

Modulation-Mode and Power-Assignment for SVD- and GMD-assisted Downlink MIMO Systems

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Abstract—Multiuser multiple-input multiple-output (MIMO) downlink (DL) transmission schemes experience both multiuser interference (MUI) as well as inter-antenna interference. However, instead of treating all the users jointly as in zero-forcing (ZF) multiuser transmission techniques, the investigated singular value decomposition (SVD) and geometric mean decomposition (GMD) assisted DL multiuser MIMO systems take the individual user's channel characteristics into account. The performed joint optimization of the number of activated MIMO layers and the number of bits per symbol along with the appropriate allocation of the transmit power shows that not necessarily all user-specific MIMO layers have to be activated in order to minimize the overall BER under the constraint of a fixed data throughput.

I. INTRODUCTION

Adaptive modulation is a promising technique to increase the spectral efficiency of wireless transmission systems by adapting the signal parameters dynamically to changing channel conditions. However, in order to comply with the demand on increasing available data rates in particular in wireless technologies, systems with multiple transmit and receive antennas, also called MIMO systems, have become indispensable and can be considered as an essential part of increasing both the achievable capacity and integrity of future generations of wireless systems [1], [2]. However, single-user MIMO transmission schemes for both non-frequency and frequency selective MIMO channels have attracted a lot of attention and reached a state of maturity [1], [3]. By contrast, MIMO-aided multiple-user systems require substantial further research where both multiuser as well as multi-antenna interferences have to be taken into account. In this work, SVD- and GMD-assisted downlink (DL) multiuser multiple-input multiple-output (MIMO) systems are considered, which take the individual user's channel characteristics into account [4], [5] rather than treating all users channels jointly as in ZF multiuser transmission techniques. Whereas SVD transforms a MIMO channel into multiple single-input single-output (SISO) channels having unequal gains, GMD with further transmit preprocessing (e. g. Tomlinson-Harashima Precoding) is able to decompose the MIMO channel into multiple SISO channels having equal gains, which might be more convenient for

the design of transmit transmission schemes. Against this background, in this paper SVD- and GMD-assisted multiuser MIMO schemes are investigated, where both multiuser interferences as well as multi-antenna interferences are perfectly eliminated. The novel contribution of this paper is that we demonstrate the benefits of amalgamating a suitable choice of activated MIMO layers and number of bits per symbol along with the appropriate allocation of the transmit power under the constraint of a given fixed data throughput. The remaining part of this paper is organized as follows: Section II introduces the multiuser system model, while the proposed uncoded solutions are discussed in section III. The associated performance results are presented and interpreted in section IV. Finally, section V provides some concluding remarks.

II. MULTIUSER SYSTEM MODEL

The system model considered in this work consists of a single base station (BS) supporting K mobile stations (MSs). The BS is equipped with n_T transmit antennas, while the k th (with $k = 1, \dots, K$) MS has n_{Rk} receive antennas, i. e. the total number of receive antennas including all K MSs is given by $n_R = \sum_{k=1}^K n_{Rk}$. The $(n_{Rk} \times 1)$ user specific symbol vector \mathbf{c}_k to be transmitted by the BS is given by

$$\mathbf{c}_k = (c_{k,1}, c_{k,2}, \dots, c_{k,n_{Rk}})^T. \quad (1)$$

The vector \mathbf{c}_k is preprocessed before its transmission by multiplying it with the $(n_T \times n_{Rk})$ DL preprocessing matrix \mathbf{R}_k and results in the $(n_T \times 1)$ user-specific transmit vector $\mathbf{s}_k = \mathbf{R}_k \mathbf{c}_k$. After DL transmitter preprocessing, the n_T -component signal \mathbf{s} transmitted by the BS to the K MSs results in

$$\mathbf{s} = \sum_{k=1}^K \mathbf{s}_k = \mathbf{R} \mathbf{c}, \quad (2)$$

with the $(n_T \times n_R)$ preprocessing matrix

$$\mathbf{R} = (\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_K). \quad (3)$$

In (2), the overall $(n_R \times 1)$ transmitted DL data vector \mathbf{c} combines all K DL transmit vectors \mathbf{c}_k (with $k = 1, 2, \dots, K$)

and is given by

$$\mathbf{c} = (\mathbf{c}_1^T, \mathbf{c}_2^T, \dots, \mathbf{c}_K^T)^T. \quad (4)$$

