

## Stability analysis for dusty plasma under grain charge fluctuations due to non-Maxwellian electron distributions

J. M. Donoso, A. Sánchez-Torres and L. Conde

*Dpto. Física Aplicada, E.T.S.I Aeronáuticos, Univ. Politécnica de Madrid, 28040 Madrid, Spain.*

**Introduction.** The most relevant characteristic of a complex plasma is the dust charge as well as the dust charging process itself which controls different collective and individual behaviors of the plasma. The dust charging has been exhaustively studied providing several theoretical approaches that have improved the early Orbital Motion Limited (OML) description [1, 2]. The OML is considered as a suitable model and, at least, it provides a certain perspective of the main plasma parameters involved in the charging processes. Recent works have stressed the importance of the electron and ion velocity distribution functions in addressing the description of plasma stability analysis under the frame of plasma fluid description, including dust charge fluctuations. The consideration of non-Maxwellian distribution functions has been proved to induce plasma departures from the usual Maxwellian equilibrium, specially under the development of intense electric fields that can accelerate charges till superthermal velocities [3]. In these cases, the distribution function tails fit well to a power-law dependence even for electrons. In this communication, we devote attention to the effect of non Maxwellian electron distribution functions on the collective plasma behavior through a linear analysis of perturbed fluid equations. The stability of a partially ionized complex plasma with a non Maxwellian electron population is studied by including this feature on the dust charge fluctuation for infinite and finite dust grain mass.

**Non-Maxwellian distributions in fluid and charging equations.** We study a dusty plasma that could be described by its species distribution functions  $f_j(\mathbf{v}, \mathbf{r})$ , ( $j = e, i, d, n$ ) each one governed by a kinetic Vlasov-Boltzmann like equation with source-sink contributions. The evolution equation of electron and ion distribution functions  $df/dt$ , i.e. the Vlasov term, should have a sink term in form  $-c_0 \int \sigma_c(\mathbf{r}, \mathbf{v}') \mathbf{v}' f(\mathbf{r}, \mathbf{v}', t) f_d(\mathbf{r}, \mathbf{v} - \mathbf{v}') d\mathbf{v}'$  accounting to the losses of charging particles due to the impact onto the dust grains having distribution  $f_d$ . This simple collisional-like term assumes total (or partial) absorption of charges by the grain with relative velocity  $\mathbf{v} - \mathbf{v}'$  proportional to the number of encounters in phase space. The effective cross section  $\sigma_c$  can be taken from any charging model, as for instance, the OML approximation.

Therefore, the dust charging contributes to the plasma species fluid equations gives rise some terms proportional to both, the charging currents  $I_j$  and the dust density  $n_d$ . This contribution should be properly stated before carrying out a stability analysis. For the charging process we consider Maxwellian ions but non-Maxwellian electron distribution  $f_e$  which would satisfy the unperturbed steady state Vlasov equation. Hence, we would have  $f_e(v) = f_{e0}(\sqrt{|v^2 - 2e\phi/m_e|})$  if a plasma potential perturbation  $\phi$  is considered. As a simple application, we have used the so-called kappa-distribution [3]  $f_{e0} = n_{e0}N_\kappa [1 + (m_e v^2/T_{e0}) \cdot (2\kappa - 3)]^{-\kappa-1}$  ( $\kappa > 3/2$  and  $N_\kappa$  is the normalization constant) which tends to the Maxwellian distribution for infinite  $\kappa$ . From this function, the electron density and average kinetic energy profiles can be obtained, giving

$$n_e = n_{e0} \xi_\phi^{1/2-\kappa}, \quad T_e = T_{e0} \xi_\phi, \quad \text{with} \quad \xi_\phi = 1 - 2e\phi/T_{e0}(2\kappa - 3). \quad (1)$$

The dust charge currents  $I_j$  due to electrons and ions can be obtained by applying, for instance, the usual theoretical OML charging model which states the dependence on the dust charge fluctuation and dust density on the electron and ion fluid equations. So that, for the OML effective charging cross section  $\sigma_c$  for grains of radius  $a$ , using the unperturbed function  $f_{j0}$  of species  $j$ , the charging current can be expressed as

$$I_j = q_j \pi a^2 \int_{v_{mj} \varepsilon_j}^{\infty} v \left(1 - 2 \frac{q_j \Phi_d}{m_j v^2}\right) f_{j0}(v) 4\pi v^2 dv \quad (2)$$

