Ranking Alternatives on the Basis of a Dominance Intensity Measure

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Abstract. The additive multi-attribute utility model is widely used within Multi-Attribute Utility Theory (MAUT), demanding all the information describing the decision-making situation. However, these information requirements can obviously be far too strict in many practical situations. Consequently, incomplete information about input parameters has been incorporated into the decision-making process. We propose an approach based on a dominance intensity measure to deal with such situations. The approach is based on the dominance values between pairs of alternatives that can be computed by linear programming. These dominance values are transformed into dominance intensities from which a dominance intensity measure is derived. It is used to analyze the robustness of a ranking of technologies for the disposition of surplus weapons-grade plutonium by the *Department of Energy* in the USA, and compared with other dominance measuring methods.

Keywords. Additive Multi-Attribute Utility Model, Dominance Intensity Measure, Linear Programming.

Introduction

For the reasons described in Raiffa (1982) and Stewart (1996), the additive model is considered to be a valid approach in most real decision-making problems, and is widely used. The functional form of the additive model is $u(A_i) = \sum_j w_j\, u_j(x_{ij})$, where x_{ij} is the specific performance of attribute X_j for alternative A_i, $u_j(x_{ij})$ is the utility associated with the above performance for u_j, the corresponding component utility function representing the decision maker's preferences over the possible attribute performances, and w_j are the weights or scaling constants for each attribute representing their relative importance.

However, complex decision-making problems are usually plagued with uncertainty, and it is impossible to predict with certainty how each alternative under consideration will perform. For instance, performances may be intangible or non-monetary as they reflect social or environmental impacts. Also, performances may be taken from statistics or measurements. Additionally, it is often not easy to elicit precise values for the scaling weights representing the relative importance of criteria. They are often

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described within prescribed bounds. The DM may find it difficult to compare criteria or may not want to reveal his or her preferences in public. Moreover, the decision could be taken in a group decision-making situation, where incomplete information, such as weight intervals, is usually derived from a negotiation process (Jiménez et al., 2005; Mateos et al., 2006).

Many researchers have dealt with imprecise information. Sage and White (1984) proposed the model of imprecisely specified multi-attribute utility theory (ISMAUT), where preference information about both weights and utilities is not assumed to be precise. Malakooti (2000) suggested a new efficient algorithm for ranking alternatives when there is incomplete information about the preferences and the value of the alternatives. This involves solving a single mathematical programming problem many times. Ahn (2003) extends Malakooti's work. Eum et al. (2001) provided linear programming characterizations of dominance and potential optimality for decision alternatives when information about performances and/or weights is incomplete, extended the approach to hierarchical structures (Lee et al., 2002), and developed the concepts of weak potential optimality and strong potential optimality (Park, 2004). More recently, Mateos et al. (2007) considered the more general case where imprecision appears in the alternative performances, as well as in weights and utilities, described by means of fixed bounds. Sarabando and Dias (2009) gave a brief overview of approaches proposed by different authors to deal with incomplete information.

In this paper we consider a decision-making problem with m alternatives, A_i, i=1,…,m, and n attributes, X_j, j=1,…,n, where incomplete information about input parameters has been incorporated into the decision-making process:

- Alternative performances are described under uncertainty ($x_{ij} \in \lfloor x_{ij}^L, x_{ij}^U \rfloor$, i= 1,…,m, j=1,…,n) where x_{ij}^L and x_{ij}^U are the lower and the upper performances of the attribute X_j for the alternative A_i, respectively.
- Imprecision concerning component utilities ($u_j(\cdot) \in \lfloor u_j^L(\cdot), u_j^U(\cdot) \rfloor$, j = 1,…,n), where $u_j^L(\cdot)$ and $u_j^U(\cdot)$ are the lower and the upper utility functions of the attribute X_j, and
- Imprecision concerning weights ($w_j \in \lfloor w_j^L, w_j^U \rfloor$, j = 1,…,n), where w_j^L and w_j^U are the lower and the upper weights for the attribute X_j.

