EXIT Chart Analysis of Iteratively Detected and SVD-assisted Broadband MIMO-BICM Schemes

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***Abstract*—In this contribution the number of activated MIMO layers and the number of bits per symbol are jointly optimized under the constraint of a given fixed data throughput and integrity. In general, non-frequency selective MIMO links have attracted a lot of research and have reached a state of maturity. By contrast, frequency selective MIMO links require substantial further research, where spatio-temporal vector coding (STVC) introduced by RALEIGH seems to be an appropriate candidate for broadband transmission channels. In analogy to bit-interleaved coded irregular modulation, a broadband MIMO-BICM scheme is introduced, where different signal constellations and mappings are used within a single codeword. Extrinsic Information Transfer (EXIT) charts are used for analyzing and optimizing the convergence behaviour of the iterative demapping and decoding. Our results show that in order to achieve the best bit-error rate, not necessarily all MIMO layers have to be activated.**

I. INTRODUCTION

Iterative demapping and decoding aided bit-interleaved coded modulation (BICM-ID) was designed for bandwidth efficient transmission over fading channels [1], [2]. The BICM philosophy has been extended by using different signal constellations and bit-to-symbol mapping arrangements within a single codeword, leading to the concept of bit-interleaved coded irregular modulation (BICIM) schemes, offering an improved link adaptation capability and an increased design freedom [3]. Since the capacity of multiple-input multiple-output (MIMO) systems increases linearly with the minimum number of antennas at both, the transmitter as well as the receiver side, MIMO-BICM schemes have attracted substantial attention and can be considered as an essential part of increasing both the achievable capacity and integrity of future generations of wireless systems [4], [5]. However, non-frequency selective MIMO links have reached a state of maturity [6]. By contrast, frequency selective MIMO links require substantial further research, where spatio-temporal vector coding (STVC), introduced by RALEIGH, seems to be an appropriate candidate for broadband MIMO transmission channels [7], [8]. In general, the choice of the number of bits per symbol and the number of activated MIMO layers combined with powerful error correcting codes offer a certain degree of design freedom, which substantially affects the performance of MIMO systems. Against this background, the novel contribution of this paper is that we jointly optimize the number of activated MIMO layers and the number of bits per symbol combined with powerful error correcting codes under the constraint of a given fixed data throughput and integrity.

The remaining part of this contribution is organized as follows: Section II introduces our system model, while the proposed uncoded solutions are discussed in Section III. In Section IV the channel encoded MIMO system is introduced. The associated performance results are presented and interpreted in Section V. Finally, Section VI provides our concluding remarks.

II. BROADBAND MIMO SYSTEM MODEL

When considering a frequency selective SDM (spatial division multiplexing) MIMO link, composed of n_{T} transmit and n_{R} receive antennas, the block-oriented system is modelled by

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{w} \quad . \tag{1}$$

In (1), $\mathbf{c}$ is the $(N_{\mathrm{T}} \times 1)$ transmitted signal vector containing the complex input symbols transmitted over n_{T} transmit antennas in K consecutive time slots, i. e., $N_{\mathrm{T}} = K\, n_{\mathrm{T}}$. This vector can be decomposed into n_{T} antenna-specific signal vectors $\mathbf{c}_\mu$ according to

$$\mathbf{c} = \left(\mathbf{c}_1^{\mathrm{T}}, \ldots, \mathbf{c}_\mu^{\mathrm{T}}, \ldots, \mathbf{c}_{n_{\mathrm{T}}}^{\mathrm{T}}\right)^{\mathrm{T}} \quad . \tag{2}$$

In (2), the $(K \times 1)$ antenna-specific signal vector $\mathbf{c}_\mu$ transmitted by the transmit antenna μ (with $\mu = 1, \ldots, n_{\mathrm{T}}$) is modelled by

$$\mathbf{c}_\mu = \left(c_{1\,\mu}, \ldots, c_{k\,\mu}, \ldots, c_{K\,\mu}\right)^{\mathrm{T}} \quad . \tag{3}$$

