Ranking Alternatives on the Basis of Intensity of Dominance and Fuzzy Logic within MAUT

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Abstract. We introduce a dominance measuring method to derive a ranking of alternatives to deal with incomplete information in multi-criteria decision-making problems on the basis of multi-attribute utility theory (MAUT) and fuzzy sets theory. We consider the situation where the alternative performances are represented by intervals and there exists imprecision concerning the decision-makers' preferences, leading to classes of component utility functions and trapezoidal fuzzy weights. An additive multi-attribute utility model is used to evaluate the alternatives under consideration. The approach we propose is based on the dominance values between pairs of alternatives that can be computed by linear programming. These values are then transformed into dominance intensities from which a dominance intensity measure is derived. The performance of the proposed method is analyzed using MonteCarlo simulation techniques.

Keywords: Decision Analysis, Fuzzy Sets, Environmental Studies.

INTRODUCTION

The additive model is considered a valid approximation in most real decision-making problems (Raiffa, 1982; Stewart, 1996). The functional form of the additive model is

$$u(A_i) = \sum_{j=1}^{n} w_j u_j(x_{ij}),$$

where x_{ij} is the performance in the attribute X_j of alternative A_i; $u_j(x_{ij})$ is the utility associated with value x_{ij}, and w_j are the weights of each attribute, representing their relative importance in decision-making.

Most complex decision-making problems involve imprecise information. For example, it is impossible to predict the exact performance of each alternative under consideration, being derived from statistical methods. At the same time, it is often not easy to elicit precise weights, which are represented by intervals. DMs may find it difficult to compare criteria or not want to reveal their preferences in public. Furthermore, the decision may be taken within a group, where the imprecision of the preferences is the result of a negotiation process.

A lot of work on MAUT has dealt with incomplete information. Sarabando and Dias (2009) give a brief overview of approaches proposed by different authors within the MAUT and MAVT (multi-attribute value theory) framework to deal with incomplete information.

Eum *et al* (2001) provided linear programming characterizations of dominance and potential optimality for decision alternatives when information about performances and/or weights is incomplete, extended the approach to hierarchical structures (Lee *et al*, 2002), and developed the concepts of *weak and strong potential optimality* (Park, 2004). More recently, Mateos and Jiménez (2009) and Mateos *et al* (2009) considered the more general case where imprecision, described by means of fixed bounds, appears in alternative performances, as well as in weights and utilities.

At the same time, a number of studies have been conducted concerning imprecision using *fuzzy sets theory*. These studies feed off the advances of research into arithmetic and logical operators of fuzzy numbers, like Tran and Duckstein's study proposing the comparison of fuzzy numbers by a fuzzy measure of distance (Tran and Duckstein, 2002).

Following this research line, we consider a decision-making problem with m alternatives, $A_i, i = 1, \ldots, m$, and n attributes, $X_j, j = 1, \ldots, n$, where incomplete information about input parameters has been incorporated into the decision-making process:

- Alternative performances are described under uncertainty $[x_{ij}^L, x_{ij}^U]$, $i = 1, \ldots, m$, $j = 1, \ldots, n$, where x_{ij}^L and x_{ij}^U are the lower and the upper performances of the attribute X_j for the alternative A_i, respectively.
- Component utilities are described by functions $u(\bullet)$, which belong to classes of functions of utility $[u_j^L(\bullet), u_j^U(\bullet)]$, $j = 1, \ldots, n$, where $u_j^L(\bullet)$ and $u_j^U(\bullet)$ are the lower and the upper utility functions of the attribute X_j.
- Imprecise weights are represented by trapezoidal fuzzy numbers $\widetilde{w}_j$, $j = 1, \ldots, n$.

