Satellite imagery fusion with an equalized trade-off between spectral and spatial quality

Fusión de Imágenes de satélite considerando el equilibrio entre la calidad espacial y espectral

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RESUMEN

En este trabajo se propone una estrategia para obtener imágenes fusionadas con calidad espacial y espectral equilibradas. Esta estrategia está basada en una representación conjunta MultiDirección-MultiRresolución (*MDMR*), definida a partir de un banco de filtros direccional de paso bajo, complementada con una metodología de búsqueda orientada de los valores de los parámetros de diseño de este banco de filtros. La metodología de búsqueda es de carácter estocástico y optimiza una función objetivo asociada a la medida de la calidad espacial y espectral de la imagen fusionada. Los resultados obtenidos, muestran que un número pequeño de iteraciones del algoritmo de búsqueda propuesto, proporciona valores de los parámetros del banco de filtro que permiten obtener imágenes fusionadas con una calidad espectral superior a la de otros métodos investigados, manteniendo su calidad espacial.

PALABRAS CLAVE: Fusión de Imágenes Multiespectral, Transformada Multidireccional-Multirresolución, Bancos de Filtros Direccionales, Simulación de Templado.

ABSTRACT

A methodology for obtaining fused images with an equalized trade-off between the spectral and spatial quality has been proposed. This methodology is based on a joint MultiDirection-MultiResolution representation (*MDMR*), defined through a Directional Low Pass Filter Bank (*DLPFB*) and complemented with a strategy of search *DLPFB's* parameters. This strategy uses a stochastic method, which optimizes an objective function associated to a spectral and spatial quality measure of the fused image. The results obtained in this work, show that a low number of iterations are needed for getting values of the *DLPFB's* parameters which provide fused images with a higher spectral quality than the images fused by other methods investigated here, conserving its spatial quality. It allows an equalized trade-off between the two considered qualities, against the other methods.

KEYWORDS: Multispectral Image Fusion, Multidirectional-Multiresolution Transform, Directional Filter Bank, Simulated Annealing.

INTRODUCTION

Image fusion can be understood as the synergetic combination of information provided from several sensors or by the same sensor in different scenarios (e.g. spatial, spectral and temporal). The reduction of redundant information, while emphasizing relevant information, not only improves image processing performance but it also facilitates their analysis and interpretation.

In the last decade, the most used image fusion strategies were based in multiresolution analysis techniques. Their objective was to find a discrete transform that minimizes the intrinsic uncertainty associated to the joint representation of information. From this point of view, the Discrete Wavelet Transform (DWT) can be considered as the most popular approximation (Garguet-Duport *et al.* 1996).

The recent appearance of new transforms, such as Curvelets (Candès and Donoho 1999a), Ridgelets (Candès and Donoho 1999b) and Contourlets (Do and Veterlli 2005), which improves the 2-D information representation with respect to the *DWT*, opens a new field of research in the image fusion algorithm area. Generally speaking, it can be affirmed that these new transforms (multiresolution-multidirectional) are based in the application of a double filter bank. The first one, is for stepping from a higher to a lower resolution level. The second, is a directional filter bank and it allows capturing the directional features for each one of the different resolution levels. They are highly anisotropic and produce a much more efficient extraction of spatial details in different directions, which makes them especially adequate to perform the fusion process. Different recently published works address this issue. Choi *et al.* (2005) proposed the use of the Curvelet transform, while Qiguang and Baoshu (2006) used a Contourlet transform, to fuse satellite images recorded by a panchromatic sensor and a multispectral sensor.

In order to reduce the cost involved in a double filter bank, in Lillo-Saavedra and Gonzalo (2007) a fusion method was proposed based on a new joint MultiDirectional and MultiResolution (*MDMR*) image representation that uses a single Directional Low Pass Filter Bank (*DLPFB*) defined in the frequency domain. As shown in the present paper, this new methodology has the intrinsic capacity to control the global (spatial-spectral) quality of the fused images. This control is based on the accurate tune-up of the *DLPFB*. The aim of this paper is to propose a method that objectively determines the design of the *DLPFB*. Specifically, it proposes the optimization of an objective function (*OF*) based on a fused image quality measure, using the Simulated Annealing (*SA*) search algorithm.