At the receiver side, the $(n_{Rk} \times 1)$ received signal vector \mathbf{u}_k of the k th MS results in

$$\mathbf{u}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k = \mathbf{H}_k \mathbf{R} \mathbf{c} + \mathbf{n}_k \quad (5)$$

and can be expressed by

$$\mathbf{u}_k = \mathbf{H}_k \mathbf{R}_k \mathbf{c}_k + \sum_{i=1, i \neq k}^K \mathbf{H}_k \mathbf{R}_i \mathbf{c}_i + \mathbf{n}_k, \quad (6)$$

where the MSs received signals experience both multi-user and multi-antenna interferences. In (5), the $(n_{Rk} \times n_T)$ channel matrix \mathbf{H}_k connects the n_T BS specific transmit antennas with the n_{Rk} receive antennas of the k th MS. It is assumed that the coefficients of the $(n_{Rk} \times n_T)$ channel matrix \mathbf{H}_k are independent and Rayleigh distributed with equal variance. The interference, which is introduced by the channel matrix \mathbf{H}_k , requires appropriate signal processing strategies. A popular technique is based on the SVD of the system matrix \mathbf{H}_k as described in [4]. Therein, after pre- and postprocessing of the transmitted and received signal vectors, the user-specific decision variables result in

$$\mathbf{y}_k = \mathbf{V}_{k,u} \mathbf{P}_k \mathbf{c}_k + \mathbf{w}_k, \quad (7)$$

where interferences between the different antenna data streams as well as MUI imposed by the other users are avoided as shown in [4]. In (7), the $(n_{Rk} \times n_{Rk})$ diagonal matrix $\mathbf{V}_{k,u}$ contains the non-zero square roots of the eigenvalues of $\mathbf{H}_k^H \mathbf{H}_k$ and the user-specific $(n_{Rk} \times n_{Rk})$ diagonal power allocation matrix \mathbf{P}_k is given by $\mathbf{P}_k = \sqrt{\beta} \mathbf{I}_{n_{Rk} \times n_{Rk}}$ with the parameter $\sqrt{\beta}$ taking the transmit-power constraint into account as highlighted in [4]. Finally the additive, white Gaussian noise (AWGN) vector is given by \mathbf{w}_k .

III. OPTIMIZATION OF THE UNCODED SYSTEM

In general, the user-specific quality of data transmission can be informally assessed by using the signal-to-noise ratio (SNR) at the detector's input defined by the half vertical eye opening and the noise power per quadrature component according to

$$\varrho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise Power}} = \frac{(U_A)^2}{(U_R)^2}, \quad (8)$$

which is often used as a quality parameter [3]. When applying the proposed system structure for the k th user, the applied signal processing leads to different eye openings per activated MIMO layer ℓ (with $\ell = 1, 2, \dots, L$ and $L \leq n_{Rk}$ describing the number of activated user-specific MIMO layers) and per transmitted symbol block m according to

$$U_{A_k}^{(\ell, m)} = \sqrt{p_{k,\ell}^{(m)}} \cdot \sqrt{\xi_{k,\ell}^{(m)}} \cdot U_{s_k}^{(\ell)}, \quad (9)$$

where $U_{s_k}^{(\ell)}$ denotes the half-level transmit amplitude assuming ℓ -ary QAM, $\sqrt{p_{k,\ell}^{(m)}}$ represents the corresponding positive square roots of the eigenvalues of the matrix $\mathbf{H}_k^H \mathbf{H}_k$ and

TABLE I

Investigated user-specific QAM transmission modes

| throughput | layer 1 | layer 2 | layer 3 | layer 4 |
|-------------------|-----------|-----------|----------|---------|
| 8 bit/s/Hz | 256 | 0 | 0 | 0 |
| 8 bit/s/Hz | 64 | 4 | 0 | 0 |
| 8 bit/s/Hz | 16 | 16 | 0 | 0 |
| 8 bit/s/Hz | 16 | 4 | 4 | 0 |
| 8 bit/s/Hz | 4 | 4 | 4 | 4 |

$\sqrt{p_{k,\ell}^{(m)}}$ represents the corresponding power allocation weighting parameters. Together with the noise power per quadrature component, introduced by the additive, white Gaussian noise (AWGN) vector \mathbf{w}_k in (7), the k th user-specific SNR per MIMO layer ℓ at the time m becomes

$$\varrho_k^{(\ell, m)} = \frac{(U_{A_k}^{(\ell, m)})^2}{U_R^2}. \quad (10)$$

Using the parallel transmission over L MIMO layers, the overall mean transmit power becomes $P_{s_k} = \sum_{\ell=1}^L P_{s_k}^{(\ell)}$. Considering QAM constellations, the average user-specific transmit power $P_{s_k}^{(\ell)}$ per MIMO layer ℓ may be expressed as [6]

$$P_{s_k}^{(\ell)} = \frac{2}{3} (U_{s_k}^{(\ell)})^2 (n_{k,\ell} - 1). \quad (11)$$