The $(N_{\mathrm{R}} \times 1)$ received signal vector $\mathbf{u}$, defined in (1), can again be decomposed into n_{R} antenna-specific signal vectors $\mathbf{u}_\nu$ (with $\nu = 1, \ldots, n_{\mathrm{R}}$) of the length $K + L_{\mathrm{c}}$, i. e., $N_{\mathrm{R}} = (K + L_{\mathrm{c}})\, n_{\mathrm{R}}$, and results in

$$\mathbf{u} = \left(\mathbf{u}_1^{\mathrm{T}}, \ldots, \mathbf{u}_\nu^{\mathrm{T}}, \ldots, \mathbf{u}_{n_{\mathrm{R}}}^{\mathrm{T}}\right)^{\mathrm{T}} \quad . \tag{4}$$

By taking the $(L_{\mathrm{c}}+1)$ non-zero elements of the resulting symbol rate sampled overall channel impulse response between the μth transmit and νth receive antenna into account, the antenna-specific received vector $\mathbf{u}_\nu$ has to be extended by L_{c} elements, compared to the transmitted antenna-specific signal vector $\mathbf{c}_\mu$ defined in (3). The $((K + L_{\mathrm{c}}) \times 1)$ signal vector $\mathbf{u}_\nu$ received by the antenna ν (with $\nu = 1, \ldots, n_{\mathrm{R}}$) can be constructed, including the extension through the multipath propagation, as follows

$$\mathbf{u}_\nu = \left(u_{1\,\nu}, u_{2\,\nu}, \ldots, u_{(K+L_{\mathrm{c}})\,\nu}\right)^{\mathrm{T}} \quad . \tag{5}$$

Similarly, in (1) the $(N_{\mathrm{R}} \times 1)$ noise vector $\mathbf{w}$ results in

$$\mathbf{w} = \left(\mathbf{w}_1^{\mathrm{T}}, \ldots, \mathbf{w}_\nu^{\mathrm{T}}, \ldots, \mathbf{w}_{n_{\mathrm{R}}}^{\mathrm{T}}\right)^{\mathrm{T}} \quad . \tag{6}$$

The vector $\mathbf{w}$ of the additive, white Gaussian noise (AWGN) is assumed to have a variance of U_{R}^2 for both the real and imaginary parts and can still be decomposed into n_{R} antenna-specific signal vectors $\mathbf{w}_\nu$ (with $\nu = 1, \ldots, n_{\mathrm{R}}$) according to

$$\mathbf{w}_\nu = \left(w_{1\,\nu}, w_{2\,\nu}, \ldots, w_{(K+L_{\mathrm{c}})\,\nu}\right)^{\mathrm{T}} \quad . \tag{7}$$

Finally, the $(N_{\mathrm{R}} \times N_{\mathrm{T}})$ system matrix $\mathbf{H}$ of the block-oriented system model, introduced in (1), results in

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{1\,1} & \ldots & \mathbf{H}_{1\,n_{\mathrm{T}}} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{n_{\mathrm{R}}\,1} & \cdots & \mathbf{H}_{n_{\mathrm{R}}\,n_{\mathrm{T}}} \end{bmatrix} \quad , \tag{8}$$

and consists of $n_{\mathrm{R}}\,n_{\mathrm{T}}$ single-input single-output (SISO) channel matrices $\mathbf{H}_{\nu\,\mu}$ (with $\nu = 1, \ldots, n_{\mathrm{R}}$ and $\mu = 1, \ldots, n_{\mathrm{T}}$). The channel convolution matrix $\mathbf{H}_{\nu\,\mu}$ between the μth transmit and the νth receive antenna is obtained by taking the $(L_{\mathrm{c}}+1)$ non-zero elements of the resulting symbol rate sampled overall impulse response into account [9]. Throughout this paper, it is assumed that the $(L_{\mathrm{c}}+1)$ uncorrelated channel coefficients, between the μth transmit and νth receive antenna have the same averaged power and undergo a Rayleigh distribution. Furthermore, a block fading channel model is applied, i. e., the channel is assumed to be time invariant for the duration of one SDM MIMO data vector.