FUSION METHODOLOGY BASED ON MDMR REPRESENTATION

Similar to other fusion methods for multispectral (*MULTI*) and panchromatic (*PAN*) images, the objective of the discussed fusion methodology is to coherently integrate the low frequency information from the *MULTI* image and the high frequency information from the *PAN* image, to obtain a fused image whose spatial quality would be as similar as possible to the quality of a higher resolution spatial image (PAN), while conserving the spectral characteristics of a high resolution spectral image (*MULTI*).

MDMR representation for image analysis and synthesis

The joint MDMR representation used in this work combines the simplicity of the Wavelet transform, calculated using the à trous algorithm (WAT), with the benefits of multidirectional transforms like Contourlet Transform (*CT*), using a single *DLPFB* (Lillo-Saavedra and Gonzalo 2007) . Thus, at each decomposition level (θ_n), image degradation is performed applying a directional low pass filter in the frequency domain, as shown in Eq. (1).

$$\text{Image}_{\theta_n}(x,y) = FFT^{-1}\left[FFT\left[\text{Image}_{\theta_{n-1}}\right]\cdot H_{\theta_n}(u,v)\right] \quad (1)$$

Where θ_{n-1} is the decomposition level prior to transform application and $H_{\theta_n}(u,v)$ represents the directional low pass filter transfer function, applied in level θ_n.

The directional information is extracted by the difference of the directional degraded images in two consecutive levels and is stored in the transform's coefficients at each level:

$$\text{Coef}_{\theta_n}(x,y) = \text{Image}_{\theta_n}(x,y) - \text{Image}_{\theta_{n-1}}(x,y) \quad (2)$$

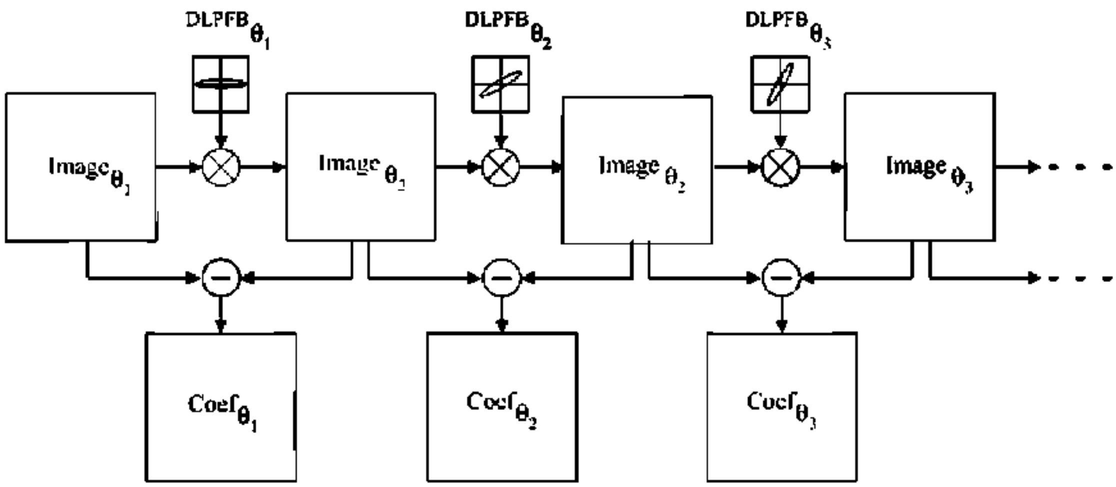

Figure 1. Flow diagram of the joint multidirectional and multiresolution images representation based on directional low pass filter bank

Fig. 1 illustrates graphically the joint *MDMR* representation.

