AN ERROR BASED MATHEMATICAL MODULE TO ENHANCE LEARNING IN SIGNALS AND SYSTEMS

L. Arriero, D. Osés, V. Osma-Ruiz, J. Gutiérrez, J. C. G. de Sande, R. Fraile, and C. Benavente.

E.U.I.T.T., Universidad Politécnica de Madrid (SPAIN)
larriero@ics.upm.es

Abstract

During the last years, the lecturers at the Circuits and Systems Engineering Department at the E.U.I.T. de Telecomunicación at the Universidad Politécnica de Madrid are observing more and more serious mathematical errors in the different exams and exercises taken by the students. Although some of these mistakes can be considered unacceptable in engineering disciplines, it is possible for a student to pass the final exam regardless of these mistakes. In this scenario, and aware that results were getting worse and worse year after year, it was considered convenient, and almost indispensable, to develop math exercises that students must practice if they want to progress following a continuous and formative assessment method along their engineering studies. The first part of this work is to analyze basic mathematical errors in final exam exercises of the course "Signals and Systems". We present and illustrate the most relevant errors detected during the last two years final exams of that course. The information obtained permits us to identify the main lacks, difficulties and defaults of the students. The second part of this work is to develop a training module in order to the students can practice as many times as they want with simple exercises dealing with the topics where frequent errors are detected. After practicing they must pass an initial test to make sure that students have acquired the adequate basic mathematical background and skills to progress successfully in the mentioned course. The questions and exercises have been written using different formats, most of them to be compatible with Moodle platform requirements.

Keywords: Errors in exams, tests questions, learning strategies.

1 INTRODUCTION

One year ago, in the fall semester of 2009, the lecturers of the EUIT de Telecomunicación began to teach according to *European Higher Education Area* as it appears in the Bologna [1] process commitments. In the course on Signals and Systems (initially denominated Linear Systems) the transition towards the student-centred strategies has been carried out in two steps. In the first step (fall 2007), we change the unique and exclusive final exam introducing a continuous and more formative assessment method including:

- Open exercises in the classroom and homework. (10%)
- More formal and rigorous exercises in the classroom to reinforce the lessons.(20%)
- Individual tests performed in a Web tool using Moodle [2] platform. (20%)
- A final and comprehensive exam. (50%)

The percentages indicate the relative contribution of the different activities to the final mark.

In this first step, different analysis and studies comparing final exam versus continuous assessment [3] and the evolution of efficiency and success rates [4] have been done too.

In the second step, we have included computer-based projects in the laboratory using Matlab, trying to change to a more integrated lecture-laboratory environment, and we have also proposed different collaborative activities for the students to work in teams of 3-4. The new assessment scenario is as follows:

- Open exercises in the classroom and homework. (8%)
- More formal and rigorous exercises in the classroom to reinforce the lessons. (20%)
- Individual tests performed in a Web tool using Moodle platform. (16%)

- Computer-based exercises in the laboratory. (8%)
- Collaborative activities in groups of 3-4 students. (8%)
- A final and comprehensive exam. (40%)

Along with this changes, the lecturers of the course in Signals and Systems began an experiment to enrich and complete the assessment process, consisting in searching the most common and serious errors made by the students in the exams. The main goal is to introduce a post-examination feedback by means of a collection of error-based exercises.

2 ANALYSIS OF ERRORS IN FINAL EXAMS

In this section we present the results obtained analysing the errors and mistakes that appear in the two main problems that students must solve in the final exam. In order to perform an exhaustive study of the mistakes found in fall 2008 and spring 2009 we prepared an excel sheet which was used by the lecturers enrolled in this experience. In this sheet we inserted the repetitions of the main errors detected during the correction and marking of the exercises.

The final exam consists of a test with 16 true/false questions (40% of the final mark), a first comprehensive problem corresponding to continuous-time concepts (30% of the final mark) and a second comprehensive problem corresponding to discrete-time concepts (30% of the final mark). In Table 1 we have listed the main concepts and learning outcomes corresponding to the first problem, in this list, we have also included items related to basic mathematical competencies where the students tend to fail.

1	Roots of a polynomial of degree 2
2	Algebraic operations on numbers and functions
3	Errors using standard mathematical notation
4	Magnitude and angle of complex numbers.
5	Magnitude of rational functions.
6	Graphical representation of functions.
7	Relationship between trigonometric functions and complex exponential functions.
8	Zero-pole representations
9	Memory of an LTI system characterized by H(s)
10	ROC of a causal LTI system
11	ROC of an stablel LTI system
12	Existence of the frequency response of a LTI system
13	Relationship between H(s) and H(jω)
14	Eigenfunction and eigenvalue.
15	Concept of frequency response.
16	Determining the Laplace transform using transform pairs.
17	Operations with unit impulse function (Dirac delta).
18	Mathematical expressions of sampling.
19	Spectral interpretation of sampling and aliasing.
20	Spectral interpretation of filtering.

