Linear stability analysis of an electron-radiative ablation front

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Hydrodynamic instabilities remain one of the limiting factors in fusion performance and efficiency. The ablation front of an inertial confinement fusion (ICF) imploding target is subject to these instabilities. In particular, during the acceleration phase of implosion, conditions for the development of the ablative Rayleigh-Taylor instability (RTI) occur. Recent work has indicated that the use of moderate-Z ablators (CHBr) significantly improve the hydrodynamic stability properties by reducing the growth of the abative RTI [1]. Besides, glass ablators (SiO2) reduce target preheat [2], improving, therefore, target compression. The structure of the ablation front (where hydrodynamic instabilities occur) is modified with the use of moderate Z-ablators, where the absorption of radiation energy and electron heat fluxes occur at two different locations. Thus, a structure of two ablation fronts separated by a region of constant density (plateau) develops [3]. This is called double ablation (DA) front. In the second front (the outer one) both, radiation flux and electron heat flux, play a fundamental role. Therefore, it is called electron-radiative ablation (ERA) front. In this work, we describe the linear stability analysis of the ERA front.

In order to build up the model of study, some hypothesis are assumed

i) The ERA front shall be isolated to study its stability. Thus, a constraint on the perturbation wavelength λ is set. Only perturbations with a wavelength much shorter that the plateau length are considered. That ensures the separation and the no-influence between both ablation fronts.

ii) The flow is subsonic (M<<1). Therefore, the isobaric approximation (P=constant) is used.

iii) Radiation temperature is constant and equal to matter termperature in the plateau [3].

The equations governing the problem are those of mass, momentum and energy conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0,$$

$$\rho \frac{\partial \bar{v}}{\partial t} + \rho \bar{v} \cdot \nabla \bar{v} = -\nabla p + \rho \bar{g}, \qquad (1.1)$$

$$\nabla \cdot \left(\frac{5}{2} p \bar{v} + k_t \left(\frac{T}{T_t} \right)^{5/2} \nabla T \right) = -4\sigma K_{pt} \left(\frac{T}{T_t} \right)^{-q_2} \left(T^4 - T_t^4 \right),$$

where an energy flux due to radiation is taken into account besides the electronic heat flux with the Spitzer conductivity $\left(k \sim T^{5/2}\right)$. The model chosen for the radiation transport is an optically thin approximation with one spectral-average absorption coefficient, the Planck mean opacity, K_P. In this way there can be distinguished three terms in the energy equation: those corresponding to convection and electronic heat fluxes (LHS) and the radiative term (RHS). This last one can be seen as a term of radiative losses since the temperature of radiation is assumed constant. The set of equation is completed with the equation of state for ideal gases. The acceleration $\overline{g} = g\,\overline{e}_y$ that appears in the momentum equation is the acceleration of the imploding target.

Introducing the steady base flow $\overline{v}(y) = v_0(y)\overline{e}_y$ in the system (1.1) and considering that the density and temperature only depend on the spatial coordinate y, we arrive to the equation of the order zero that reads

$$\frac{d}{d\eta}\left(\theta - \theta^{5/2}\frac{d\theta}{d\eta}\right) = -\beta_t\left(\theta^4 - 1\right), \tag{1.2}$$

where θ is the dimensionless temperature. A characteristic length comes naturally from the equation, $L_{sp} = \left(k_t T_t\right)/\left(\left(5/2\right)P v_t\right)$, that is used to form the dimensionless y-direction, η. The parameter that governs the equation is β, which is a mesure of the relative importance of the radiation energy flux in the problem

$$\beta_t \equiv \frac{16}{25}\frac{\left(\sigma T_t^4 K_{Pt}\right)\left(k_t T_t\right)}{\left(P_t v_t\right)^2}. \tag{1.3}$$

The condition for the development of a steep ERA is β>1. This condition is fulfilled by the use of moderate-Z ablators, for example for CHBr $\beta \simeq 6$ and for SiO_2 $\beta \simeq 20$. [3] The bigger is β, the steeper becomes the ERA front. In this way, the use of moderate-Z ablators makes the ERA front appear, and this structure is better defined when higher Z ablators are used.