One possibility described in the literature for dealing with imprecision attempts to eliminate inferior alternatives based on the concept of *pairwise dominance*. Given two alternatives A_k and A_l, the alternative A_k dominates A_l if $D_{kl} \geq 0$, D_{kl} being the optimum value of the optimization problem:

$$\begin{aligned} D_{kl} = \min \{u(A_k) - u(A_l)\} \quad &= \sum_{j=1}^{n} \widetilde{w}_j u_j(x_{kj}) - \sum_{j=1}^{n} \widetilde{w}_j u_j(x_{lj}) \\ s.t. \quad & x_{kj}^L \leq x_{kj} \leq x_{kj}^U, \ j = 1, \ldots, n \\ & x_{lj}^L \leq x_{lj} \leq x_{lj}^U, \ j = 1, \ldots, n \\ & u_j^L(x_{kj}) \leq u_j(x_{kj}) \leq u_j^U(x_{kj}), j = 1, \ldots, n \\ & u_j^L(x_{lj}) \leq u_j(x_{lj}) \leq u_j^U(x_{lj}), j = 1, \ldots, n. \end{aligned}$$

The objective function in the above optimization problem can also be represented by $D_{kl} = \min \sum_{j=1}^{n} \widetilde{w}_j \left[u_j(x_{kj}) - u_j(x_{lj})\right]$. Therefore, its resolution is equivalent to solving (Mateos *et al*, 2007)

$$D_{kl} = \sum_{j=1}^{n} \widetilde{w}_j z_{klj}^*, \tag{1}$$

where z^*_{klj} is the optimum value of the following optimization problem

$$
\begin{array}{ll}
\min z_{klj} & = u_j(x_{kj}) - u_j(x_{lj}) \\
 & x^L_{kj} \leq x_{kj} \leq x^U_{kj},\ j = 1, \ldots, n \\
s.t. & x^L_{lj} \leq x_{lj} \leq x^U_{lj},\ j = 1, \ldots, n \\
 & u^L_j(x_{kj}) \leq u_j(x_{kj}) \leq u^U_j(x_{kj}), j = 1, \ldots, n \\
 & u^L_j(x_{lj}) \leq u_j(x_{lj}) \leq u^U_j(x_{lj}), j = 1, \ldots, n.
\end{array}
\tag{2}
$$

The optimal solution of problem (2) can be determined in a very simple way for certain types of utility functions (Mateos *et al*, 2007). If the utility function is monotonically increasing or decreasing, then $z^*_{klj} = u^L_j(x^L_{kj}) - u^U_j(x^U_{lj})$ or $z^*_{klj} = u^U_j(x^U_{kj}) - u^L_j(x^L_{lj})$, respectively.

A recent approach is to use information about each alternative's intensity of dominance, known as *dominance measuring methods*. Ahn and Park (2008) compute both dominating and dominated measures, from which they derive a *net dominance*. This is used as a measure of the strength of preference in the sense that a greater net value is better.

In the next section we introduce a dominance measuring method accounting for fuzzy weights. Section 3 analyzes the performance of the proposed method using MonteCarlo simulation techniques. Finally, some conclusions are provided in Section 4.

MEASUREMENT METHOD FOR FUZZY DOMINANCE

The proposed dominance measuring method adapts the proposal by Mateos and Jiménez (2009) and Mateos *et al* (2009) to account for fuzzy weights making use of work by Tran and Duckstein (2002) on distances between fuzzy numbers based on the generalization of the left and right fuzzy numbers ($GLRFN$) (Dubois and Prade, 1980; Bárdossy and Duckstein, 1995).

A fuzzy set $\widetilde{a} = (a_1, a_2, a_3, a_4)$ is called *GLRFN* when its membership function is defined as

$$\mu_{\widetilde{a}}(x) = \begin{cases} L\left(\dfrac{a_2 - x}{a_2 - a_1}\right) & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ R\left(\dfrac{x - a_3}{a_4 - a_3}\right) & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise,} \end{cases}$$

where L and R are strictly decreasing functions defined in $[0, 1]$ and satisfying the conditions:

$$L(x) = R(x) = 1 \text{ if } x \leq 0$$

$$L(x) = R(x) = 0 \text{ if } x \geq 0.$$

For $a_2 = a_3$, you have Dubois and Prade classic definition of right and left fuzzy numbers (Dubois and Prade, 1980). The trapezoidal fuzzy numbers are a special case of *GLRFN* with $L(x) = R(x) = 1 - x$. A *GLRFN* is denoted as $\tilde{a} = (a_1, a_2, a_3, a_4)_{L\tilde{a}-R\tilde{a}}$ and a α-cut of $\tilde{a}$ is defined as

$$\tilde{a}(\alpha) = (\tilde{a}_L(\alpha), \tilde{a}_R(\alpha)) = (a_2 - (a_2 - a_1)a_3 L_{\tilde{a}}^{-1}(\alpha), a_3 - (a_4 - a_3)a_3 R_{\tilde{a}}^{-1}(\alpha),).$$