From Eqs. (1) and (2), the original image can be exactly reconstructed by Eq. (3):

$$\text{Image}(x,y) = \text{Image}_{\theta_k}(x,y) + \sum_{n=1}^{k} \text{Coef}_{\theta_n}(x,y) \qquad (3)$$

In other words, it adds to the corresponding image at the higher decomposition level (θ_k) all the directional coefficients, $\sum_{n=1}^{k} \text{Coef}_{\theta_n}(x,y)$, in a procedure analogous to the one used in *WAT*.

Lakshmanan (2004) demonstrated that a low pass filter that is simultaneously separable and directional could not exist. However, it is possible to define a directional low pass filter as the sum of two separable filters as shown in Eq. (4):

$$H(u,v) = H_1(u) \times H_2(v) - \alpha u H_1(u) \times v H_2(v) \qquad (4)$$

Where α is given by the relation $(a^2-b^2)\cdot\sin(2\theta)/(a^2\cdot b^2)$, being θ, a and b the orientation, scale and elongation of the filter, respectively:

$$H_1(u) = \exp\left(-u^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right)\right) \qquad (5)$$

$$H_2(v) = \exp\left(-v^2\left(\frac{\cos^2\theta}{b^2} + \frac{\sin^2\theta}{a^2}\right)\right) \qquad (6)$$

The most interesting filter characteristic is not its elliptic form, but rather its directional character by which it assigns higher weights to the corresponding values in a determined direction and lower weights to their orthogonal direction.

It is important to note that the values of a and b determine the geometry of the low pass filters that conform *DLPFB*. From an image representation perspective, the values that these parameters take will determine the quantity of image information contained in the coefficients, and in each one of the degraded images, which in the case being studied is determinant of the final quality of the fused image.

Formal MDMR fusion methodology

Under the previous considerations, it is posible to formalize a fusion methodology based on *MDMR* (Lillo-Saavedra and Gonzalo 2007):

$$FUS^i(x,y) = MULTI^i_{\theta_k}(x,y) + \sum_{n=1}^{k} Coef^{PAN}_{\theta_n}(x,y) \quad (7)$$

Where $FUS^i(x,y)$ represents the i^th band of the fused image, $MULTI^i{}_{\theta k}$ represents the i^th band of the *MULTI* image degraded in k directions, and $\sum_{n=1}^{k} Coef^{PAN}_{\theta_n}(x,y)$ represents the addition of *PAN* image coefficients (Eq. (2)).

The two most relevant characteristics of this methodology are its high anisotropy and the control of the inherent compromise between spatial and spectral quality of the fused image; in particular, the possibility to obtain fused image with an equalized trade-off between both qualities. As indicated earlier, the parameters a and b determine filters geometry and consequently the information selected in the filtering process for each particular image. In this sense, the potential of the proposed fusion methodology would be strengthened if a filter parameters tune-up method would be available.

In the next section a method based on SA for optimizing an OF defined through the measures of the spatial and spectral quality of the fused images, is proposed. These measures have been defined using the *ERGAS* spectral (Erreur Relative Globale Adimensionnelle de Synthèse, Wald 2000) and spatial (Lillo-Saavedra *et al.* 2005) quality indexes. It is important to note that an *ERGAS* value close to zero indicates a good fused image quality.