Hydrodynamic instabilities are studied by perturbating the equations of the model to determine whether small-amplitude perturbations have a tendency to either grow (be unstable) or decay (be stable) in time. For a steady base flow, the decomposition $v(x,y,t)=v_0(y)+\varepsilon v_1(x,y,t)$ can be adopted where v_0 (y) is the steady base flow, v_1 (x,y,t) the perturbation and ε<<1 an amplitude parameter, considered small for linearisation. An analogous expression is also adopted for the temperature T(x,y,t). Perturbation is expressed as v_1 (x,y,t) $=\hat{u}_1(y)\exp(kx+\gamma t)$, where k is the wavenumber of the perturbation and γ the growth rate on time. Thus the condition for stability is Re(γ)<0. The wavenumber for which Re(γ)=0 is called the "cut-off". Wavenumbers bigger than the cut-off are stable. By linearising we obtain a five-order linear differential equations system.

A. Numerical solution

The adopted approach for the stability is to solve an eigenvalue problem (EVP) for the linearised perturbation equations. Modal analysis indicates that, in this problem, there are the same bounded modes (2) in the limit $\eta \to -\infty$ (left side) than unbounded modes in $\eta \to \infty$ (right side). The boundary condition at the left is a linear combination of the bounded modes at that side. The integration of the 5-order differential equations system leads to a solution that explodes, since the unbounded modes at the right develop. Thus, the dispersion relation (the relation between the growth rate γ of a perturbation with wavenumber k) comes from the

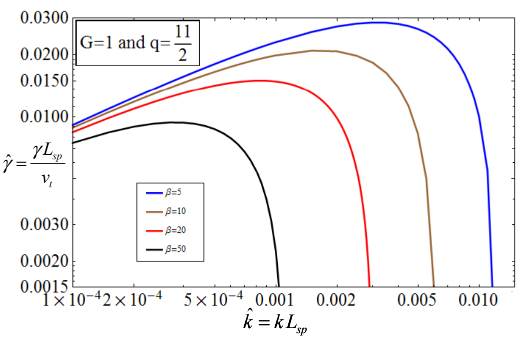

FIG. 1 Dispersion relation for G=1, q_2=11/2 and different β (β =5,10,20,50)

the compatibility equation[4]-[5], which imposes the solution to be bounded at the right side as well. The solution depends on 3 parameters: β, q_2 and the inverse of the Froude number $G=\left(gL_{sp}\right)/v_t^2$.

The effect of the parameter β is clearly stabilizing. From β=5 (CHBr) to β=20 (SiO_2), the cut-off wavenumber is reduced by a factor of 4 and the maximum growth rate is divided by 2.

B. Analytical solution. Limit β>>1

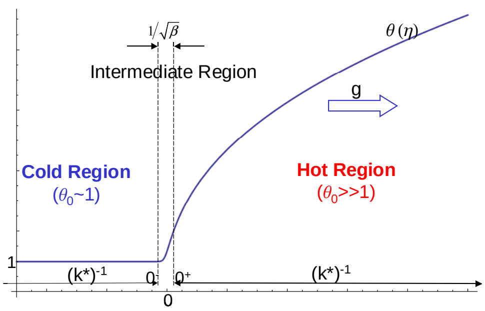

FIG. 2 Dimensionless density profile with the schematic representation of the three regions in which the problem is divided.

The bigger is β, the smaller are the unstable wavenumbers. That allows us to separate the problem into three regions: cold, intermediate and hot regions. Each of them has different scaling laws that give us information about the physics inside. The characteristic length of the problem is $\left(k^*\right)^{-1} \equiv \beta^{7/2(4n-7)} \gg 1$ where $n=\left(2q_2-1\right)/4$.

Thus, the normalization is $\hat{k}=k^*\tilde{k}$ and $\hat{\gamma}=\left(k^*\right)^{1/2}\tilde{\gamma}$. The analysis of the cold region leads to the form of the analytic dispersion relation, but still with the unknowns of the mass flux and the total momentum in the matching region (C_1 and C_2, respectively) that shall be determined with the analysis of the hot region. The analysis of the intermediate region let us make the matching between the cold and hot region. The analysis of the hot region gives us the information to close the problem. In this region, the vector of state V can be decomposed:

