angle φ.

We use $\mathbf{k} \simeq \mathbf{k}_\perp\ (k_x, k_y)$ except at $\mathcal{D}$, allowing to integrate over k_z poles at $\mathcal{D} = 0$, with $\omega \to \omega + i\nu\ \ (\nu \to 0^+)$.

For fast magnetosonic waves, we obtain

$$Z_{FM} = 2\pi \int_0^{2\pi} \frac{d\varphi}{2\pi} \int \frac{d\mathbf{k}_\perp \, |g(\mathbf{k}_\perp)|^2}{k_\perp \sqrt{\omega_{pe}^2 - \omega^2}} \frac{\omega}{\sqrt{\omega^2 - \omega_{LH}^2}}$$

with $\omega \approx V_{orb,y} k_y - V_{pl} k_x$.

Considering polar coordinates $(k_\perp, \xi)$, the FM impedance becomes

$$\begin{aligned} Z_{FM} &= \frac{2}{\pi^2 R^2 \omega_{pe}} \int_0^{2\pi} \frac{d\xi}{k_{LH}} \int_{k_{LH}}^{k_{pe}} \frac{dk_\perp}{k_\perp} \frac{\sin^2(k_\perp R)}{\sqrt{1 - k_\perp^2/k_{pe}^2}\sqrt{k_\perp^2/k_{LH}^2 - 1}} \\ &\times \int_0^{\pi} d\varphi \sin^2\left(k_\perp \frac{L}{2} \sin\varphi \sin\xi\right) \end{aligned}$$

where $|V_{orb,y} \sin\xi - V_{pl} \cos\xi| \times k_{pe} \ (k_{LH}) = \omega_{pe} \ (\omega_{LH})$.

Using $k_{LH} L, k_{pe} L \gg 1$ the φ-integral yields $\pi/2$.

Using similarly $k_{LH} R, k_{pe} R \gg 1$, we set $\sin^2(k_\perp R) \approx 1/2$ in the $k_\perp$-integral, which then yields $\pi/4$.

Finally, evaluating the ξ-integral as

$$\int_0^{2\pi} \frac{d\xi}{k_{LH}(\xi)} = \int_0^{2\pi} \frac{d\xi}{\omega_{LH}} \left| V_{orb,y} \sin\xi - V_{pl} \cos\xi \right| = \frac{4}{\omega_{LH}} \sqrt{V_{orb,y}^2 + V_{pl}^2}$$

then the FM impedance is

$$Z_{FM} = \frac{\tilde{V}_{rel}}{R^2 \omega_{pe} \omega_{LH}}, \quad \tilde{V}_{rel} \equiv \sqrt{V_{orb,y}^2 + V_{pl}^2}$$

Nonlinear effects would adjust contactor areas to an effective value the effective contactor surface

$$4\pi R^2 = \frac{I_s}{j_{th}}, \quad j_{th} \equiv e N_e \left(k_B T_e / 2\pi m_e\right)^{1/2}.$$

with j_{th} the umperturbed random current density and N_e and T_e given by the *Divine-Garrett* model. The voltaje drop for the FM impedance is

$$\Delta V_{FM} = Z_{FM} I_s \sim \frac{4\pi \tilde{V}_{rel}\, j_{th}}{\omega_{pe}\omega_{pi}\sqrt{1 + \Omega_i^2/\omega_{pi}^2}} = \sqrt{\frac{m_i \tilde{V}_{rel}^2 T_e}{2\pi e^2 \left(1 + \Omega_i^2/\omega_{pi}^2\right)}}$$

The power radiated in the FM mode $Z_{FM} I_s^2$ will increase just linearly with I_s.

For Alfven waves, the radiation impedance is

$$Z_A = \frac{2\pi V_A}{c^2} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int \frac{|g(\mathbf{k})|^2 \; d\mathbf{k}_\perp}{k_\perp^2} \frac{\sqrt{1 - \omega^2/\Omega_i^2}\ \sqrt{1 + c^2 k_\perp^2/\omega_{pe}^2}}{\sqrt{1 - \tilde{V}_A^2 \omega^2/\Omega_i^2}}$$

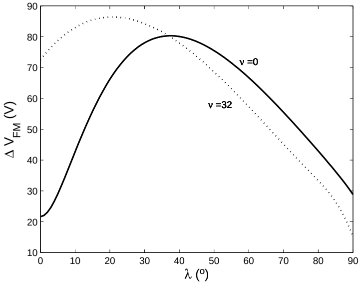
Carring out the integral of $\sin^2(\frac{L}{2} k_y \sin\varphi)$ over ϕ analitically and changing to polar coordiantes, the double integral can be evaluated numerically.

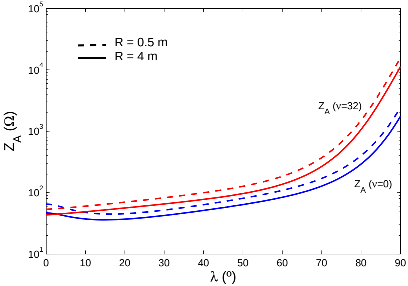
Complete numerical results fit (within 10 %, roughly) an analytical law

$$Z_A \simeq \frac{2 V_A}{c^2 \sqrt{1 + V_A^2/c^2}} \left\{ \ln\left(\frac{L e^{\gamma} \omega_{pe}}{2\, c} \right) + \frac{\Omega_i\, c}{2\omega_{pe} \tilde{V}_{rel}} \right.$$

$$\left. \times \quad \left(1 + \frac{\Omega_i^2/8}{\Omega_i^2 + \omega_{pi}^2} \right) \ln\left(\frac{2 \tilde{V}_{rel}\, e^{2-\gamma}}{R\, \Omega_i} \right) \right\}$$

Notice that this law recovers previous results (Adv. Space Res. **45**, 1050, 2010) for $\nu = 0$, and $\lambda = 0$ and $\pi/2$, where $\tilde{V}_{rel} = V_{pl}$ and $V_{orb}/\sqrt{2}$ respectively.





CONCLUSIONS

- Both Alfven and FM radiation impedances have been analytically determined.

- The typical voltaje drop for FM in JUNO would be two orders higher than in LEO ($\Delta V_{FM}^{LEO} \sim 0{,}4$ V).

- FM radiation $Z_{FM} I_s^2$ will dominate Alfven radiation, except at large current.