The interference, introduced by the off-diagonal elements of the channel matrix $\mathbf{H}$, requires appropriate signal processing strategies. A popular technique is based on the singular-value decomposition (SVD) [10] of the system matrix $\mathbf{H}$, which can be written as $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^{\mathrm{H}}$, where $\mathbf{S}$ and $\mathbf{D}^{\mathrm{H}}$ are unitary matrices and $\mathbf{V}$ is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\,\mathbf{H}$ sorted in descending order[1]. The SDM MIMO data vector $\mathbf{c}$ is now multiplied by the matrix $\mathbf{D}$ before transmission. In turn, the receiver multiplies the received vector $\mathbf{u}$ by the matrix $\mathbf{S}^{\mathrm{H}}$. Thereby neither the transmit power nor the noise power is enhanced. The overall transmission relationship is defined as

$$\mathbf{y} = \mathbf{S}^{\mathrm{H}}\left(\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{w}\right) = \mathbf{V} \cdot \mathbf{c} + \tilde{\mathbf{w}} \,. \tag{9}$$

As a consequence of the processing in (9), the channel matrix $\mathbf{H}$ is transformed into independent, non-interfering layers having unequal gains [9].

[1]The transpose and conjugate transpose (Hermitian) of $\mathbf{D}$ are denoted by $\mathbf{D}^{\mathrm{T}}$ and $\mathbf{D}^{\mathrm{H}}$, respectively.

III. Quality Criteria

In general, the quality of data transmission can be informally assessed by using the signal-to-noise ratio (SNR) at the detector's input defined by the half vertical eye opening and the noise power per quadrature component according to

$$\varrho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise Power}} = \frac{(U_{\mathrm{A}})^2}{(U_{\mathrm{R}})^2} \quad , \tag{10}$$

which is often used as a quality parameter [6]. The relationship between the signal-to-noise ratio $\varrho = U_{\mathrm{A}}^2/U_{\mathrm{R}}^2$ and the bit-error probability evaluated for AWGN channels and M-ary Quadrature Amplitude Modulation (QAM) is given by [11]

$$P_{\mathrm{BER}} = \frac{2}{\log_2(M)}\left(1 - \frac{1}{\sqrt{M}}\right)\operatorname{erfc}\left(\sqrt{\frac{\varrho}{2}}\right) \quad . \tag{11}$$

When applying the proposed system structure, the SVD-based equalization leads to different eye openings per activated MIMO layer ℓ (with $\ell = 1, 2, \cdots, L$) at the time k (with $k = 1, 2, \cdots, K$) within the SDM MIMO signal vector according to

$$U_{\mathrm{A}}^{(\ell,k)} = \sqrt{\xi_{\ell,k}} \cdot U_{\mathrm{s}\,\ell} \quad , \tag{12}$$

where $U_{\mathrm{s}\,\ell}$ denotes the half-level transmit amplitude assuming M_ℓ-ary QAM and $\sqrt{\xi_{\ell,k}}$ represents the corresponding positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\,\mathbf{H}$. Together with the noise power per quadrature component, the SNR per MIMO layer ℓ at the time k becomes

$$\varrho^{(\ell,k)} = \frac{\left(U_{\mathrm{A}}^{(\ell,k)}\right)^2}{U_{\mathrm{R}}^2} = \xi_{\ell,k}\,\frac{(U_{\mathrm{s}\,\ell})^2}{U_{\mathrm{R}}^2} \quad . \tag{13}$$