(Tran and Duckstein, 2002) define the distance between two *GLFRN* fuzzy numbers $\tilde{a}$ and $\widetilde{b}$ as

$$D^2(\tilde{a}, \widetilde{b}, f) = \left\{ \int_0^1 \left\{ \begin{array}{c} \left[\frac{\tilde{a}_L(\alpha)+\tilde{a}_R(\alpha)}{2} - \frac{\widetilde{b}_L(\alpha)+\widetilde{b}_R(\alpha)}{2}\right]^2 + \\ +\frac{1}{3}\left[\left(\frac{\tilde{a}_L(\alpha)+\tilde{a}_R(\alpha)}{2}\right)^2 + \left(\frac{\widetilde{b}_L(\alpha)+\widetilde{b}_R(\alpha)}{2}\right)^2\right] \end{array} \right\} \times f(\alpha)(d\alpha) \right\} / \int f(\alpha)(d\alpha).$$

The function $f(\alpha)$, which serves as a function of weights, is positive continuous in $[0, 1]$, the distance being computed as a weighted sum of distances between two intervals along all of the α-cuts from 0 to 1. The presence of function f permits the DM to participate in a flexible way. For example, when the DM is risk-neutral, $f(\alpha) = \alpha$ seems to be reasonable. A risk-averse DM might want to put more weight on information at a higher α level by using other functions, such as $f(\alpha) = \alpha^2$ or a higher power of α. A constant ($f(\alpha) = 1$), or even a decreasing function f, can be utilized for a risk-prone DM.

For the particular case of the distance of a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ to a constant (specifically 0), we have:

1. If $f(\alpha) = \alpha$ then $d(\widetilde{a}, 0)^2 = \left(\frac{a_2 + a_3}{2}\right)^2 + \frac{1}{3}\left(\frac{a_2 + a_3}{2}\right)[(a_4 - a_3) - (a_2 - a_1)] + \frac{2}{3}\left(\frac{a_3 - a_2}{2}\right)^2 + \frac{1}{9}\left(\frac{a_3 - a_2}{2}\right)[(a_4 - a_3) + (a_2 - a_1)] + \frac{1}{18}\left[(a_4 - a_3)^2 + (a_2 - a_1)^2\right] - \frac{1}{18}\left[(a_2 - a_1)(a_4 - a_3)\right].$

2. If $f(\alpha) = 1$ then $d(\widetilde{a}, 0)^2 = \left(\frac{a_2 + a_3}{2}\right)^2 + \frac{1}{2}\left(\frac{a_2 + a_3}{2}\right)[(a_4 - a_3) - (a_2 - a_1)] + \frac{1}{3}\left(\frac{a_3 - a_2}{2}\right)^2 + \frac{1}{6}\left(\frac{a_3 - a_2}{2}\right)[(a_4 - a_3) + (a_2 - a_1)] + \frac{1}{9}\left[(a_4 - a_3)^2 + (a_2 - a_1)^2\right] - \frac{1}{9}\left[(a_2 - a_1)(a_4 - a_3)\right].$

As trapezoidal fuzzy numbers are used to represent weights, the objective function in (1) can be now represented by

$$D_{kl} = \widetilde{d}_{kl} = \sum_{j=1}^{n} \widetilde{w}_j z^*_{klj} = \sum_{j=1}^{n} (w_{j1}, w_{j2}, w_{j3}, w_{j4}) z^*_{klj} = (d_{kl1}, d_{kl2}, d_{kl3}, d_{kl4}).$$

The first step in the proposed method, then, consists of computing the above trapezoidal fuzzy numbers. Consequently, the strength of dominance of alternative A_k can be defined as

$$\widetilde{d}_k = (d_{k1}, d_{k2}, d_{k3}, d_{k4}) = \sum_{\substack{l=1 \\ l \neq k}} D_{kl} = \left(\sum_{\substack{l=1 \\ l \neq k}}^{n} d_{kl1}, \sum_{\substack{l=1 \\ l \neq k}}^{n} d_{kl2}, \sum_{\substack{l=1 \\ l \neq k}}^{n} d_{kl3}, \sum_{\substack{l=1 \\ l \neq k}}^{n} d_{kl4} \right).$$