Assuming that the transmit power is uniformly distributed over the number of activated MIMO layers, i. e., $P_{s_k}^{(\ell)} = P_{s_k}/L$, the layer-specific signal-to-noise ratio at the time m results with the ratio of symbol energy to noise power spectral density $E_s/N_0 = P_{s_k}/(2U_R^2)$ and (10), (11) and (9) in

$$\varrho_k^{(\ell, m)} = p_{k,\ell}^{(m)} \xi_{k,\ell}^{(m)} \frac{3}{L(n_{k,\ell} - 1)} \frac{E_s}{N_0}. \quad (12)$$

In order to transmit at a fixed data rate while maintaining the best possible integrity, i. e., bit-error rate, an appropriate number of user-specific MIMO layers has to be used, which depends on the specific transmission mode, as detailed in Table I for the exemplarily investigated two-user multiuser-system ($n_{Rk} = 4$ (with $k = 1, 2$), $K = 2$, $n_R = n_T = 8$). However, the user-specific BER of the uncoded MIMO system is dominated by the specific layers having the lowest SNR's. As a remedy, a MIMO-layer transmit power allocation (PA) scheme is required for minimizing the overall BER under the constraint of a limited total MIMO transmit power. The proposed PA scheme scales the k th user half-level transmit amplitude $U_{s_k}^{(\ell)}$ of the ℓ th MIMO layer by the factor $\sqrt{p_{k,\ell}^{(m)}}$. This results in a MIMO layer-specific transmit amplitude of $U_{s_k}^{(\ell)} \sqrt{p_{k,\ell}^{(m)}}$ for the QAM symbol of the transmit data vector transmitted at the time m over the MIMO layer ℓ . Together with the DL preprocessing design, the layer-specific power allocation parameter at the time m results in:

$$\sqrt{p_{k,\ell}^{(m)}} = \sqrt{\beta^{(m)}} \sqrt{\tilde{p}_{k,\ell}^{(m)}}. \quad (13)$$

A natural choice is to opt for a PA scheme, which results in an identical signal-to-noise ratio

$$\rho_{\text{PA}k}^{(\ell,m)} = \frac{\left(U_{\text{PA}k}^{(\ell,m)}\right)^2}{U_{\text{R}}^2} = \tilde{P}_{k,\ell}^{(m)} \frac{3\xi_{k,\ell}^{(m)}\beta^{(m)}E_s}{L(k_{\ell}-1)N_0} \quad (14)$$

for all activated MIMO layers at the time m , i. e., in $\rho_{\text{PA}k}^{(\ell,m)}$ = constant for $\ell = 1, 2, \dots, L$. The power to be allocated to each activated MIMO layer at the time m can be shown to be calculated as follows [3]:

$$\tilde{P}_{k,\ell}^{(m)} = \frac{(k_{\ell}-1)}{\xi_{k,\ell}^{(m)}} \cdot \frac{L}{\sum_{\nu=1}^L \frac{(M_{k,\nu}-1)}{\xi_{k,\nu}^{(m)}}} \quad (15)$$

Another decomposition that has attracted a lot of interest within the last years is called geometric mean decomposition (GMD) [7], [8] and avoids the unequal weighting of the user-specific MIMO layers introduced by the SVD at the cost of remaining interferences between the different antenna data streams. The GMD of the system matrix \mathbf{H}_k results in: $\mathbf{H}_k = \mathbf{U}_{Gk} \cdot \mathbf{V}_{Gk} \cdot \mathbf{D}_{Gk}^H$, where \mathbf{U}_{Gk} and \mathbf{D}_{Gk}^H have orthonormal columns and \mathbf{V}_{Gk} is a real-valued upper triangular matrix. The $(n_{\text{R}k} \times n_{\text{T}})$ matrix \mathbf{V}_{Gk} can be decomposed into a $(n_{\text{R}k} \times n_{\text{R}k})$ real-valued upper triangular matrix \mathbf{V}_{Gku} and a $(n_{\text{R}k} \times (n_{\text{T}} - n_{\text{R}k}))$ zero-matrix \mathbf{V}_{Gkn} according to

$$\mathbf{V}_{Gk} = (\mathbf{V}_{Gku} \mathbf{V}_{Gkn}) = (\mathbf{V}_{Gku} \mathbf{0}) \quad (16)$$