Using the parallel transmission over $L \leq \min(n_{\mathrm{T}}, n_{\mathrm{R}})$ MIMO layers, the overall mean transmit power becomes $P_{\mathrm{s}} = \sum_{\ell=1}^{L} P_{\mathrm{s}\,\ell}$, where the number of readily separable layers is limited by $\min(n_{\mathrm{T}}, n_{\mathrm{R}})$. Considering QAM constellations, the average transmit power $P_{\mathrm{s}\,\ell}$ per MIMO layer ℓ may be expressed as $P_{\mathrm{s}\,\ell} = 2/3\,U_{\mathrm{s}\,\ell}^2\,(M_\ell - 1)$ [11]. Together with (13), the layer-specific SNR at the time k results in

$$\varrho^{(\ell,k)} = \xi_{\ell,k}\,\frac{3}{2\,(M_\ell - 1)}\,\frac{P_{\mathrm{s}\,\ell}}{U_{\mathrm{R}}^2} \quad . \tag{14}$$

In order to transmit at a fixed data rate while maintaining the best possible integrity, i. e., bit-error rate, an appropriate number of MIMO layers $L \leq \min(n_{\mathrm{T}}, n_{\mathrm{R}})$ has to be used, which depends on the specific transmission mode, as detailed in Table I. However, in order to avoid any signalling overhead, fixed transmission modes are used in this contribution regardless of the channel quality [9].

IV. Coded MIMO System

The channel encoded transmitter structure is depicted in Fig. 1. The encoder employs a half-rate nonrecursive, non-systematic convolutional (NSC) code using the generator polynomials $(7, 5)$ in octal notation. The uncoded information is organized in blocks of N_{i} bits, consisting of at least 3000 bits, depending on the specific QAM constellation used. Each data

TABLE I
Investigated transmission modes

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	64	4	0	0
8 bit/s/Hz	**16**	**16**	**0**	**0**
8 bit/s/Hz	**16**	**4**	**4**	**0**
8 bit/s/Hz	4	4	4	4

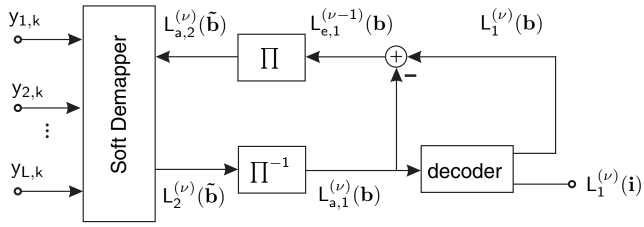

Fig. 2. Iterative demodulator structure

block $\mathbf{i}$ is encoded and results in the block $\mathbf{b}$ consisting of $N_\mathrm{b} = 2\,N_\mathrm{i} + 4$ encoded bits, including 2 termination bits. The encoded bits are interleaved using a random interleaver and stored in the vector $\tilde{\mathbf{b}}$. The encoded and interleaved bits are then mapped to the MIMO layers. The task of the multiplexer and buffer block of Fig. 1 is to divide the vector of encoded and interleaved information bits $\tilde{\mathbf{b}}$ into subvectors $(\tilde{\mathbf{b}}_{1,k}, \tilde{\mathbf{b}}_{2,k}, \cdots, \tilde{\mathbf{b}}_{L,k})$, each consisting of 8 bits according to the chosen transmission mode (Table I). The individual binary data vectors $\tilde{\mathbf{b}}_{\ell,k}$ are then mapped to the QAM symbols $c_{\ell,k}$ according to the specific mapper used [12], [4]. The iterative demodulator structure is shown in Fig. 2. When using the iteration index ν, the first iteration of $\nu = 1$ commences with the soft-demapper delivering the N_b log-likelihood ratios (LLRs) $L_2^{(\nu=1)}(\tilde{\mathbf{b}})$ of the encoded and interleaved information bits, whose de-interleaved version $L_{\mathrm{a},1}^{(\nu=1)}(\mathbf{b})$ represents the input of the convolutional decoder as depicted in Fig. 2 [13], [4]. This channel decoder provides the estimates $L_1^{(\nu=1)}(\mathbf{i})$ of the original uncoded information bits as well as the LLRs of the N_b NSC-encoded bits in the form of

$$L_1^{(\nu=1)}(\mathbf{b}) = L_{\mathrm{a},1}^{(\nu=1)}(\mathbf{b}) + L_{\mathrm{e},1}^{(\nu=1)}(\mathbf{b}) \ . \qquad (15)$$