Next, a *dominance intensity*, DI_k, for each alternative A_k is computed as the proportion of the positive part of the fuzzy number $\widetilde{d}_k$ by the distance of the fuzzy number to zero. Specifically, the dominance intensity for alternative A_k is computed according to the location of $\widetilde{d}_k$ as follows:

1. If $\widetilde{d}_k$ is completely located at the left of zero, then DI_k is minus the distance of $\widetilde{d}_k$ to zero, because there is no positive part in $\widetilde{d}_k$.

2. If $\widetilde{d}_k$ is completely located at the right of zero, then DI_k is the distance of $\widetilde{d}_k$ to zero, because there is no negative part existing in $\widetilde{d}_k$.

3. If $\widetilde{d}_k$ includes the zero in its base, then the fuzzy number will have a part on the right of zero that we denote $\widetilde{d}_k^R$ and another part on the left of zero that we denote $\widetilde{d}_k^L$. DI_k is the proportion that represents $\widetilde{d}_k^R$ with respect to $\widetilde{d}_k$ by the distance of $\widetilde{d}_k$ to zero less the proportion that represents $\widetilde{d}_k^L$ with respect to $\widetilde{d}_k$ by the distance of $\widetilde{d}_k$ to zero.

Next, we analyze each one of these cases in more detail.

- If $d_{k4} < 0$, see Figure 1a), then the dominance intensity of alternative A_k is defined as $DI_k = -d(\widetilde{d}_k, 0, f)$.

- Figure 1 -

- If $d_{k1} > 0$, see Figure 1b), then the dominance intensity of alternative A_k is defined as $DI_k = d(\widetilde{d}_k, 0, f)$.

- If $d_{k1} < 0$ and $d_{k2} > 0$, see Figure 1c), the corresponding trapezoidal fuzzy number is divided by the vertical axis (at zero) into two parts. The left part $\widetilde{d}_k^L$ represents the proportion

$$\frac{\dfrac{-d_{k1}\dfrac{-d_{k1}}{d_{k2}-d_{k1}}}{2}}{\dfrac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{(d_{k1})^2}{(d_{k4}+d_{k3}-d_{k2}-d_{k1})\,(d_{k2}-d_{k1})},$$

whereas the right part $\widetilde{d}_k^R$ represents the proportion

$$\frac{\dfrac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}-\dfrac{(-d_{k1})(-d_{k1})}{2(d_{k2}-d_{k1})}}{\dfrac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{d_{k2}(-d_{k2}+d_{k3}+d_{k4})-d_{k1}(d_{k3}+d_{k4})}{(d_{k2}-d_{k1})\,(d_{k4}+d_{k3}-d_{k2}-d_{k1})}.$$

The dominance intensity of alternative A_k is defined as

$$DI_k = \frac{d_{k2}(-d_{k2}+d_{k3}+d_{k4})-d_{k1}(d_{k3}+d_{k4})}{(d_{k2}-d_{k1})(d_{k4}+d_{k3}-d_{k2}-d_{k1})}d(\widetilde{d}_k,0,f) - \frac{(d_{k1})^2}{(d_{k4}+d_{k3}-d_{k2}-d_{k1})(d_{k2}-d_{k1})}d(\widetilde{d}_k,0,f).$$

- If $d_{k3} < 0$ and $d_{k4} > 0$, Figure 1d), the corresponding trapezoidal fuzzy number is again divided by the vertical axis into two parts, $\widetilde{d}_k^L$ and $\widetilde{d}_k^R$, represented by the proportions

$$\frac{\frac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}-\frac{d_{k4}\frac{d_{k4}}{d_{k4}-d_{k3}}}{2}}{\frac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{d_{k4}(d_{k4}-d_{k2}-d_{k1})-d_{k3}(d_{k3}-d_{k2}-d_{k1})}{(d_{k4}+d_{k3}-d_{k2}-d_{k1})(d_{k4}-d_{k3})},$$

and

$$\frac{\frac{d_{k4}\frac{d_{k4}}{d_{k4}-d_{k3}}}{2}}{\frac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{(d_{k4})^2}{(d_{k4}-d_{k3})(d_{k4}+d_{k3}-d_{k2}-d_{k1})},$$

