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Modelling of atmospheric boundary layer: Generation of shear profile in wind tunnel

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ABSTRACT

Roughness length, z_0 and friction velocity, u_* are the defining parameters of wind log profile that must be matched in wind tunnel simulation. To fully understand the role of these parameters, the basics and review from the primitive equations and its relation to the logarithmic profile obtained for wind tunnel conditions were discussed. The problem of roughness, although well known, still needs to be addressed more rigorously especially when determining values of z_0 and u_* from wind tunnel data and their relation to the roughness element geometry. A review of classic literature and new published material were carried out, focusing on the applicability to wind tunnel modelling.

KEYWORDS

log profile, roughness length, roughness height, friction velocity

1 INTRODUCTION

1.1 Logarithmic Profile

In describing the atmosphere, the set of seven equations configured by the Navier-Stokes ones plus continuity, energy, state equation and conservation of humidity is the most rigorous model known. The full set of equations, while giving the best solution, is practically impossible to solve due to non-linearity and large number of complex initial and boundary conditions that need to be considered. Starting from these equations, performing the ensemble averaging and applying the corresponding averaging rules to obtain the RANS equations, and confining the problem to wind tunnel conditions (channel flow hypothesis, see Schlichting and Gersten, 2000 and Wyngaard, 2010), therefore adopting a few assumptions; isentropic flow, dry air considered and incompressibility (density, ρ = constant); the problem is reduced to a much simpler approximation though still leaves much to be solved. The classic logarithmic profile equation is attained in the case of a flow on a smooth, flat surface,

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for the mean value of the longitudinal component of the wind speed, U (see expression (0.1)), in terms of the well known non-dimensional "+" variables, after using as characteristic length, $l_+ = \nu/u_*$, being ν the kinematic viscosity, and as characteristic velocity, the friction velocity, u_* (u_* is a reference velocity applied to the motion near the surface where the shear stress is not a function of the distance from the wall and defined as $u_* = \sqrt{\tau/\rho}$, where τ is the Reynolds stress and ρ is the density of the fluid).

$$U(z_+) = \frac{u_*}{\kappa}\ln(z_+) + Bu_* \tag{0.1}$$

z_+ is the non-dimensional distance from the wall, κ is the von Karman constant (taken as 0.4) and B is an integration constant. In the considered case of smooth wall, B is usually taken to be 5.1 though experimental values give a range 5.0 – 5.5 (Raupach, 1991). In the figure 1, the theoretical result (0.1) (log law in the figure) is included along with experimental data from different authors, the viscous region (viscous sub-layer in the figure, where $U = u_* z_+$) and the buffer region (buffer layer in the figure, adapted from Garratt, 1992). For a deeper explanation on the viscous and buffer regions, see Jimenez (2004) and Durbin & Pettersson Reif (2001).

1.2 Rough Wall Boundary Layer

For rough walls, the logarithmic law can be written as

$$U(z_+) = \frac{u_*}{\kappa}\ln(z_+) + B_h(h_+)u_* \tag{0.2}$$

where the constant B is replaced by a function B_h that depends on the non-dimensional roughness geometry. A basic dependency on the roughness geometry for B_h can be defined in terms of the non-dimensional equivalent sand grain roughness height $h_+ = h/l_+$ (being h the dimensional equivalent sand grain roughness height, see Durbin & Peterson-Reif, 2001). The equation above can also be written as

$$U(z_+) = \frac{u_*}{\kappa}\ln(z_+) + \tilde{\mathrm{B}} \tag{0.3}$$

where $\tilde{\mathrm{B}}$ is a function of roughness height in the form of

$$\tilde{\mathrm{B}}(h_+) = \frac{1}{\kappa}\ln(h_+) + B_h(h_+) \tag{0.4}$$

The logarithmic profile for rough wall can now be rewritten alternatively as

$$U(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{0.5}$$

with z_0 the hydrodynamic roughness length (also known as aerodynamic roughness length or simply roughness length), which is defined from (0.3) and (0.5) as:

$$z_0 = h\exp(-\kappa\tilde{\mathrm{B}}) \qquad (0.6)$$

For smooth walls, expressions (0.1) and (0.5) give the minimum value of hydrodynamic roughness length, $z_{0,min} = \exp(-\kappa B)\,\nu/u_* \approx 0.14\nu/u_*$. For fully rough surface values $\tilde{B} = 8.5$ and $z_0 = 0.033h$ are obtained (see Ligrani & Moffat, 1986). This practice of defining roughness influence was first introduced by Schlicting (1936). The effect of roughness on the logarithmic profile is a matter of intense study since the pioneering work of Schlicting (1936). As an example, in figure 2, the measurements obtained by Schultz and Flack (2007) are shown. It can be observed that the higher the roughness height, h_+ the lower the wind speed in the log region. This is a common result but it is not universal since some types of roughness geometry can lead to flow acceleration (see Jimenez, 2004).