The original definition of the ERGAS index was proposed by Wald (2000) through the Eq. (8):

$$ERGAS = 100\frac{h}{l}\sqrt{\frac{1}{N_{Bands}}\sum_{i=1}^{N_{Bands}}\left(\frac{RMSE^2(Band_i)}{(M^i_{MULTI})^2}\right)} \quad (8)$$

Where h and l represent the spatial resolution of the *PAN* and *MULTI* images, respectively; N_{Bands} is the number of bands of the fused image; $M^i{}_{MULTI}$ is the mean radiance value of the i^th band of the *MULTI* image. The RMSE (Root Mean Square Error) is evaluated through Eq. (9).

$$RMSE(Band_i) = \frac{1}{NP}\sqrt{\sum_{k=1}^{NP}\left(I^i_{REF}(k) - I^i_{FUS}(k)\right)^2} \quad (9)$$

NP is the number of pixels of the fused image; $I^i{}_{REF}$ represents the i^th spectral band of the reference multispectral image and $I^i{}_{FUS}$ the corresponding spectral band of fused image. It is clear, from its definition, that low *ERGAS* index values represent high quality of the fused images.

Although *ERGAS* is defined as a global quality index, Lillo-Savedra and Gonzalo (2005) showed that their behaviour is rather that of a spectral quality index. In this senses, Wald-ERGAS to be called $ERGAS_{spectral}$.

On the other hand, in previous paper was proposed a new index with the objective of evaluating the distance between the *PAN* image and the *FUS* image (spatial quality) (Lillo-saavedra *et al.* 2005). This index has been named spatial *ERGAS*, since it is based in the same concept that the original *ERGAS* (Wald 2000) . In its definition, a spatial *RMSE* has been included, which is defined as in (10):

$$RMSE_{Spatial}(Band_i) = \frac{1}{NP}\sqrt{\sum_{k=1}^{NP}\left(I^i_{PAN}(k) - I^i_{FUS}(k)\right)^2} \quad (10)$$

Where $I^i{}_{PAN}$ is the image obtained by adjusting the histogram of the original PAN image to the histogram of the i^th band of the FUS image. In this way the spectral differences between the PAN and fused images are minimized. Therefore, replacing $M^i{}_{MULTI}$ by $M^i{}_{PAN}$ in the Eq. (8), the next expression is obtained:

$$ERGAS_{Spatial} = 100\frac{h}{l}\sqrt{\frac{1}{N_{Bands}}\sum_{i=1}^{N_{Bands}}\left(\frac{RMSE^2_{Spatial}(Band_i)}{(M^i_{PAN})^2}\right)} \quad (11)$$

This index is able to quantify the spatial quality of fused images by measuring the *PAN* and fused image distances, in the same senses of *ERGAS* discussed above does for spectral quality.

Fig. 2 presents the spatial and spectral *ERGAS* values obtained when a PAN image and a *MULTI* image have been fused with a and b varying between 0 and 5 and k=23. In this figure, it can be observed how a set of a and b values exist that establish an equalized trade-off between spatial and spectral quality of the images fused.

Therefore, the *OF* is defined as the absolute value of the difference between both indexes as shown in Eq. (12):

$$\Delta E = \left| ERGAS_{Spatial} - ERGAS_{Spectral} \right| \quad (12)$$

Fig. 3 displays the surface corresponding to the *OF* (Eq. (12)) for a particular image for a and b varying between 0 and 10 and k=23.

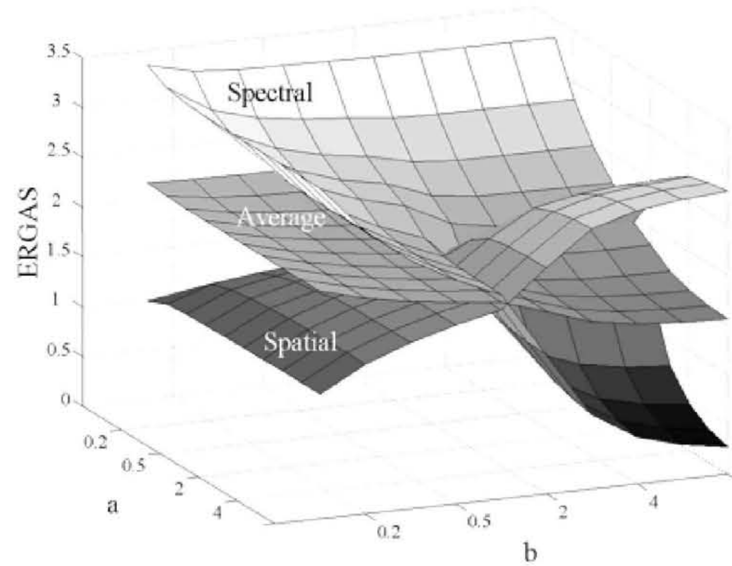

Figure 2. Surfaces of the spatial and spectral *ERGAS* and their average values for a fused image with $k=2^3$ and different values of a and b parameters