One possibility described in the literature for dealing with imprecision attempts to eliminate inferior alternatives based on the concept of *pairwise dominance*. It is often worthwhile to output the non-dominated alternatives because of the following property. If one alternative A_k dominates another A_l, then A_l can be discarded as a worthwhile solution for the DM. Given two alternatives A_k and A_l, alternative A_k dominates A_l if $D_{kl} \geq 0$, D_{kl} being the optimum value of the optimization problem:

$$D_{kl} = \min \{u(A_k) - u(A_l) = \sum_j w_j u_j(x_{kj}) - \sum_j w_j u_j(x_{lj})\}$$

$$\text{s.t.} \quad w_j^L \leq w_j \leq w_j^U, j=1,\ldots,n$$

$$x_{kj}^L \leq x_{kj} \leq x_{kj}^U, j=1,...,n \quad (1)$$

$$x_{lj}^L \leq x_{lj} \leq x_{lj}^U, j=1,...,n$$

$$u_j^L(x_{kj}) \leq u_j(x_{kj}) \leq u_j^U(x_{kj}), j=1,...,n$$

$$u_j^L(x_{lj}) \leq u_j(x_{lj}) \leq u_j^U(x_{lj}), j=1,...,n.$$

The objective function in the above optimization problem can also be represented by $\boldsymbol{w}(\boldsymbol{u}_k - \boldsymbol{u}_l)$, where $\boldsymbol{w} \in \boldsymbol{W} = \{(w_1, \ldots, w_n): w_j \in [w_j^L, w_j^U], j = 1, \ldots, n\}$, $\boldsymbol{u}_k \in \boldsymbol{U}_k = \{(u_1(x_{k1}), \ldots, u_n(x_{kn})): u_j(x_{kj}) \in [u_j^L(x_{kj}), u_j^U(x_{kj})], x_{kj} \in [x_{kj}^L, x_{kj}^U], j = 1, \ldots, n\}$, and $\boldsymbol{u}_l \in \boldsymbol{U}_l = \{(u_1(x_{l1}), \ldots, u_n(x_{ln})): u_j(x_{lj}) \in [u_j^L(x_{lj}), u_j^U(x_{lj})], x_{lj} \in [x_{lj}^L, x_{lj}^U], j = 1, \ldots, n\}$.

A recent approach is to use information about each alternative's intensity of dominance, known as *dominance measuring methods*. Ahn and Park (2008) compute both dominating and dominated measures, from which they derive a *net dominance*. This is used as a measure of the strength of preference in the sense that a greater net value is better.

In this paper we introduce a dominance measuring method. This method first computes dominance intensities depending on the dominance among alternatives. Then, a dominance intensity measure is derived for each alternative as the sum of the dominance intensities of that alternative regarding the others. This is used as a measure of the strength of the preference.

The paper is organized as follows. Section 1 proposes the dominance measuring method. Section 2 shows how this approach can be applied by solving linear problems. Section 3 illustrates the approach with an example. Finally, we outline our conclusions in Section 4.

1. Dominance Measuring Method

The dominance measuring method we introduce is based on the one proposed in Mateos et al. (2009), specifically, on the fact that $\boldsymbol{w}(\boldsymbol{u}_k - \boldsymbol{u}_l) \in [D_{kl}, -D_{lk}]$, $\forall \boldsymbol{w} \in \boldsymbol{W}$, $\forall \boldsymbol{u}_k \in \boldsymbol{U}_k$, $\forall \boldsymbol{u}_l \in \boldsymbol{U}_l$. Then, the *dominance intensity* can be defined as follows:

- If $-D_{lk} \leq 0 \Leftrightarrow D_{kl} < 0$ and $D_{lk} \geq 0 \Leftrightarrow$ alternative A_l dominates $A_k \Rightarrow$ the dominance intensity of alternative A_k over A_l can be defined as $-DI_{kl}$, where

$$DI_{kl} = d(0, [D_{kl}, -D_{lk}]) = \sqrt{\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\frac{D_{kl} - D_{lk}}{2} + x(-D_{lk} - D_{kl})\right]^2 dx}$$

$$= \sqrt{\left(\frac{D_{kl} - D_{lk}}{2}\right)^2 + \frac{1}{3}\left(\frac{-D_{lk} - D_{kl}}{2}\right)^2}$$

and $d(0, [D_{kl}, -D_{lk}])$ is the distance from the interval $[D_{kl}, -D_{lk}]$ to 0 (see Mateos et al., 2007). Note that all values in interval $[D_{kl}, -D_{lk}]$ are negative.

- If $D_{kl} \geq 0 \Leftrightarrow D_{kl} \geq 0$ and $D_{lk} < 0 \Leftrightarrow$ alternative A_k dominates $A_l \Rightarrow$ the dominance intensity of alternative A_k over A_l is $DI_{kl} = d([D_{kl}, -D_{lk}], 0)$. Note that in this case all values in $[D_{kl}, -D_{lk}]$ are positive.