Therein, the diagonal elements of the real-valued upper triangular matrix \mathbf{V}_{Gku} are equal to the geometric mean of the positive square roots of the eigenvalues of the matrix $\mathbf{H}_k^H \mathbf{H}_k$. For a given time m , the diagonal elements $v_{\ell,\ell}$ of the matrix \mathbf{V}_{Gku} can be obtained by

$$v_{\ell,\ell} = \left(\prod_{\ell=1}^{n_{\text{R}k}} \sqrt{\xi_{k,\ell}^{(m)}} \right)^{1/n_{\text{R}k}} \quad \ell = 1, 2, \dots, n_{\text{R}k} \quad (17)$$

If the GMD is constrained to the best $L \leq n_{\text{R}k}$ layers, the diagonal elements of the matrix \mathbf{V}_{Gku} result in

$$v_{\ell,\ell} = \left(\prod_{\ell=1}^L \sqrt{\xi_{k,\ell}^{(m)}} \right)^{1/L} \quad \ell = 1, 2, \dots, L \quad (18)$$

and the corresponding real-valued upper triangular matrix \mathbf{V}_{Gku} changes to

$$\mathbf{V}_{Gku} = \begin{pmatrix} v_{1,1} & v_{1,2} & \dots & v_{1,L} & 0 \\ 0 & v_{2,2} & \dots & v_{2,L} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & v_{L,L} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (19)$$

The remaining interferences between the different antenna data streams can be removed by combining GMD with conventional zero-forcing VBLAST (Vertical Bell Laboratories Layered Space-Time Architecture [9]), where sequential nulling and cancellation is applied. However, in order to avoid the error-propagation at the receiver side, dirty paper precoding could be implemented at the transmitter side, if channel state information is available. The proposed iterative transmitter preprocess-

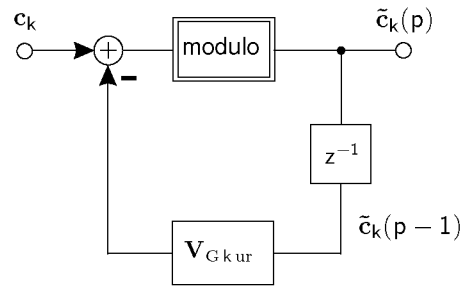


Fig. 1. Iterative user-specific DL interference cancellation scheme

ing structure is depicted in Fig. 1. The modulo operation has to be performed according to the chosen modulation alphabet [10] and the matrix \mathbf{V}_{Gkur} results in

$$\mathbf{V}_{Gkur} = \frac{1}{v_{1,1}} (\mathbf{V}_{Gku} - \text{diag}(\mathbf{V}_{Gku})) \quad (20)$$

The preprocessed transmit data vector $\tilde{c}_k(p)$ can be obtained iteratively with $p \geq 1$ and $\tilde{c}_k(0) = 0$.

IV. RESULTS

In this contribution fixed transmission modes are used regardless of the channel quality. Assuming predefined transmission modes, a fixed data rate can be guaranteed. Considering a non-frequency selective SDM (spatial division multiplexing) single-user MIMO link ($K = 1$) composed of $n_{\text{T}} = 4$ transmit and $n_{\text{R}} = 4$ receive antennas, the obtained BER curves are depicted in Fig. 2 for the different QAM constellation sizes and MIMO configurations of Table I, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs.

PA can be used to balance the bit-error probabilities in the different number of activated MIMO layers. As shown in Fig. 2, unequal PA is only effective in conjunction with the

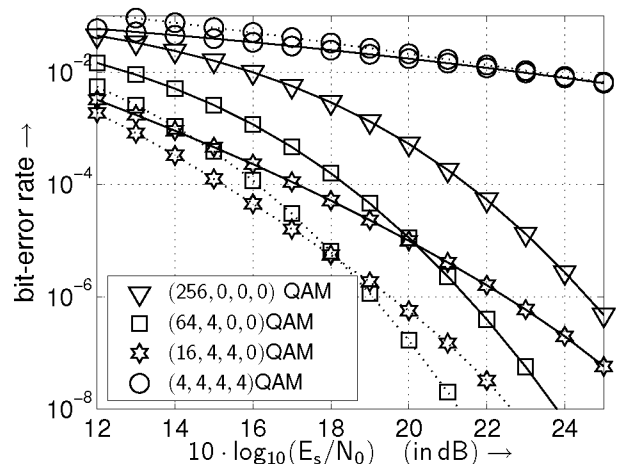


Fig. 2. BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Table I and transmitting 8 bit/s/Hz over non-frequency selective channels