As seen in Fig. 2 and (15), the LLRs of the NSC-encoded bits consist of the receiver's input signal itself plus the extrinsic information $L_{\mathrm{e},1}^{(\nu=1)}(\mathbf{b})$, which is generated by subtracting $L_{\mathrm{a},1}^{(\nu=1)}(\mathbf{b})$ from $L_1^{(\nu=1)}(\mathbf{b})$. The appropriately ordered, i. e. interleaved extrinsic LLRs are fed back as *a priori* information $L_{\mathrm{a},2}^{(\nu=2)}(\tilde{\mathbf{b}})$ to the soft demapper of Fig. 2 for the second iteration.

V. Results

In this contribution fixed transmission modes are used regardless of the channel quality. Assuming predefined transmission modes, a fixed data rate can be guaranteed. The obtained

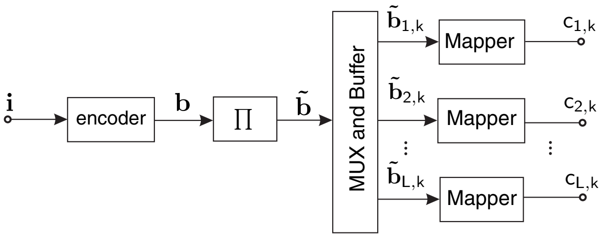

Fig. 1. The channel-encoded MIMO transmitter structure

uncoded BER curves are depicted in Fig. 3 for the different QAM constellation sizes of Table I, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs.

More explicitly, our goal is to find that specific combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at a given fixed bit/s/Hz bandwidth efficiency. However, the lowest BERs can only be achieved by using bit auction procedures leading to a high signalling overhead [9].

Using the half-rate constraint-length $K_\mathrm{cl} = 3$ NSC code, the BER performance is analyzed for an effective user throughput of 4 bit/s/Hz. The BER investigations using the NSC code are based on the best uncoded schemes of Table I. The information word length is 3000 bits and a random interleaver is applied.

In addition to the number of bits per symbol and the number of activated MIMO layers, the achievable performance of the iterative decoder is substantially affected by the specific mapping of the bits to both the QAM symbols as well as to the MIMO layers. While the employment of the classic Gray-mapping is appropriate in the absence of *a priori* information, the availability of *a priori* information in iterative receivers

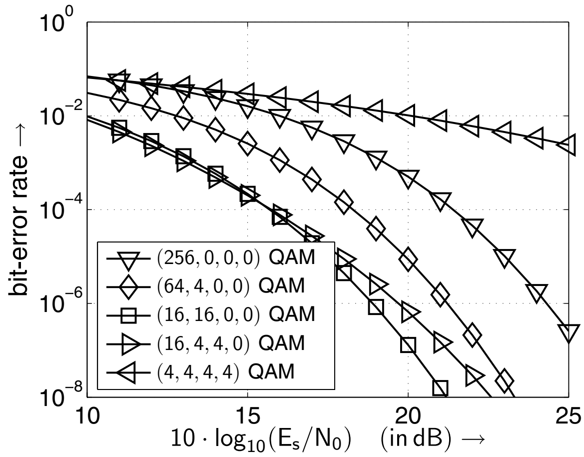

Fig. 3. BER when using the transmission modes introduced in Table I and transmitting 8 bit/s/Hz over frequency-selective channels with $L_\mathrm{c} = 1$ (two-path channel model)