- If $D_{kl} < 0$ and $D_{lk} < 0$, then interval $[D_{kl}, -D_{lk}]$ will consist of a positive subinterval with positive values in which alternative A_k dominates A_l and a negative subinterval in which alternative A_l dominates A_k. Thus, the dominance intensity of alternative A_k over A_l is $DI_{kl} = d([0, -D_{lk}], 0) - d([D_{kl}, 0], 0) = (-D_{lk} + D_{kl})/\sqrt{3}$, i.e. both positive and negative values in interval $[D_{kl}, -D_{lk}]$ are considered.

On the basis of this idea, paired dominance values D_{kl} are first transformed into dominance intensities DI_{kl} depending on the dominance among alternatives A_k and A_l. Then a dominance intensity measure (DIM_k) is derived for each alternative A_k as the sum of the dominance intensities of alternative A_k regarding the other alternatives. This is used as a measure of the strength of preference in the sense that a greater dominance intensity measure is better.

The method can be implemented in the following three steps:

1. Obtain the paired dominance values D_{kl} by solving problem (1) and consider the following matrix:

$$\begin{pmatrix} - & D_{12} & \dots & D_{1m-1} & D_{1m} \\ D_{21} & - & \dots & D_{2m-1} & D_{2m} \\ D_{31} & D_{32} & \dots & D_{3m-1} & D_{3m} \\ & & \dots & & \\ D_{m1} & D_{m2} & \dots & D_{mm-1} & - \end{pmatrix}.$$

2. Obtain the dominance intensity of alternative A_k over $A_{k_l l}$, DI_{kl}, by applying the following method and consider the matrix:

$$\begin{pmatrix} - & DI_{12} & \dots & DI_{1m-1} & DI_{1m} \\ DI_{21} & - & \dots & DI_{2m-1} & DI_{2m} \\ DI_{31} & DI_{32} & \dots & DI_{3m-1} & DI_{3m} \\ & & \dots & & \\ DI_{m1} & DI_{m2} & \dots & DI_{mm-1} & - \end{pmatrix}.$$

If $D_{kl} \geq 0$, then alternative A_k dominates alternative A_l, i.e. the dominance intensity of alternative A_k over A_l is $DI_{kl} = d([D_{kl}, -D_{lk}], 0)$.
Else ($D_{kl} < 0$):

- If $D_{lk} \geq 0$, then alternative A_l dominates alternative A_k, therefore, the dominance intensity of alternative A_k over A_l is - DI_{kl}.
- Else note that alternative A_l is preferred to alternative A_k for those values in $\boldsymbol{w}$ that satisfy $D_{kl} \leq \boldsymbol{w}(\boldsymbol{u}_k - \boldsymbol{u}_l) \leq 0$, and A_k is preferred to A_l

for those values in $\boldsymbol{w}$ that satisfy $0 \leq \boldsymbol{w}(\boldsymbol{u}_k - \boldsymbol{u}_l) \leq -D_{lk} \Rightarrow$ the dominance intensity of A_k over A_l is $DI_{kl} = (-D_{lk} + D_{kl})/\sqrt{3}$.

3. Compute a dominance intensity measure (DIM) for each alternative A_k

$$DIM_k = \sum_{l=1, l\neq k}^{m} DI_{kl}.$$

Rank alternatives according to the DIM values, where the best (rank 1) is the alternative with greatest DIM and the worst is the alternative with the least DIM.

A similar simulation study to the one in Mateos et al. (2009) remains to be done to compare the proposed method with *surrogate weighting methods* (Stillwell et al. 1981; Barron and Barrett, 1996), the modification of *three classical decision rules* to encompass an imprecise decision context (Puerto et al., 2000), and with dominance measuring methods proposed in Ahn and Park (2008) and Mateos et al. (2009). The performance of the proposed approach will be analyzed in terms of the best alternative and the overall ranking of alternatives.

2. Using linear programming to compute dominance values

In the first step of the proposed dominance intensity measure paired dominance values D_{kl} must be computed solving the optimization problem (1). In this section we show that, although the optimization problems to be solved in this context with incomplete information are not linear, they can be transformed and then solved by linear programming.

As mentioned earlier, we consider that the decision-making problem includes incomplete information about input parameters. Alternative performances are described under uncertainty by intervals ($x_{ij} \in [x_{ij}^L, x_{ij}^U]$, i=1,…,m, j=1,…,n) and attribute weights representing their relative importance are imprecise ($w_j \in [w_j^L, w_j^U]$, j = 1,…,n).

We also allow for imprecise information in the procedure for assessing of the individual utility functions (probability equivalence/certainty equivalence) (Farquhar, 1984), obtaining a class of utility functions rather than a unique function for each attribute.