respectively, and the dominance intensity of alternative A_k is

$$DI_k = \frac{(d_{k4})^2}{(d_{k4}-d_{k3})(d_{k4}+d_{k3}-d_{k2}-d_{k1})}d(\widetilde{d}_k,0,f) - \frac{d_{k4}(d_{k4}-d_{k2}-d_{k1})-d_{k3}(d_{k3}-d_{k2}-d_{k1})}{(d_{k4}+d_{k3}-d_{k2}-d_{k1})(d_{k4}-d_{k3})}d(\widetilde{d}_k,0,f).$$

- If $d_{k2} < 0$ and $d_{k3} > 0$, see Figure 1e), $\widetilde{d}_k^L$ and $\widetilde{d}_k^R$ are

$$\frac{\frac{d_{k4}-d_{k3}}{2}+d_{k3}}{\frac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{d_{k4}+d_{k3}}{d_{k4}+d_{k3}-d_{k2}-d_{k1}},$$

and

$$\frac{\frac{d_{k2}-d_{k1}}{2}-d_{k2}}{\frac{d_{k4}+d_{k3}-d_{k2}-d_{k1}}{2}} = \frac{-d_{k2}-d_{k1}}{d_{k4}+d_{k3}-d_{k2}-d_{k1}},$$

respectively, and the dominance intensity of alternative A_k is

$$DI_k = \frac{-d_{k2}-d_{k1}}{d_{k4}+d_{k3}-d_{k2}-d_{k1}}d(\widetilde{d}_k,0,f) - \frac{d_{k4}+d_{k3}}{d_{k4}+d_{k3}-d_{k2}-d_{k1}}d(\widetilde{d}_k,0,f).$$

Once the dominance intensity, DI_k, has been computed for each alternative A_k, the alternatives are ranked, where the best (rank 1) is the alternative with greatest DI_k and the worst is the alternative with the least DI_k.

PERFORMANCE ANALYSIS BASED ON MONTECARLO SIMULATION TECHNIQUES

We carried out a simulation study of the above method to analyze its performance. Four different levels of alternatives ($m = 3, 5, 7, 10$) and five different levels of attributes ($n = 3, 5, 7, 10, 15$) were considered in order to validate the results. Also, 5000 trials were run for each of the 20 design elements.

We used two measures of efficacy, hit ratio and rank-order correlation (Ahn and Park, 2008; Mateos *et al*, 2009). The *hit ratio* is the proportion of all cases in which the method selects the same best alternative as in the TRUE ranking. *Rank-order correlation* represents how similar the overall structures ranking alternatives are in the TRUE ranking and in the ranking derived from the method. It is calculated using Kendall's τ (Winkler and Hays, 1985): $\tau = 1 - 2 \times$ (number of pairwise preference violations)/(total number of pair preferences).

First, component utilities for each alternative in each attribute from a uniform distribution in $(0, 1)$ are randomly generated, leading to an $m \times n$ matrix. The columns in this matrix are normalized to make the smallest value 0 and the largest 1, and dominated alternatives are removed. Next, attribute weights representing their relative importance are generated. Note that these weights are the TRUE weights and the derived ranking of alternatives will be denoted as the TRUE ranking. Those weights are then transformed into triangular fuzzy numbers applying a 5% deviation from the original weight. Finally, the ranking of alternatives is computed and compared with the TRUE ranking. Table 1 exhibits the measures of efficacy for each of the 20 design elements for the cases of a risk-prone and a risk-neutral DM.

- Table 1 -

We can conclude that the hit ratio and the correlation coefficient are very similar for the cases of a risk-prone and a risk-neutral DM; the hit ratio is greater than 78% for all the design elements, whereas Kendall's τ is greater than 86%. Both are increasing against the number of attributes. Whereas the hit ratio decreases when the number of alternatives is increased, Kendall's τ increases.

CONCLUSIONS

Dominance measuring methods are becoming widely used in a decision-making context with incomplete information and have been proved to outperform other approaches, like most surrogate weighting methods or the modification of classical decision rules to encompass an imprecise decision context.