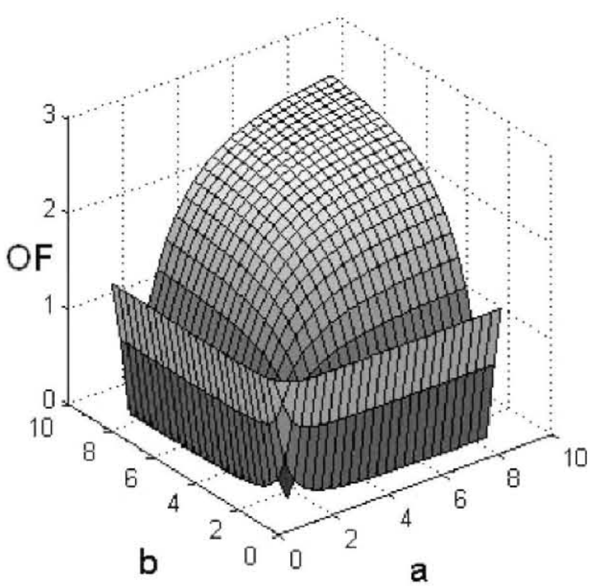

Figure 3. Objective function (OF) surface

MDMR FILTER PARAMETERS TUNE-UP

The quality of images fused using the described methodology is determined by the characteristics of the *DLPFB* applied during the fusion process. Basically, there are 4 parameters that determine these characteristics: size (*m*), filter scale (*a*), filter elongation (*b*), and the number of partitions of the frequency space (*k*).

Given the symmetrical nature of Fourier space where the *DLPFB* is applied, this filter must be symmetric.

Experimentally, it was observed that m=5 is the minimal number of samples required to define a symmetric kernel of *H(u,v)* type filters (Eq. (4)). Other kernel sizes that maintain the symmetry are m=11 and m=21, which present similar behaviour. However, an increase in size implies an elevated increase in computational complexity. Under these considerations, a filter kernel size of m=5 has been used.

Empirical studies had shown that for frequency space partitions (*k*) varying between 2^2 and 2^6, there is a pair of values (a, b) that provides very similar spatial and spectral qualities. In fact, it was determined that a frequency space division in 2^3 directions provides a good compromise between the process's computational complexity and fused image quality.

From Fig. 3, it can be noted that the parameters *a* and *b* present a symmetrical behaviour with respect to the principal diagonal of the space defined by these parameters. This symmetrical behaviour has been checked for a large number of cases. As a result, the condition that $b > a$ in the search space has been imposed.

Search algorithm

The search algorithm proposed in this paper is based on the SA optimization method developed by Kirpatrick *et al.* (1983) and it pertains to a wide class of local search algorithms, known as Threshold Algorithms (Ingber 1993).

In Fig. 4 the methodology for searching a and b parameters is shown schematically. As can be seen, once the source images have been pre-processed, a pair of initial values is assigned (a_{ini} and b_{ini}). With this first filter, a fused image is obtained and the *ERGAS* spatial and spectral quality values are determined in order to calculate the initial value of the ΔE_{ini} (Eq. (12)).