Each class of utility functions will be made up of all the utility functions $u_j(\cdot)$, comprised between $u_j^L(\cdot)$ and $u_j^U(\cdot)$, where x_j^L and x_j^U are the least and most preferred values by the DM for attribute X_j; x_j^1, x_j^2 and x_j^3 are three intermediate values for the attribute X_j; and $[u_j^{1L}, u_j^{1U}]$, $[u_j^{2L}, u_j^{2U}]$ and $[u_j^{3L}, u_j^{3U}]$ are the utility intervals that, for the DM, represent the values x_j^1, x_j^2 and x_j^3, respectively.

The implementation of the imprecise weight and utility assessments combining different methods is explained in detail for a variety of real applications in Ríos et al. (2000) and Mateos et al. (2001).

The functions $u_j^L(\cdot)$ and $u_j^U(\cdot)$ are obtained by fitting the cubic splines that go through points $(x_j^L,0)$, (x_j^1, u_j^{1L}), (x_j^2, u_j^{2L}), (x_j^3, u_j^{3L}) and $(x_j^U,1)$ and points $(x_j^L,0)$, (x_j^1, u_j^{1U}), (x_j^2, u_j^{2U}), (x_j^3, u_j^{3U}) and $(x_j^U,1)$, respectively. The procedure is as follows. We define a cubic spline for each interval, and we get

$$\text{for each } x \in [x_j^L, x_j^1] \Rightarrow a_{1j} + b_{1j}\, x + c_{1j}x^2 + d_{1j}x^3$$
$$\text{for each } x \in [x_j^1, x_j^2] \Rightarrow a_{2j} + b_{2j}x + c_{2j}x^2 + d_{2j}x^3$$
$$\text{for each } x \in [x_j^2, x_j^3] \Rightarrow a_{3j} + b_{3j}x + c_{3j}x^2 + d_{3j}x^3$$
$$\text{for each } x \in [x_j^3, x_j^U] \Rightarrow a_{4j} + b_{4j}x + c_{4j}x^2 + d_{4j}x^3.$$

For each attribute X_j, we have 16 unknown quantities, a_{kj}, b_{kj}, c_{kj}, d_{kj} with k = 1,2,3,4. We, therefore, need the same number of equations to set out a system and then obtain the cubic splines by intervals. The equations/constraints for obtaining $u_j^L(\cdot)$ are:

1. On the value taken by the utility function at the extremes of the interval:
   a. If the utility function is increasing:
      $a_{1j} + b_{1j}\, x_j^L + c_{1j}(x_j^L)^2 + d_{1j}(x_j^L)^3 = 0$
      $a_{4j} + b_{4j}\, x_j^U + c_{4j}(x_j^U)^2 + d_{4j}(x_j^U)^3 = 1$
   b. If the utility function is decreasing:
      $a_{1j} + b_{1j}\, x_j^U + c_{1j}(x_j^U)^2 + d_{1j}(x_j^U)^3 = 1$
      $a_{4j} + b_{4j}\, x_j^L + c_{4j}(x_j^L)^2 + d_{4j}(x_j^L)^3 = 0$

2. On the values taken by the utility function at the intermediate points of the interval:
   $a_{1j} + b_{1j}\, x_j^1 + c_{1j}(x_j^1)^2 + d_{1j}(x_j^1)^3 = u_j^{1L}$
   $a_{2j} + b_{2j}\, x_j^2 + c_{2j}(x_j^2)^2 + d_{2j}(x_j^2)^3 = u_j^{2L}$
   $a_{3j} + b_{3j}\, x_j^3 + c_{3j}(x_j^3)^2 + d_{3j}(x_j^3)^3 = u_j^{3L}$

3. On the continuity of the utility function at the intermediate points of the interval:
   $a_{1j} + b_{1j}\, x_j^1 + c_{1j}(x_j^1)^2 + d_{1j}(x_j^1)^3 = a_{2j} + b_{2j}\, x_j^1 + c_{2j}(x_j^1)^2 + d_{2j}(x_j^1)^3$
   $a_{2j} + b_{2j}\, x_j^2 + c_{2j}(x_j^2)^2 + d_{2j}(x_j^2)^3 = a_{3j} + b_{3j}\, x_j^2 + c_{3j}(x_j^2)^2 + d_{3j}(x_j^2)^3$
   $a_{3j} + b_{3j}\, x_j^3 + c_{3j}(x_j^3)^2 + d_{3j}(x_j^3)^3 = a_{4j} + b_{4j}\, x_j^3 + c_{4j}(x_j^3)^2 + d_{4j}(x_j^3)^3$