Table 1: Learning outcomes and skills evaluated in the first problem of the final exam

In fig. 1 appear the ratios of errors corresponding to the concepts and skills listed in Table 1. The results have been obtained processing two sets of abut 250 exercises each, from fall 2008 and spring 2009 final exams.

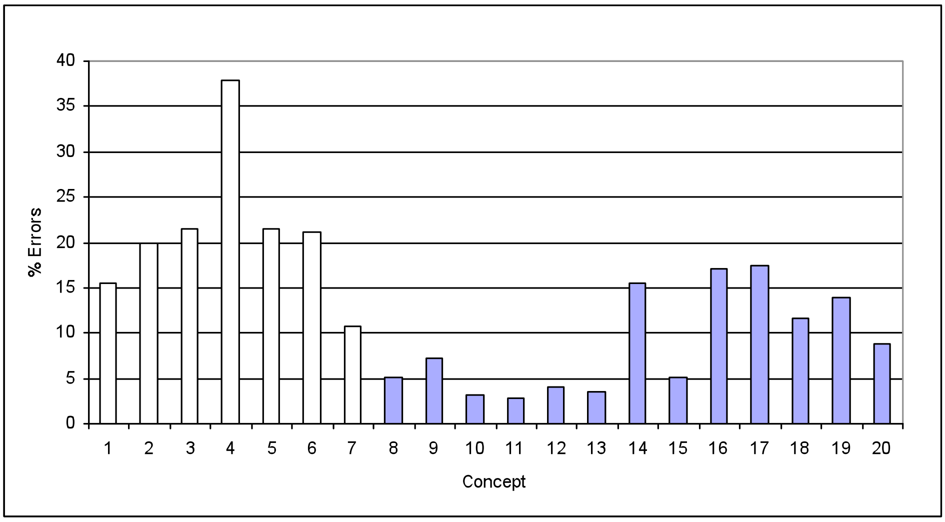


Fig 1: Ratios of errors corresponding to concepts of table 1

In a similar form was prepared Table 2, including the main concepts and learning outcomes of discrete-time signals and systems along with basic mathematical skills.

1	Roots of a polynomial of degree 2
2	Magnitude of rational functions.
3	Graphical representation of functions.
4	Graphical approximation of the frequency response.
5	Study of causality in time domain.
6	Study of stability in time domain.
7	Study of memory.
8	ROC of non causal system
9	Series association of systems
10	Prallel association of systems
11	Determining the Z-transform using transform pairs
12	Study of causality in Z-transform domain.
13	Study of stability in Z-transform domain
14	Concept of FIR and IIR.
15	Existence of the frequency response.

Table 2: Learning outcomes and skills evaluated in the second problem of the final exam

The error ratios obtained for the items of Table 2 can be seen in fig. 2. The results have been obtained processing the same exams commented before for fig. 1.

Analysing the results obtained, we can conclude the following:

- The errors and mistakes corresponding to specific concepts of signals and systems (coloured bars) tend to be less than those corresponding to mathematical skills (white bars).
- The same kind of mathematical mistakes can be found in the solutions given by some students to problems 1 and 2 (continuous-time and discrete-time). So, we consider that these errors can not be attributed simply to forgetfulness or exam anxiety.
- Some of the mathematical errors can be considered unacceptable in a second year university course. So, the lecturers must pay special attention to them.
- It is possible to pass the final exam making serious mathematical mistakes, because these errors are, in general, penalized lightly.