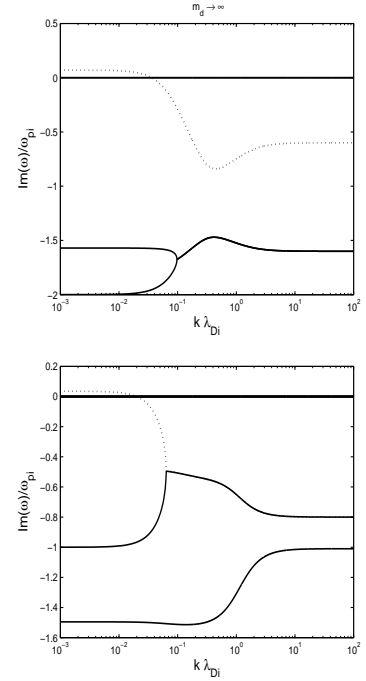
where  $\Phi_d \approx q_d/a$  is the surface grain potential with respect to the plasma and  $\varepsilon_j = (\text{sign}(q_j \Phi_d) + 1)/2$  and  $v_{mj}^2 = 2|\Phi_d q_j|/m_j$ . Considering the charge conservation equation for an isolated plasma  $\partial(en_i - en_e + q_d n_d)/\partial t + \nabla \cdot (en_i \mathbf{u}_i - en_e \mathbf{u}_e + q_d n_d \mathbf{u}_d) = 0$  for a fluctuating dust charge  $q_d = -eZ$ , ( $\varepsilon_e = 1, \varepsilon_i = 0$ ) the dust charging equation is

$$\partial q_d / \partial t + \mathbf{u}_d \cdot \nabla q_d = I_e + I_i = [-en_e V_{Te} \zeta_e(T_e, Z) + en_i V_{Ti} \zeta_i(T_i, Z)] \pi a^2, \quad (3)$$

where the dimensionless functions

$$\zeta_j(T_j, Z) = \int_{\varepsilon_j w_{mj}}^{\infty} w \left(1 - \frac{w_{mj}^2}{w^2}\right) \hat{f}_j(w) 4\pi dw \quad (4)$$

have been defined in terms of the dimensionless distribution  $\hat{f}_j(w) = V_{Tj}^3 f_{j0}(V_{Tj} w)/n_{j0}$ , where  $w_j = v_{mj}/V_{Tj}$ , for species  $j$  with thermal velocity  $V_{Tj} = \sqrt{T_j/m_j}$ . From these expressions, and because the dependence on  $I_j n_d$  ( $I_e n_d$ ) on the fluid equations sink terms, it is convenient to define a characteristic charging frequency  $\omega_i$ , due to  $n_i$  fluctuation, as  $e\omega_i/n_d = \partial I_i / \partial n_i$  which allows us to write  $\partial I_e / \partial n_e = -e\omega_i n_i / (n_e n_d)$  (computed at equilibrium) which, together with the two charging frequencies  $\nu_{qj} = -\partial I_j / \partial q_j$  ( $j = e, i$ ), the effects of the charging currents and dust charge fluctuations in the coupled perturbed equations can be included by means of these three frequencies with  $\delta I_i + \delta I_e = I_{i0} (\delta n_i / n_i - \delta n_e / n_e) - (\nu_{qe} + \nu_{qi}) \delta q_d$ .



**The model.** We are now in position to establish the fluid model equation for the stability analysis of a partially ionized complex plasma accounting with the dust charging. Hence, for single charged positive ions, we have the ions continuity equation

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i \mathbf{u}_i) = -\frac{I_i n_d}{e} + \nu_I n_e - \nu_l n_i \quad (5)$$

where we have included a source of ions for ionization proportional to the electron density  $n_e$ , although the frequency  $\nu_I$  can also be a function of  $n_e$  [4]. The sink term with frequency  $\nu_l$  accounts itself with the ion losses due to several mechanisms as recombination. The remaining sink comes from the dust charging and it explicitly depends upon dust and ion densities, the latter is included on  $I_i$ . The time evolution equation for the ion momentum density  $n_i \mathbf{u}_i$ , taking into account the frequencies  $\nu_{ij}$  for collisions between species  $i$  and  $j$  and the previous discussion on kinetic descriptions, reads :