4. On the continuity of the first derivative of the utility function at the intermediate points of the interval:
   $b_{1j} + 2c_{1j}x_j^1 + 3d_{1j}(x_j^1)^2 = b_{2j} + 2c_{2j}x_j^1 + 3d_{2j}(x_j^1)^2$
   $b_{2j} + 2c_{2j}x_j^2 + 3d_{2j}(x_j^2)^2 = b_{3j} + 2c_{3j}x_j^2 + 3d_{3j}(x_j^2)^2$
   $b_{3j} + 2c_{3j}x_j^3 + 3d_{3j}(x_j^3)^2 = b_{4j} + 2c_{4j}x_j^3 + 3d_{4j}(x_j^3)^2$

5. On the continuity of the second derivative of the utility function at the intermediate points of the interval:
   $2c_{1j} + 6d_{1j}x_j^1 = 2c_{2j} + 6d_{2j}x_j^1$
   $2c_{2j} + 6d_{2j}x_j^2 = 2c_{3j} + 6d_{3j}x_j^2$
   $2c_{3j} + 6d_{3j}x_j^3 = 2c_{4j} + 6d_{4j}x_j^3$

6. On the first derivative of the utility function at the extremes of the interval for the range of each attribute:
   a. If the utility function is increasing:
      $b_{1j} + 2c_{1j}\, x_j^L + 3d_{1j}(x_j^L)^2 = 0$
      $b_{4j} + 2c_{4j}\, x_j^U + 3d_{4j}(x_j^U)^2 = 0$
   b. If the utility function is decreasing:
      $b_{1j} + 2c_{1j}\, x_j^U + 3d_{1j}(x_j^U)^2 = 0$
      $b_{4j} + 2c_{4j}\, x_j^L + 3d_{4j}(x_j^L)^2 = 0.$

The $u_j^U(\cdot)$ are obtained by solving the same equations after substituting u_j^{1L}, u_j^{2L} and u_j^{3L} by u_j^{1U}, u_j^{2U} and u_j^{3U}, respectively, in Eq. 2. Once the extreme utility functions $u_j^L(\cdot)$ and $u_j^U(\cdot)$ have been determined, we know that the condition $u_j^L(\cdot) \leq u_j(\cdot) \leq u_j^U(\cdot)$ must hold for the true, albeit unknown, utility function $u_j(\cdot)$ for attribute X_j .

We know that given two alternatives A_k and A_l, alternative A_k dominates A_l if $D_{kl} \geq 0$, D_{kl} being the optimum value of the optimization problem (1).

The objective function of the optimization problem is nonlinear, because the variables w_j multiply to the variables a_{kj}, b_{kj}, c_{kj}, d_{kj}, with k = 1,2,3,4, which correspond to the utility function $u_j(.)$. Even so, all the constraints are linear, although, due to the notation used, they, and specially the last two blocks, do not appear to be at first glance. This is because the constraints

$$u_j^L(x_{kj}) \leq u_j(x_{kj}) \leq u_j^U(x_{kj})\ , j=1,...,n$$

$$u_j^L(x_{lj}) \leq u_j(x_{lj}) \leq u_j^U(x_{lj})\ , j=1,...,n \qquad (2)$$

represent equations/constraints 1a or 1b - 6a or 6b if constraints 2 are substituted by

$$\begin{gathered}
u_j^{1L} \leq a_{1j} + b_{1j}\, x_j^1 + c_{1j}(x_j^1)^2 + d_{1j}(x_j^1)^3 \\
u_j^{2L} \leq a_{2j} + b_{2j}\, x_j^2 + c_{2j}(x_j^2)^2 + d_{2j}(x_j^2)^3 \\
u_j^{3L} \leq a_{3j} + b_{3j}\, x_j^3 + c_{3j}(x_j^3)^2 + d_{3j}(x_j^3)^3 \\
a_{1j} + b_{1j}\, x_j^1 + c_{1j}(x_j^1)^2 + d_{1j}(x_j^1)^3 \leq u_j^{1U} \\
a_{2j} + b_{2j}\, x_j^2 + c_{2j}(x_j^2)^2 + d_{2j}(x_j^2)^3 \leq u_j^{2U} \\
a_{3j} + b_{3j}\, x_j^3 + c_{3j}(x_j^3)^2 + d_{3j}(x_j^3)^3 \leq u_j^{3U},
\end{gathered}$$

i.e. constraints (2) represent 19 linear constraints.