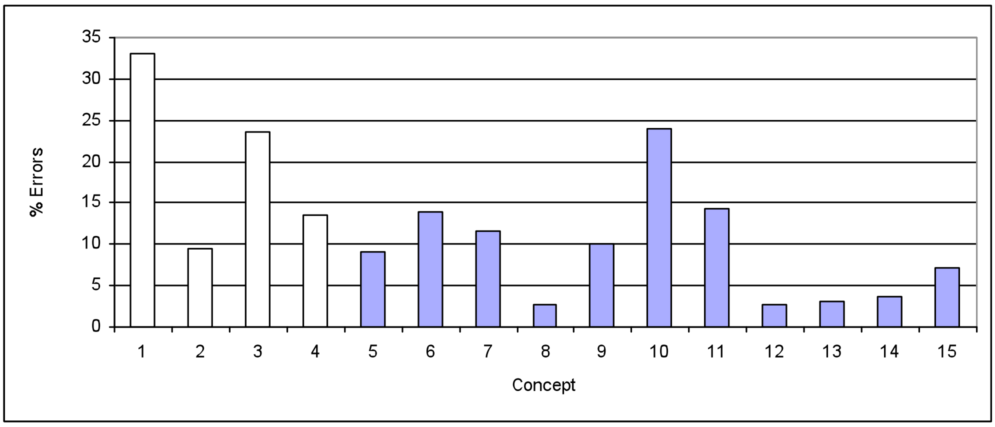


Fig 2: Ratios of errors corresponding to concepts of table 2

The above mentioned conclusions lead us to believe that something is going wrong when the students tend to consider that mathematical knowledge is irrelevant and must not be considered as a necessary part of the course in signals and systems. So, in order to improve the final marks and the pass rates, we have begun to incorporate mathematical test and exercises. Many of these exercises are based on the errors made by the students in exams.

3 ERROR-BASED QUESTIONS

In this section we present an example of error-based test prepared to be taken by students at the beginning of the course. The main idea is that students begun to consider mathematics as a relevant and very important part of the course.

Once we have identified the most important and usual errors (see figs. 1 and 2), a set of questions is derived. In table 3 appear a selection of five errors found during the correction of the exams. From these mistakes, we have prepared the five questions listed in Table 4. In order to enrich the resulting test, the questions can be of different types:

- True-false questions. These are the simplest kind and can be very useful to review some fundamental concepts.
- True-false and, if false, cross out what is wrong. In this case the student is required to analyze the expression provided and identify if some part is wrong.
- True-false and, if false, write the correct version below. These are the more important and comprehensive questions. These exercises allow us to review, in deep, concepts and mathematical skills.

Obviously, the final test must also include true questions, so, the resulting exercises will contain 10 questions to be solved in about 30 min.

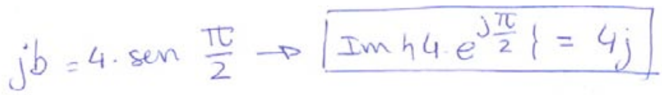
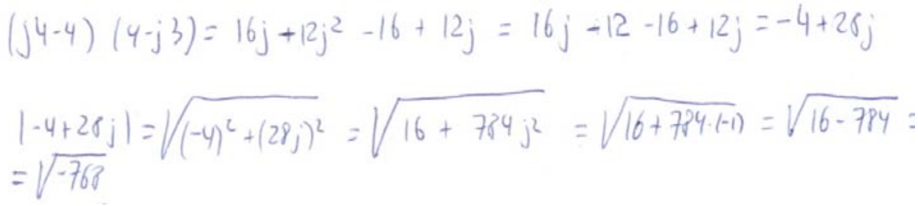
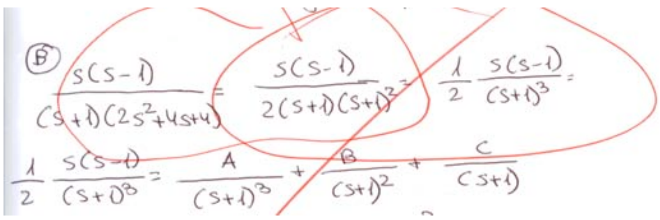
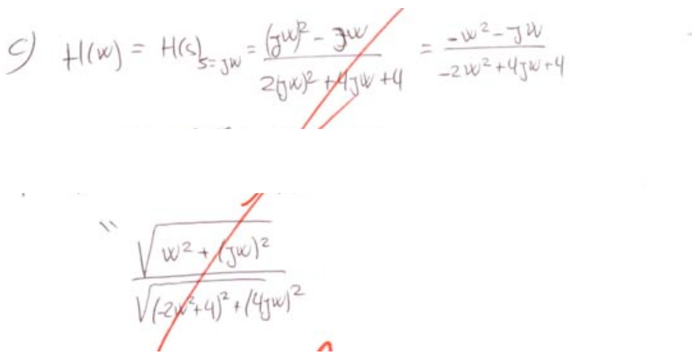
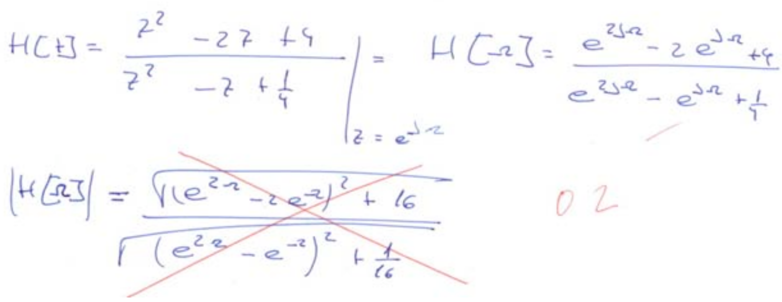
1	
2	
3	
4	
5	