$$\frac{\partial n_i \mathbf{u}_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i \mathbf{u}_i + \mathbb{P}/m_i) = -e \nabla \phi / m_i - I_i n_d \mathbf{u}_d / e - n_i \sum_j \nu_{ij} (\mathbf{u}_i - \mathbf{u}_j) \quad (6)$$

where  $j$  is referred to electrons, dust grains and neutrals. The only relevant collision frequency considered here is  $\nu_i = \nu_{ia}$  due to the interaction of ions with cold neutrals at rest. Although similar equations hold for the electrons, in this contribution we assume the electron population to satisfy the relations  $n_e(\phi)$  and for  $T_e(\phi)$  directly derived from the non-Maxwellian distribution (1) that should replace the usual Maxwellian representation. So that,  $n_e$  can be approximated by a linear function of the perturbation plasma potential  $\delta\phi$  as

$$n_e = \int f_e d\mathbf{v} \approx \int (f_{e0} - \frac{e\delta\phi}{m_e} \frac{1}{v} \frac{\partial}{\partial v} f_{e0}) 4\pi v^2 dv \quad \text{giving} \quad \delta n_e \approx -n_{e0} \frac{e\delta\phi}{m_e} 4\pi \int f_{e0} dv \quad (7)$$

For the kappa distribution, these relations can be obtained by Taylor expansion of (1) up to first order in  $\phi = \delta\phi$ , giving  $n_e - n_{e0} = \delta n_e = n_{e0} e \delta\phi (2\kappa - 1) / (2\kappa - 3) T_{e0}$  which, if compared with the usual Maxwellian relation  $n_{e0} e \delta\phi / T_e$ , we find that an effective electron temperature can be defined as  $T'_e = (2\kappa - 3) T_{e0} / (2\kappa - 1) < T_{e0}$ . Disregarding ion-electron and dust-electron collisions and neglecting also the electron inertia, the electron equations become decoupled from the rest, entering into the analysis through  $\delta n_e$ . Due to the fact that many collective effects involve grains oscillations, it is noteworthy to deal with dust and momentum density fluctuations in the linear stability analysis. For simplicity, we assume for this work the grains to satisfy a continuity equation with no source-sink terms, whereas for the momentum density, further considerations are stated in the following. First, let us stress that, by the same reasoning leading to (3), an equivalent relation holds for the dust mass  $m_d$  variation as  $\partial m_d / \partial t + \mathbf{u}_d \cdot \nabla m_d = (m_e |I_e| + m_i I_i) / e$  meaning that the grain mass still increases although the charging equilibrium  $I_e + I_i = 0$  is reached. Therefore, we write the momentum density equation as  $m_d d(n_d \mathbf{u}_d) / dt = -q_d n_d \nabla \phi - m_d \nu_d \mathbf{u}_d \simeq -q_d n_d \nabla \phi$ , where we have neglected the dust pressure term  $\mathbb{P}$  and the collision frequencies between grains and the other lighter species. The linearized equations for finite mass oscillating grains, with constant charge sign, are

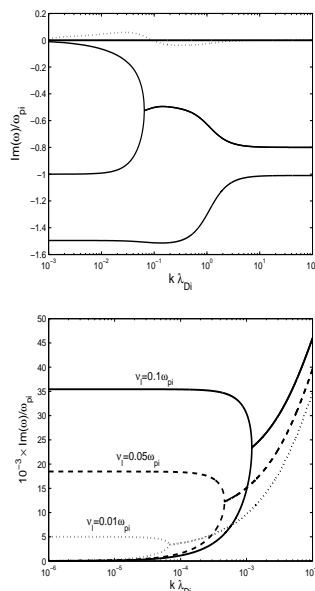
$$\frac{\partial \delta n_d}{\partial t} + n_{d0} \nabla \cdot \delta \mathbf{u}_d = 0 \quad \text{and} \quad m_{d0} n_{d0} \frac{\partial \delta \mathbf{u}_d}{\partial t} + q_{d0} n_{d0} \nabla \delta \phi = 0 \quad (8)$$

The set of equations is closed by using the Poisson equation for the plasma potential  $\nabla^2 \phi = 4\pi e (n_e + Z n_d - n_i)$ .