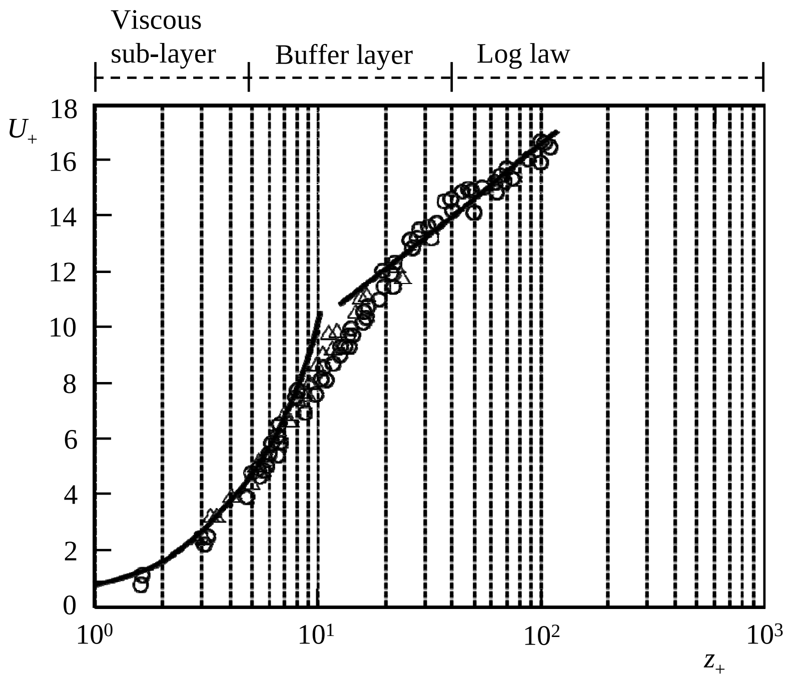

Figure 1: Velocity distribution above a smooth surface, from laminar flow to fully turbulent flow (viscous sub-layer to log law regions). Experimental data represented by symbols. The curve at low Re number represents a linear velocity profile while at higher Re. represents the logarithmic law. (Schlichting, 2000, Garratt, 1992). Observe that $z_+ = zu_*/\nu$ can be interpreted as a Reynolds number based on the friction velocity and the height on the wall.

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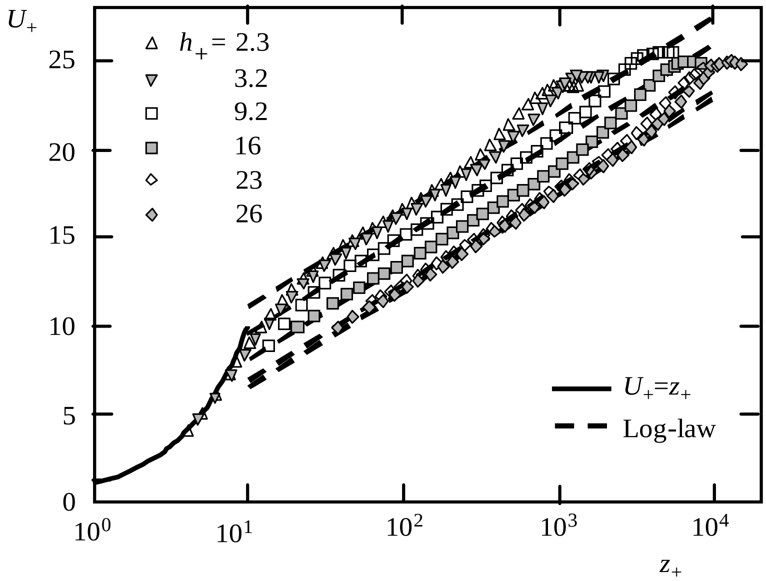

Figure 2: Mean velocity profile for different roughness heights compared to logarithmic profile of McKeon et al. (2004). (Schultz and Flack, 2007)

The relation between the hydrodynamic roughness length, z_0, and the roughness geometry has been an issue of main concern up to know. In the figure 3, the so called roughness function, $\Delta U/u_*$ (see Jimenez, 2004), which is related to the hydrodynamic roughness length by (see Raupach et al. 1991)

$$z_0 = \frac{\nu}{u_*}\exp\left(-B\kappa + \frac{\Delta U}{u_*}\kappa\right) \quad (0.7)$$

is presented for different values of non-dimensional roughness height in lab and the free atmosphere.