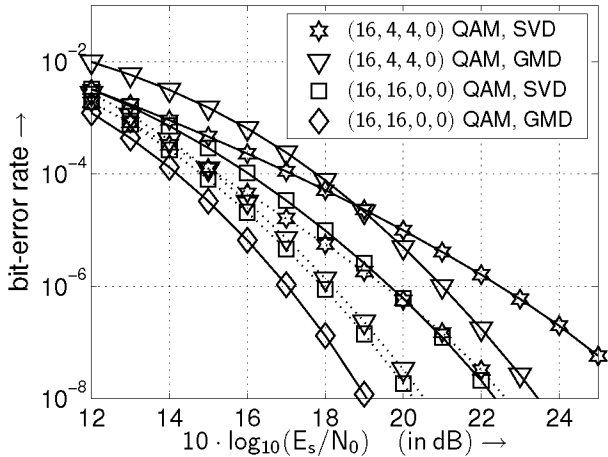


Fig. 3. BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Table I and transmitting 8 bit/s/Hz over non-frequency selective channels

optimum number of MIMO layers and at high SNR. Using all MIMO layers, our PA scheme would assign much of the total transmit power to the specific symbol positions per data block having the smallest singular values and hence the overall performance would deteriorate.

Next to the investigated SVD approach, GMD concentrates the channel energy on the number of activated MIMO layers at the cost of remaining interferences. However, the remaining interferences can easily be compensated by dirty paper precoding such as the investigated iterative Tomlinson-Harashima precoding, which transfers the whole MIMO link into a number of multiple SISO channels having equal gains. The obtained BER curves are depicted in Fig. 3 and show the potential of GMD based signal processing in conjunction with the investigated iterative Tomlinson-Harashima precoding. Having layers with equal gains, power allocation can be avoided as long as equal QAM constellation sizes are used on all activated MIMO layers.

Comparing the SVD and GMD-based results it turns out that not necessarily all MIMO layers have to be activated in order to get the lowest BERs. However, GMD-based signal processing offers the advantage of multiple SISO channels per SDM MIMO data block with equal gains regardless of the channel quality that makes power allocation unnecessary and outperforms the SVD-assisted solutions.

The parameters of the exemplarily studied two-users MIMO system are chosen as follows: $P_{s_k} = 1 \text{ V}^2$, $n_{R_k} = 4$ (with $k = 1, 2$), $K = 2$, $n_R = n_T = 8$. In this contribution a power with the dimension (voltage)² (in V^2) is used. At a real, constant resistor this value is proportional to the physical power (in W). The obtained user-specific BER curves are depicted in Fig. 4 for the different QAM constellation sizes and MIMO configurations of Table I and confirm the obtained results within the single-user system ($K = 1$). Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it still turns out that not all MIMO layers have to be activated in order to achieve the best BERs.

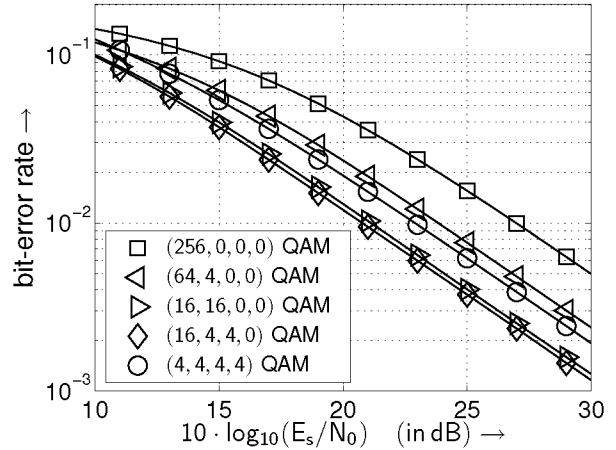


Fig. 4. SVD-based user-specific BERs without PA when using the transmission modes introduced in Table I and transmitting 8 bit/s/Hz over non-frequency selective channels

V. CONCLUSION

Single- and multiuser MIMO systems in conjunction with SVD and GMD assisted signal processing were investigated in this work. It turned out, that the choice of the number of bits per symbol as well as the number of activated MIMO layers substantially affects the performance of a MIMO system, suggesting that not all MIMO layers have to be activated in order to achieve the best BERs. The main goal was to find that specific combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at a given fixed bit/s/Hz bandwidth efficiency. The E_s/N_0 value required by each scheme at BER 10^{-2} ($K = 2$) and a BER 10^{-4} ($K = 1$) was extracted from computer simulations and the best systems are shown in bold in Table I.

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