In this paper a new dominance measuring method has been proposed, in which weights representing the relative importance of decision-making criteria are described by fuzzy numbers. The method is based on the distance of a fuzzy number to a constant, where the generalization of the left and right fuzzy numbers, *GLRFN*, is used.

The application of Monte Carlo simulation techniques has demonstrated that the proposed method performs well in terms of two measures of efficacy: hit ratio and rank-order correlation.

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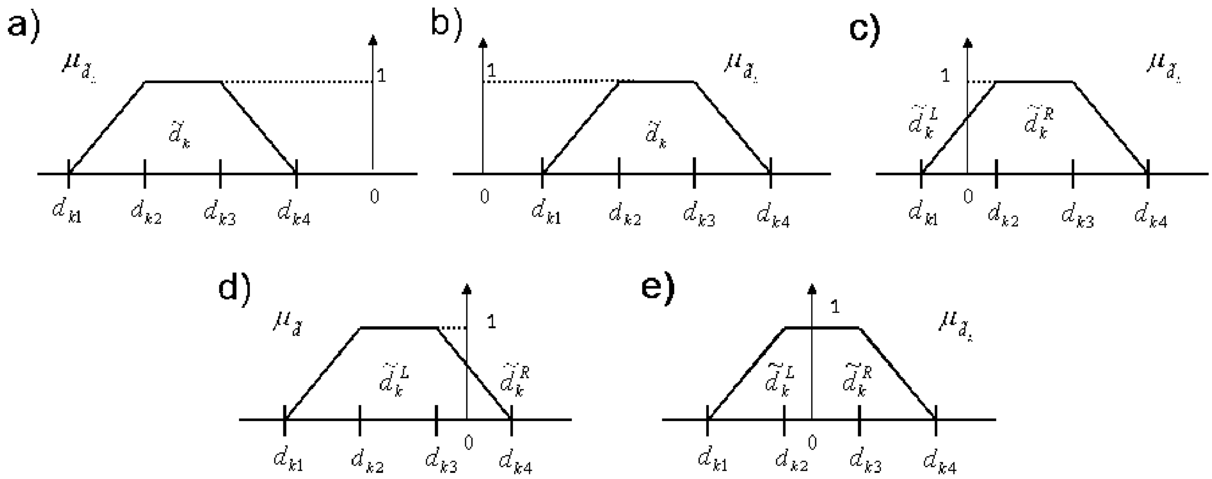

a)
1
0
d_{k1} d_{k2} d_{k3} d_{k4}
b)
1
0
d_{k1} d_{k2} d_{k3} d_{k4}
c)
1
0
$\tilde{d}_k^L$ $\tilde{d}_k^R$
d_{k1} d_{k2} d_{k3} d_{k4}
d)
1
0
$\tilde{d}_k^L$ $\tilde{d}_k^R$
d_{k1} d_{k2} d_{k3} d_{k4}
e)
1
0
$\tilde{d}_k^L$ $\tilde{d}_k^R$
d_{k1} d_{k2} d_{k3} d_{k4}

Figure 1

Table 1

Alternatives	Attributes	Hit ratio		Kendall's τ	
		risk-prone	risk-neutral	risk-prone	risk-neutral
3	3	90.78	90.72	88.90	88.85
3	5	91.42	91.34	88.77	88.66
3	7	91.02	91	87.97	87.97
3	10	90.9	90.96	87.08	87.15
3	15	89.78	89.78	86.24	86.27
5	3	85.02	84.98	89.8	89.79
5	5	85.8	85.8	89.60	89.59
5	7	85.16	85.12	89.29	89.28
5	10	83.64	83.66	88.11	88.18
5	15	83.74	83.66	86.72	86.70
7	3	84.4	84.38	92.16	92.15
7	5	83.72	83.66	91.48	91.47
7	7	83.48	83.34	91.07	91.04
7	10	82.98	82.94	90.21	90.26
7	15	81.14	81.16	88.66	88.67
10	3	85.22	85.08	93.89	93.87
10	5	83.08	82.96	93.51	93.49
10	7	82.28	82.18	93.01	93.01
10	10	82.74	82.66	92.08	92.12
10	15	78.36	78.54	90.41	90.45

Captions of Figures and Tables

Figure 1: Computing dominance intensities

Table 1. Measures of efficacy