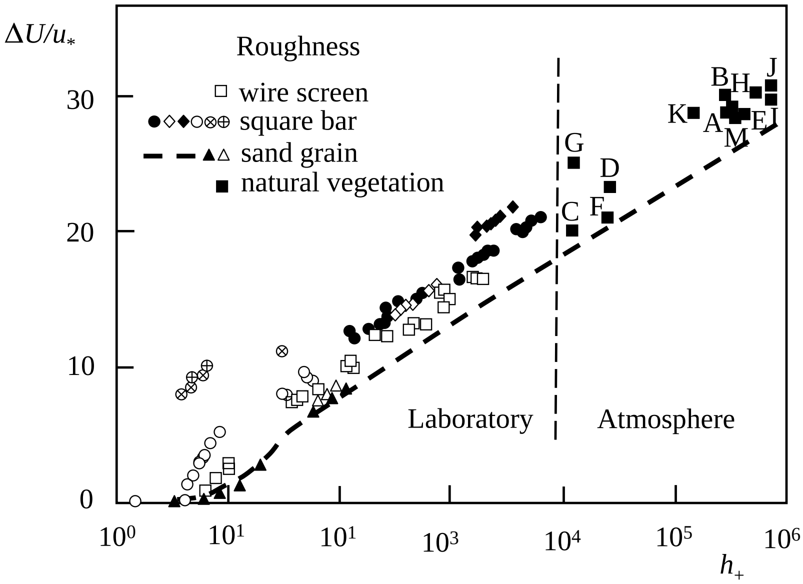

Figure 3: Roughness function, $\Delta U/u_*$, as a function of the roughness Reynolds number, $h_+=hu_*/\nu$ for different types of roughness elements in the wind tunnel and in the atmosphere (from Raupach et al., 1991). Codes for natural vegetation are described in the reference.

Studies of different roughness elements by Lettau (1969), Wooding (1973) and Raupach et al. (1991), amongst others, established a relation between the hydrodynamic roughness length, z_0, roughness height, h, and roughness density, λ, which is the total projected frontal roughness area per unit area on the wind tunnel floor corresponding to a single roughness element of a homogenous roughness layout, in the form

$$z_0 / h = 0.5\lambda \qquad (0.8)$$

The relation (0.8) is widely used to define the density, λ, and height, h, of homogeneously distributed roughness elements to match a given value of hydrodynamic roughness length, z_0. (Dyrbye and Hansen, 1996) A scheme on the definition of roughness density, λ, is presented in figure 4.

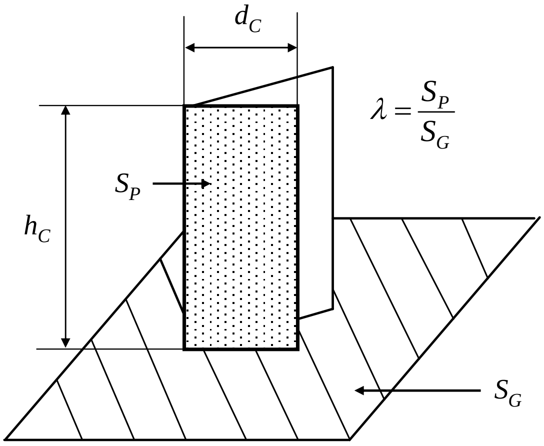

Figure 4: Schematic for determining the roughness density, λ with S_P being the projected frontal area to the averaged wind speed, and S_G being the unit area on the wind tunnel floor corresponding to a single roughness element.

Therefore, with this method, a simple estimation of roughness element needed to generate the required value of z_0 can be obtained. Here there is a maximum value of λ_{max} where the equation holds which has been studied to be ≈0.15. Beyond λ_{max}, z_0/h decreases with further increase of λ, which is attributed to the mutual sheltering of roughness elements (Wooding, 1973). However, changes of the value of λ_{max} depending on the roughness geometry, suggest a need to study different aspect ratios or additional characteristic lengths (Raupach et al. 1991), see figure 5. Lettau (1969) who first proposed equation (0.8), also remarked that a sufficient extent of roughness in the upwind direction is needed for the equation (0.8) to be valid. This was confirmed by Counehan (1971) who suggested a minimum length of 1000h needed to achieve an almost equilibrium boundary layer in the wind tunnel.