**Discussion and conclusions.** Finally, dropping the zero subscript for noting equilibrium values, with the dimensionless parameters  $\delta$  and  $\tau$  derived from  $n_i = (1 + \delta)n_e$ ,  $T_i = \tau T_e'$ , the grain mass-to-charge ratio  $\gamma_d = m_d/Zm_i$  and with the plasma ion and dust frequencies related by  $\gamma_d \omega_{pd}^2 = \omega_{pi}^2 \delta / (1 + \delta)$ , the linear equations can be cast into a matrix form as

$$\begin{pmatrix} -i\omega + \nu_l + \omega_i & ikn_e(1 + \delta) & \nu_{qi} n_e \frac{\delta}{Z} & -n_e \left[ \nu_l \tau + \frac{k^2}{\omega^2} \frac{1 + \delta}{\gamma_d} V_{Ti}^2 \omega_i \right] \\ ik \frac{V_{Ti}^2}{n_e(1 + \delta)} & -i\omega + \nu_i & 0 & ikV_{Ti}^2 \left[ 1 + i \frac{\omega_i}{\omega} \frac{1}{\gamma_d} \right] \\ \frac{\omega_i Z}{n_e \delta} & 0 & -i\omega + \nu_{qe} + \nu_{qi} & -\omega_i \tau (1 + \delta) \frac{Z}{\delta} \\ -1 & 0 & n_e \frac{\delta}{Z} & n_e \left[ \tau + k^2 \frac{V_{Ti}^2}{\omega_{pi}^2} (1 + \delta) \left( 1 - \frac{\omega_{pd}^2}{\omega^2} \right) \right] \end{pmatrix} \begin{pmatrix} \tilde{n}_i \\ \tilde{u}_i \\ \tilde{Z} \\ \tilde{\phi} \frac{e}{T_i} \end{pmatrix} = 0 \quad (9)$$

from which the wave dispersion relation can be extracted, after linearizing and Fourier transforming the perturbations  $\delta X$  to  $\tilde{X}$  by the phasor  $e^{-i\omega t + kx}$ .



In agreement with previous works [5] for massive grains ( $1/\gamma_d = 0$ ) an instability emerges (in the  $1/\omega_{pi}$  time scale) in one of the three modes because of the non zero ionization which has to satisfy  $\nu_l = (1 + \delta)(\nu_l + \omega_i)$  because of the initial equilibrium condition. However for  $\nu_i > \nu_{qe} + \nu_{qi} + \omega_i$  we find a quite different branches disposal, since the unstable-stable mode disappears and a new stable-stable one emerges, as shown in the previous figures. For finite  $m_d$  ( $\gamma_d < \infty$ ) two new branches come out, giving rise to a bifurcation at the origin ( $\text{Im}(\omega) = 0$ ,  $k = 0$ ). This remarkable behavior does not allow to cross the  $\omega = 0$  axis with continuity, as in the previous case (see the last figure frames). The unstable mode length for low  $k$  is governed not only for  $\nu_l$  but also by  $\nu_{qe}$  with no significant change for different tested electron distributions, although  $\nu_{qe}$  is slightly greater for Maxwellian electrons. For large  $k$  there are always two (or

three) stable modes corresponding to the asymptotic values  $\nu_i$  and  $\nu_{qe} + \nu_{qi}$  for  $\text{Im}(\omega)$ . Thus, there is always an instability due to charging that would only disappear for zero  $\omega_i$  and  $\nu_l$ .

**Acknowledgments:** This work is supported by the Spanish Ministry of Science and Innovation under projects ENE2007-67406-C02-01, AYA2008-04769 and the FPI grant BES-2009-013319.

## References

- [1] V. N. Tsytovich, G. E. Morfill, S. V. Vladimirov, and H. Thomas, *Elementary Physics of Complex Plasmas*. Springer-Verlag, Berlin(2008).
- [2] J. R. Bhatt and B. P. Pandey, *Phys. Rev. E* **50**, 3980(1994).
- [3] N. Rubab and G. Murtaza *Phys. Scr.* **73**,178-183(2006).
- [4] L. Conde, *Phys. Plasmas*. **13**, 032104 (2006).
- [5] D. Else, K. Kompaneets and S. V. Vladimirov *Phys. Rev. E*, **80**, 016303-1, 5(2009).