$$V = V_0(\varepsilon_0)+\varepsilon_1 V_1(\varepsilon_0)+\varepsilon_2\left(GV_{2g}(\varepsilon_0)+\alpha V_{2\alpha}(\varepsilon_0)\right), \qquad (1.4)$$

where, $\varepsilon_0 \equiv (k^*)^{\frac{1}{14}}\tilde{k}^{\frac{-(n-1)}{n}}\tilde{\gamma}\approx O(1)$, $\varepsilon_1 \equiv (k^*)^{\frac{6}{14}}\tilde{k}^{\frac{5-2n}{2n}} \ll 1$, $\varepsilon_2 \equiv (k^*)^{\frac{8}{14}}\tilde{k}^{\frac{9-4n}{2n}} \ll 1$ and $\alpha \equiv \frac{\tilde{\gamma}^2}{\tilde{k}} \approx O(1)$.

When all the coefficients corresponding to the mass flux and the total momentum (functions with only ε_0-dependency) are computed, they are introduced into the expression coming from the cold region to obtain the dispersion relation, that reads

$$\tilde{\gamma}^2+\tilde{\gamma}\frac{(k^*)^{\frac{1}{14}}\tilde{k}^{\frac{4n-5}{2n}}\tilde{C}_{10}+(k^*)^{\frac{1}{2}}\tilde{k}(1+\tilde{C}_{11})}{1+\mu_\alpha}-\tilde{k}G\frac{1-\mu_g}{1+\mu_\alpha}+\frac{\tilde{k}^{\frac{6n-7}{2n}}\tilde{C}_{20}+(k^*)^{\frac{6}{14}}\tilde{k}^{\frac{4n+5}{2n}}\tilde{C}_{21}-(k^*)^{\frac{8}{14}}\tilde{k}^{\frac{6n-5}{2n}}\tilde{C}_{10}}{1+\mu_\alpha}=0, \qquad (1.5)$$

where $\mu_g \equiv (k^*)^{8/14}\tilde{k}^{1/n}\tilde{C}_{22g}$ and $\mu_\alpha \equiv (k^*)^{8/14}\tilde{k}^{1/n}\tilde{C}_{22\alpha}$.

Notice that the factor $(1-\mu_g)/(1+\mu_\alpha)$ can be interpreted as the Atwood number of this problem and that the coefficient $\tilde{C}_{20}$ gives the dominant order for the cut-off wavenumber.

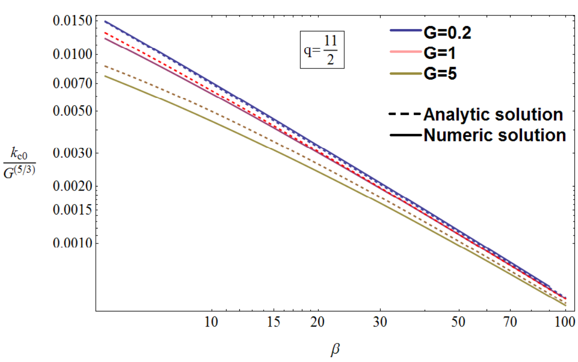

FIG. 3 Comparison between the analytical and numerical solutions of the cut-off wavenumber Vs β for q_2=11/2 and different values of G.

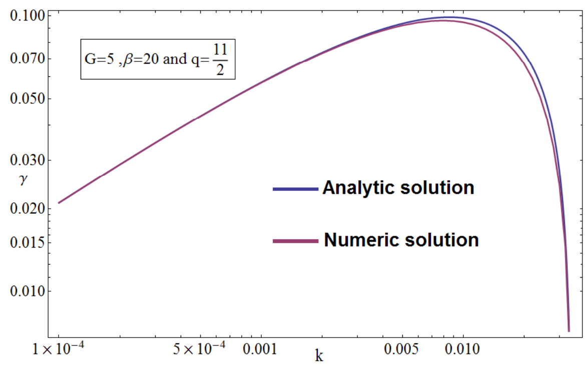

FIG. 4 Comparison between the analytical and the numerical solution of the dispersion relation for G=5, β=20 and q_2=11/2.

Linear stability theory for an electron-radiative ablation front is presented. Radiation losses, modelized by the parameter β, stabilize the ERA front. The bigger is β, the more stable behaviour of the ERA front is found. An expression for the dispersion relation is obtained, in good agreement with the numerical results.

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