Table 3: Examples of errors detected in different exams

1	$j\,b = 4sen\left(\dfrac{\pi}{2}\right) \rightarrow \operatorname{Im}\left\{4e^{j\frac{\pi}{2}}\right\} = 4j$	T/F ☐
2	$(4j-4)(4-3j) = 16j + 12j^2 - 16 + 12j = 16j - 12 - 16 + 12j =$ $-4 + 28j$ $\lvert -4+28j\rvert = \sqrt{(-4)^2 + (28j)^2}\ \sqrt{16+784j^2} = \sqrt{16+784(-1)} =$ $\sqrt{16-784} = \sqrt{-768}$	T/F ☐

3	$$\frac{s(s-1)}{(s+1)(2s^2+4s+4)}=\frac{s(s-1)}{2(s+1)((s+1)^2}=\frac{1}{2}\frac{s(s-1)}{(s+1)^3}=$$ $$\frac{A}{(s+1)^3}+\frac{B}{(s+1)^2}+\frac{C}{(s+1)}$$ If false, cross out what is wrong.	T/F ☐
4	$$H(\omega)=H(s=j\omega)=\frac{(j\omega)^2-j\omega}{2(j\omega)^2+4j\omega+4}=\frac{-\omega^2-j\omega}{-2\omega^2+4j\omega+4}=$$ $$\frac{\sqrt{\omega^2+(j\omega)^2}}{\sqrt{(-2\omega^2+4)+(4j\omega)^2}}$$ If false, cross out what is wrong.	T/F ☐
5	$$H(z)=\left.\frac{z^2-2z+4}{z^2-z+\frac{1}{4}}\right\|_{z=e^{j\Omega}}=H(\Omega)=\frac{e^{2j\Omega}-2e^{j\Omega}+4}{e^{2j\Omega}-e^{j\Omega}+\frac{1}{4}}$$ $$\|H(\Omega)\|=\frac{\sqrt{\left(e^{2\Omega}-2e^{\Omega}\right)^2+16}}{\sqrt{\left(e^{2\Omega}-e^{\Omega}\right)^2+\frac{1}{16}}}$$ If false, write the correct version below.	T/F ☐

Table 4: Questions derived from the mathematical errors listed in table 3

4 CONCLUSIONS

In this paper we have described a new experience in the “EUIT de Telecomunicación” at the Universidad Politécnica de Madrid. The work is based in identifying errors during the students assessment. With these errors and mistakes, lecturers generate a feedback based in new exams or exercises to reinforce the learning in the lacks detected.

We have found that students do not conceive of mathematics as a fundamental part of the course on "Signals and Systems" which is a major reason of repeated mathematical mistakes. We believe that the solution to this situation is not to let the students to solve their math deficiencies alone. On the other hand, we consider that it is necessary some orientation and some extra work in this direction.

Different exercises and tests derived from the errors detected in exams have been prepared and included as a compulsory part of the course.

The results obtained until now are encouraging and allow us to have hope in the new methodology. For the future it will be particularly important to continue improving the tests and adapting them to the new lacks that will be detected in the next years.

REFERENCES

[1] Confederation of EU Rectors' Conference. “The Bologna Declaration on the European space for higher education: an explanation”. On line http://ec.europa.eu/education/policies/educ/bologna/bologna.pdf

[2] “Moodle: A free, open source code course management system for online learning” . On line http://moodle.org.

[3] J. C. González de Sande, L. Arriero, C. Benavente, R. Fraile, J. I. Godino-Llorente, J. Gutiérrez, D. Osés, V. Osma-Ruiz, ”A case study: final exam versus continuous assessment marks for electrical and electronic engineering students”, *International Conference of Education, Research and Innovation. ICERI 2008.*

[4] J. C. G.de Sande, L. Arriero, C. Benavente, R. Fraile, J. I. Godino-Llorente, J.Gutiérrez, D. Osés, V. Osma-Ruiz. “Evolution of efficiency and success rates for electrical and electronic engineering students at EUITT from Universidad Politécnica de Madrid”. *International Technology, Education and Development Conference. INTED 2009.*