Examining the objective function, we find that it can be rewritten as

$$\sum_j w_j\, [u_j(x_{kj}) - u_j(x_{lj})],$$

where $u_j(x_{kj}) - u_j(x_{lj})$ does not depend on the weights w_j. Moreover, if we carefully observe the constraints, we discover that variables w_j are independent of the other variables. So, taking into account that the weights w_j are nonnegative, solving (1) is equivalent to solving the optimization problem

$$\begin{aligned}
D_{kl} \quad &= \min \textstyle\sum_j w_j\, z_{klj}^* \\
\text{s.t.} \quad &w_j^L \leq w_j \leq w_j^U, j=1,...,n,
\end{aligned}$$

where z_{klj}^* are the optimal values of the optimization problem

$$\begin{aligned}
z_{klj}^* \quad &= \min\ u_j(x_{kj}) - u_j(x_{lj}) \\
\text{s.t.} \quad &x_{kj}^L \leq x_{kj} \leq x_{kj}^U, j=1,...,n
\end{aligned}$$

$$x_{lj}^L \leq x_{lj} \leq x_{lj}^U, j=1,...,n \qquad (3)$$
$$u_j^L(x_{kj}) \leq u_j(x_{kj}) \leq u_j^U(x_{kj}), j=1,...,n$$
$$u_j^L(x_{lj}) \leq u_j(x_{lj}) \leq u_j^U(x_{lj}), j=1,...,n.$$

The solution to (3) can be determined depending on what the characteristics of the utility function for attribute X_j are (Mateos et al., 2007):

- If the utility function is increasing monotone, we demonstrated that $z_{klj}^* = u_j^L(x_{kj}^L) - u_j^U(x_{lj}^U)$
- If the utility function is decreasing monotone, we demonstrated that $z_{klj}^* = u_j^L(x_{kj}^U) - u_j^U(x_{lj}^L)$.

3. Example

This example is concerned with the selection of a technology for the disposition of surplus weapons-grade plutonium by the *Department of Energy* in the USA. A major objective of this decision was to further US efforts to prevent the proliferation of nuclear weapons, but other concerns that were addressed include economic, technical, institutional, schedule, environmental, and health and safety issues. This problem has been studied in depth; see Dyer et al. (1996, 1997, 1998) and Jiménez et al. (2006).

An objective hierarchy was built with three main objectives at the highest level: *non-proliferation*; *environmental, safety and health*; and *operational effectiveness*. Each of them was split into other sub-objectives and so on. Finally, 37 lowest level objectives were identified. A complete description of the objectives and attributes is given in Dyer et al. (1996, 1997).

A set of thirteen technologies, denoted by S^k, k=1,…,13, were proposed for analysis (see Table 1). *Reactor alternatives* would use surplus plutonium to fabricate mixed oxide fuel for nuclear reactors that generate electric power; while *immobilitation alternatives* would immobilitate surplus plutonium by mixing it with different substances, like non-radioactive and radioactive glass, or ceramic, and pouring it in recipients, like cans, canisters; and *direct disposal alternatives* would place it in a borehole.

Table 1. Technologies to be evaluated

Reactor alternatives
S^1: Existing light water reactors, existing facilities
S^2: Existing light water reactors, greenfield facilities
S^3: Partially completed light water reactors
S^4: Evolutionary light water reactors
S^5: CANDU reactors
Direct disposal alternatives
S^6: Deep borehole (immobilitation)
S^7: Deep borehole (direct emplacement)
Inmobilization alternatives
S^8: Vitrification greenfield

S^9: Vitrification can-in-canister
S^{10}: Vitrification adjunt melter
S^{11}: Ceramic greenfield
S^{12}: Ceramic can-in-canister
S^{13}: Electrometallurgical treatment

The performances of the thirteen technologies in terms of the 37 attributes were identified and uncertainty was introduced by means of an attribute deviation of 5% leading to intervals, aimed at analyzing the robustness of the final ranking, Jiménez et al. (2006).

On the other hand, the procedures for assessing component utilities and eliciting weights did allow for imprecision concerning the decision-makers' responses. This led to classes of utility functions (Fig 1) and weight intervals, respectively. For a detailed description of the preference assessment, see Jiménez et al. (2006).

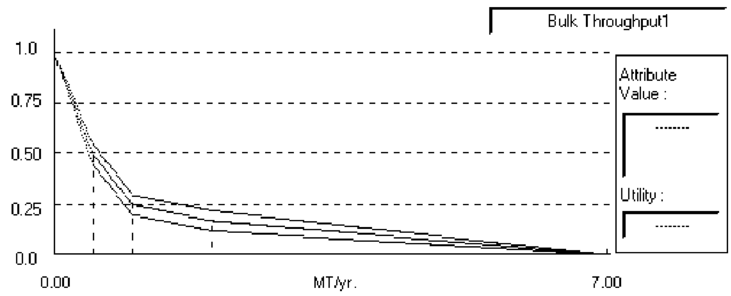

Figure 1. Class of utility functions.