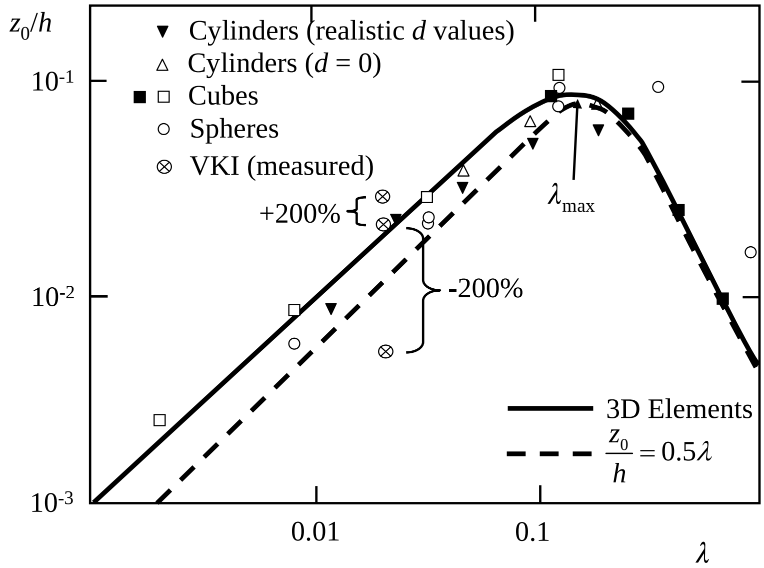

Figure 5: Normalized roughness length z_0/h as a function of roughness density, λ for three dimensional elements. The empirical correlation $z_0/h = 0.5\ \lambda$, is shown by the dashed line. Measured values of z_0/h from VKI initial wind tunnel tests (Buckingham, 2010) are included along with the uncertainty levels due to different methods of determining z_0 from Iyengar & Farell (2001).

Some authors have also paid attention to the limits related to the geometrical roughness height, h, for instance Jimenez (2004) remarks that the ratio of the total boundary layer height, δ, to the roughness element height, h, must satisfy $\delta/h > 80$ if one wants to apply the previous considerations. For larger heights, the flow must be better analysed as flow over obstacles.

2 FRICTION VELOCITY AND ROUGHNESS LENGTH DETERMINATION

Some studies have been devoted to compare the different methods to estimate the friction velocity, u_* and hydrodynamic roughness length, z_0 in wind tunnel flows (see for instance, Iyengar & Farell, 2001). These authors compared indirect methods of obtaining z_0, u_* and d, zero-displacement height, (Schlichting, 2000) (using Hama's law fits and log-power law fits) with values obtained from direct measurements of u_* (from Reynolds shear stress and balance measurements). Estimates by indirect methods, used successfully in smooth wall flow studies, can give differences of up to 200% in rough wall cases due to the possibility of having several sets of values z_0, u_* and d giving comparable fits to the same velocity profile (Iyengar & Farell, 2001). Direct measurements give errors of up to 15 % which is mainly due to the X-wire probe errors in measuring turbulent flows. The predicted value of z_0/h from the roughness elements (33mm cups with base to top diameter variation of 40mm to 30mm) used in the initial wind tunnel study in VKI is shown in the figure 5 along with possible value range of z_0/h according to the indirect method uncertainty factor of 2 documented in (Iyengar & Farell, 2001).

2.1 Scaling Factor Effects

After deciding the model scale, *S*, at the beginning and considering an objective value for $z_{0(real)}$, the value of the required hydrodynamic roughness length in the wind tunnel, $z_{0(WT)}$ is determined. As a preliminary attempt, the relation (0.8) can be used for determining the geometry of the roughness element and its density to be used in the wind tunnel. By taking $\lambda = S_P/S_G$ as shown in figure 5, a range of possible roughness element dimensions can be calculated (see figure 6).