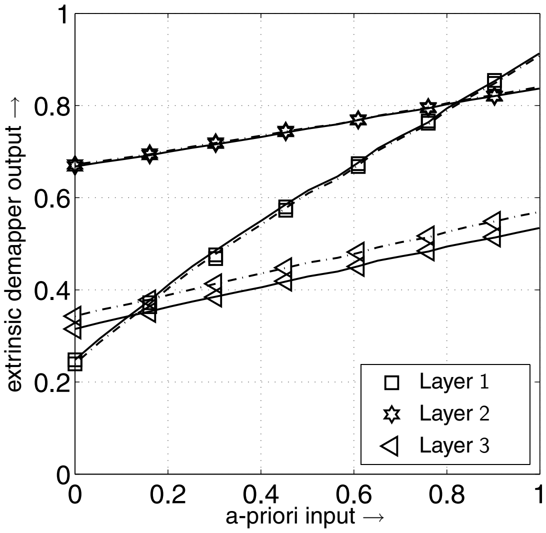

Fig. 4. Layer-specific transfer characteristic when using anti-Gray mapping and the $(16, 4, 4, 0)$ transmission mode over frequency-selective MIMO links ($10\ \log_{10}(E_{\mathrm{s}}/N_0) = 2$ dB, two-path channel model (solid line, $L_{\mathrm{c}} = 1$), five-path channel model (dashed line, $L_{\mathrm{c}} = 4$))

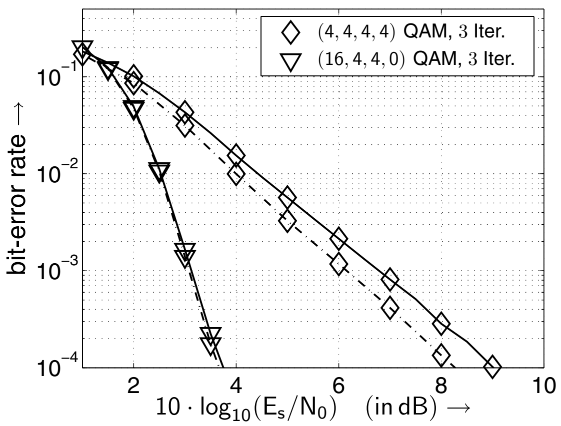

Fig. 5. BER for an effective user throughput of 4 bit/s/Hz and anti-Gray mapping in combination with different transmission modes (two-path channel model (solid line, $L_{\mathrm{c}} = 1$), five-path channel model (dashed line, $L_{\mathrm{c}} = 4$)) and the half-rate NSC code with the generator polynomials of $(7, 5)$ in octal notation

requires an exhaustive search for finding the best non-Gray – synonymously also referred to as anti-Gray – mapping scheme [2].

Activating only an appropriate number of MIMO layers, as determined by analyzing the uncoded transmission, the layer-specific transfer characteristics are depicted in Fig. 4. However, taking frequency selective MIMO links rather than non-frequency selective MIMO links into account, large delay-spreads seems to be highly beneficial and lead to further degree's of design freedom as investigated in [7] or [8].

The results, depicted in Fig. 4, highlight that at low SNR an increased delay spread is only beneficial for MIMO layers carrying a low number of bits, whereas the characteristic of the strongest MIMO layers remains nearly unchanged. Therefore, in combination with an appropriate number of MIMO layers, as determined by optimizing the uncoded transmission scheme, only minor improvements can be expected. The corresponding BER curves are shown in Fig. 5 and confirm the EXIT charts results. The best uncoded solutions seems also to be useful in the coded scenario.

VI. Conclusion

The choice of the number of bits per symbol and the number of MIMO layers combined with powerful error correcting codes substantially affects the performance of a MIMO system. Analyzing the uncoded system, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs. Considering the coded system, the choice of the mapping strategies combined with the appropriate number of activated MIMO layers and transmitted bits per symbol offers a certain degree of design freedom, which substantially affects the performance of MIMO-BICM systems. As demonstrated along this work, it is showed that activating an appropriate number of MIMO layers seems to be a promising solution for minimizing the overall BER characteristic.

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