If the techniques described in Section 2 are used to solve this problem, the following dominance matrix is output:

$$
\begin{pmatrix}
- & -0.039 & 0.015 & 0.371 & -0.096 & -0.086 & -0.114 & 0.020 & -0.193 & -0.063 & -0.001 & -0.192 & -0.041 \\
-0.267 & - & -0.082 & 0.280 & -0.219 & -0.209 & -0.237 & -0.088 & -0.316 & -0.173 & -0.110 & -0.315 & -0.142 \\
-0.375 & -0.248 & - & 0.212 & -0.316 & -0.305 & -0.332 & -0.184 & -0.412 & -0.269 & -0.206 & -0.411 & -0.239 \\
-0.736 & -0.611 & -0.523 & - & -0.675 & -0.656 & -0.694 & -0.525 & -0.773 & -0.625 & -0.552 & -0.772 & -0.597 \\
-0.221 & -0.109 & -0.041 & 0.326 & - & -0.140 & -0.178 & -0.028 & -0.258 & -0.115 & -0.048 & -0.257 & -0.094 \\
-0.253 & -0.142 & -0.067 & 0.311 & -0.191 & - & -0.192 & -0.032 & -0.275 & -0.133 & -0.059 & -0.275 & -0.112 \\
-0.229 & -0.117 & -0.045 & 0.319 & -0.167 & -0.130 & - & -0.024 & -0.252 & -0.109 & -0.045 & -0.252 & -0.089 \\
-0.387 & -0.263 & -0.180 & 0.209 & -0.325 & -0.292 & -0.333 & - & -0.411 & -0.261 & -0.187 & -0.410 & -0.232 \\
-0.041 & 0.051 & 0.118 & 0.468 & 0.007 & 0.025 & -0.002 & 0.124 & - & 0.049 & 0.105 & -0.053 & 0.071 \\
-0.271 & -0.146 & -0.076 & 0.296 & -0.209 & -0.176 & -0.216 & -0.052 & -0.294 & - & -0.071 & -0.294 & -0.115 \\
-0.358 & -0.233 & -0.153 & 0.231 & -0.296 & -0.263 & -0.303 & -0.131 & -0.381 & -0.231 & - & -0.381 & -0.203 \\
-0.039 & 0.053 & 0.120 & 0.470 & 0.009 & 0.026 & -0.001 & 0.125 & -0.053 & 0.050 & 0.105 & - & 0.070 \\
-0.298 & -0.173 & -0.100 & 0.275 & -0.236 & -0.203 & -0.081 & -0.081 & -0.322 & -0.172 & -0.102 & -0.322 & -
\end{pmatrix}
$$

Then, dominance intensities DI_{kl} depending on the dominance among technologies S^k and S^l are computed, and used to derive the dominance intensity measure (DIM_k) (see Table 2).

Table 2. Dominance intensities DI_{kl} and the resulting DIM_k

	S^1	S^2	S^3	S^4	S^5	S^6	S^7	S^8
S^1	-	0.132	0.221	0.563	0.072	0.097	0.067	0.229
S^2	-0.132	-	0.096	0.456	-0.064	-0.039	-0.069	0.101
S^3	-0.221	-0.096	-	0.378	-0.159	-0.137	-0.166	-0.002
S^4	-0.563	-0.456	-0.378	-	-0.511	-0.285	-0.518	-0.378
S^5	-0.072	0.064	0.159	0.511	-	0.029	-0.006	0.171
S^6	-0.097	0.039	0.137	0.285	-0.029	-	-0.036	0.150
S^7	-0.066	0.069	0.166	0.518	0.006	0.036	-	0.178
S^8	-0.229	-0.100	0.002	0.378	-0.171	-0.150	-0.178	-
S^9	0.088	0.199	0.278	0.627	0.151	0.167	0.144	0.280
S^{10}	-0.120	0.015	0.112	0.470	-0.054	-0.025	-0.062	0.121
S^{11}	-0.206	-0.071	0.031	0.402	-0.143	-0.118	-0.149	0.032
S^{12}	0.088	0.199	0.278	0.627	0.151	0.167	0.145	0.280
S^{13}	-0.149	-0.018	0.080	0.446	-0.082	-0.053	-0.089	0.087