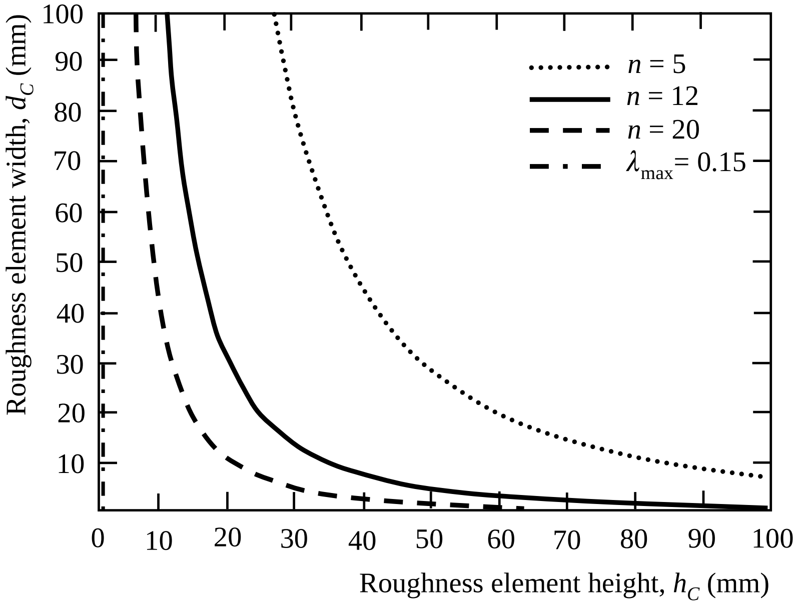

Figure 6: Relation roughness element dimensions, d_C and h_C, and roughness density λ for a chosen model scale, *S,* and hydrodynamic roughness length, $z_{0(real)}$ (in the figure, S=500 and $z_{0(real)}$= 50mm, therefore $z_{0(WT)} = z_{0(real)}/S$ = 0.1mm). A homogenous distribution of roughness elements in wind tunnel has been assumed, lines plotted are from the different number of elements in a row, *n* perpendicular to the mean flow speed. The limit λ = 0.15 is shown with the right side of this line for $\lambda << \lambda_{max}$.

Considering the Alaiz site, the large scaling factor proposed for wind tunnel modelling posed a new challenge as most wind tunnel modelling have been done to a scale of S <500. Considered the minimum value reproducible in the wind tunnel $z_{0,min} = 0.14\nu/u_*$ a minimum real value for the hydrodynamic roughness length can be estimated as $z_{0,min(real)} = 0.14S\nu/u_*$. If the free upflow to the Alaiz site is assumed to be between Type II and Type III terrain type according to the Eurocode classification, which corresponds to a $z_{0(real)}$ range of 0.05-0.3m, choosing the model scale to be $S \approx 5000$, the $z_{0(WT)}$ needed to be reproduced in the wind tunnel can be calculated (see table 1).

Table 1: z_0 of real terrain and wind tunnel model with a model scale, S= 5000.

	Terrain	Model
Type II: z_0 (m)	0.05	1×10^{-5}
Type III: z_0 (m)	0.3	6×10^{-5}

For flat and smooth wall boundary layers, it is quite well established that the viscous region (the region close to the wall where the log profile is no applicable) extends to non-dimensional values of height, $z_+ \in [0, \approx 40]$. In case of using large scale models, such as the one for Alaiz ($S \approx 1:5000$) this range of non-dimensional heights could correspond to values comparable to the hub height in the real case.

Roughness and non-homogeneity of the terrain lead to variations of this interval of height for which the log profile is not applicable (see figure 2). Taking as valid the estimation for smooth and flat wall, and considering typical values of kinematic viscosity and friction velocity (ν = 1.5×10^{-5} m^2/s, u_* = 0.5 m/s) the viscous limitation in the case of Alaiz might range in the interval $z \in [0, 1.2mm]$, which corresponds to an interval in the real terrain $z \in [0, 6.5m]$. So no-conclusions about the log profile from the wind tunnel model could be obtained for height values less than 6.5m in the reality. Hence, a rough estimation on the minimum analyzable height is $z_{min,(real)} = 40S\nu/u_*$.

Table 2: Wind turbine height and distance from model with scaling.

	Wind Turbine	Distance from model surface
Height: (m)	70	0.014
Height: (m)	100	0.02

The distances shown in the table above indicate that measurements taken on a model with scale S = 1:5000 are well into the logarithmic region of the wind profile (non viscous region).

In this case the limitation would be associated to the precision of the traverse system used in the case of a hot-wire system.

3 CONCLUSIONS

This paper presents a preliminary study to conduct a modelling of complex terrain in wind tunnels with large scales (≈5000). The analysis is focussed on the generation of a required log profile for the mean wind speed. The relation between the hydrodynamic roughness length and the roughness geometry has been analysed particularly for the case of large scale models. Two issues are outlined, first the fact that a large scale factor might provoke that viscous sub-layer on the model extends to heights that in the real field correspond to distances from the ground comparable to the hub height, being the minimum analyzable height $z_{min,(real)} = 40S\nu/u_*$. Secondly, there is a limit for the generation of the hydrodynamic roughness height in the wind tunnel which establishes a minimum reproducible value for the real value $z_{0(real)} = 0.14S\nu/u_*$.

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