	S^9	S^{10}	S^{11}	S^{12}	S^{13}	DIM_k	*Ranking*
S^1	-0.088	0.120	0.206	-0.088	0.149	**1.680**	**3**
S^2	-0.199	-0.015	0.071	-0.199	0.018	**0.025**	**8**
S^3	-0.278	-0.112	-0.031	-0.278	-0.080	**-1.182**	**11**
S^4	-0.627	-0.470	-0.402	-0.627	-0.446	**-5.661**	**13**
S^5	-0.151	0.054	0.143	-0.151	0.082	**0.834**	**5**
S^6	-0.167	0.025	0.118	-0.167	0.053	**0.310**	**6**
S^7	-0.144	0.062	0.149	-0.145	0.089	**0.918**	**4**
S^8	-0.280	-0.121	-0.032	0.80	-0.087	**-1.250**	**12**
S^9	-	0.186	0.256	-0.001	0.209	**2.583**	**2**
S^{10}	-0.186	-	0.160	-0.186	0.033	**0.279**	**7**
S^{11}	-0.256	-0.160	-	-0.255	-0.058	**-0.952**	**10**
S^{12}	0.001	0.186	0.255	-	0.209	**2.586**	**1**
S^{13}	0.209	-0.033	0.058	-0.209	-	**-0.167**	**9**

Finally, all we have to do is to rank technologies according to the *DIM* values. Thus the best strategy is S^{12}: *Ceramic can-in-canister* (2.586), followed by S^9: *Vitrification can-in-canister* (2.583) and S^1: *Existing light water reactors, existing facilities* (1.680). The worst strategies are S^4: *Evolutionary light water reactors* (-5.661), S^8: *Vitrification greenfield* (-1.250) and S^3: *Partially completed light water reactors* (-1.182).

These results prove that the ranking output in Jiménez et al. (2006), on the basis of average overall utilities (Figure 2), is robust. Moreover, if we applied the dominance measuring methods proposed in Ahn and Park (2008), where a *net dominance value* (***OUT I***) is derived from dominating and dominated measures or just the dominating measure is used (***OUT II***); would output the same ten best ranked technologies in both, having the same ranking in ***OUT II*** (see Table 3).

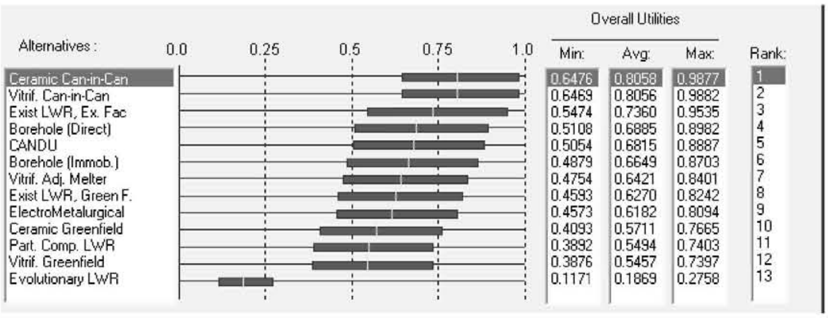

Figure 2. Ranking of technologies on the basis of their average overall utilities.

Table 3. Comparison of the ranking derived from Ahn and Park's and the proposed dominance measure

	DIM_k	*Ranking*	*OUT I*	*Ranking*	*OUT II*	*Ranking*
S^1	1.680	3	-0.417	3	3.058	3
S^2	0.025	8	-1.877	8	0.099	8
S^3	-1.182	11	-3.085	12	-2.071	11
S^4	-5.661	13	-7.738	13	-11.506	13
S^5	0.834	5	-1.163	5	1.553	5
S^6	0.310	6	-1.419	6	0.986	6
S^7	0.918	4	-1.140	4	1.706	4
S^8	-1.250	12	-3.071	11	-2.197	12
S^9	2.583	2	0.921	2	4.859	2
S^{10}	0.279	7	-1.623	7	0.429	7
S^{11}	-0.952	10	-2.702	10	-1.531	10
S^{12}	2.586	1	0.935	1	4.868	1
S^{13}	-0.10	9	-1.979	9	-0.254	9

Conclusions

Dominance measuring methods are becoming widely used in a decision-making context with incomplete information and have been proved to outperform other approaches, like most surrogate weighting methods or the modification of classical decision rules to encompass an imprecise decision context.

In this paper we have described a dominance measuring method that transforms paired dominance values into dominance intensities, then used to derive a dominance intensity measure. This is used as a measure of the strength of preference in the sense that a greater dominance intensity measure is better.

The method has been used to analyze the robustness of the ranking output in a complex decision-making problem centered on the selection of a technology for the disposition of surplus weapons-grade plutonium by the *Department of Energy* in the USA. Moreover, it is shown that the results completely match up with those output using other dominance measuring methods.

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