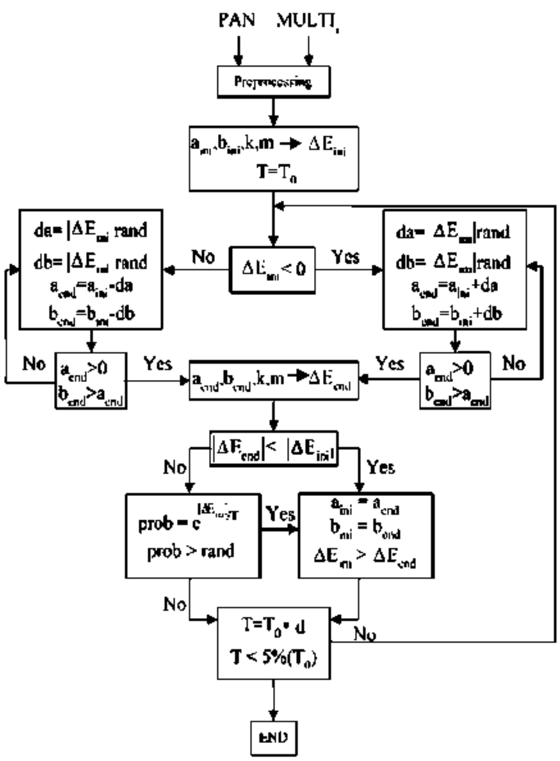

Figure 4. Flow diagram of the filter parameters search algorithm

The study of *ERGAS* index behaviour respect to the variation of filter parameters, a and b, indicates that a growth in these parameters diminishes the spatial quality of the fused image, increasing its spectral quality and vice-versa (Fig. 2). This behaviour allows the specification of a directed search criterion: if the value of ΔE_{ini} is less than zero ($ERGAS_{Spatial}<ERGAS_{Spectral}$), the spectral quality of the fused image should be improved by decreasing the fused image's spatial quality. Consequently, the parameters a_{ini} *and* b_{ini} should increase in the values *da* and *db*. In the opposite case, for ΔE_{ini} greater than zero, the spatial quality of the fused image should be increased. That implies a reduction of the values parameters (a_{ini} and b_{ini}) in da and *db*. Once the new solution ΔE_{end} is obtained from the new parameters ($a_{end}= a_{ini}+da$ and $b_{end}= b_{ini}+db$), it is compared with ΔE_{ini}, then if it is lower the new solution is accepted, in otherwise it will be accepted or discarded according to the SA acceptance criterion, formalized in Eq. (13).

$$rand(0,1) < e^{|\Delta E_{ini}| / T} \tag{13}$$

Where *rand(0,1)* is a random number between 0 and 1 with a uniform probability distribution and *T* represents a parameter that receives the name "temperature".

The *SA* strategy begins with an initially high temperature, which provides a high probability to accept movements that worsen result quality. In each iteration, the temperature is reduced, diminishing the probability of accepting worse solutions. This temperature reduction process is known as the cooling schedule and is controlled by the temperature's decrease index (δ). A very small δ value implies a rapid convergence; however, this means that the search is not exhaustive, increasing the probability of getting confined at a local minimum. In contrast, with a high δ value, the search algorithm converges more slowly since it is more exploratory, increasing the probability of obtaining solutions close to the global minimum.

RESULTS

The data used to evaluate the performance of the fusion method based on *MDMR* correspond to two scenes registered by the panchromatic and multispectral sensors on board IKONOS and QUICKBIRD satellites, respectively. For the two scenes, the multispectral image size has been 128x128 pixels and consequently the size of *PAN* images are 512x512. The IKONOS scene was recorded on March 10, 2000, and is geographically located in the Maipo Valley, near Santiago, Chile. The QUICKBIRD scene was extracted from an image recorded on August 22, 2002, and geographically corresponds to the north-west area outside of Madrid, Spain.

The NGB (NearIR-Green-Blue) compositions of the MULTI images of these scenes are presented in Figs. 5 (a) and 6 (a), and their corresponding PAN images in Figs. 5 (b) and 6 (b).

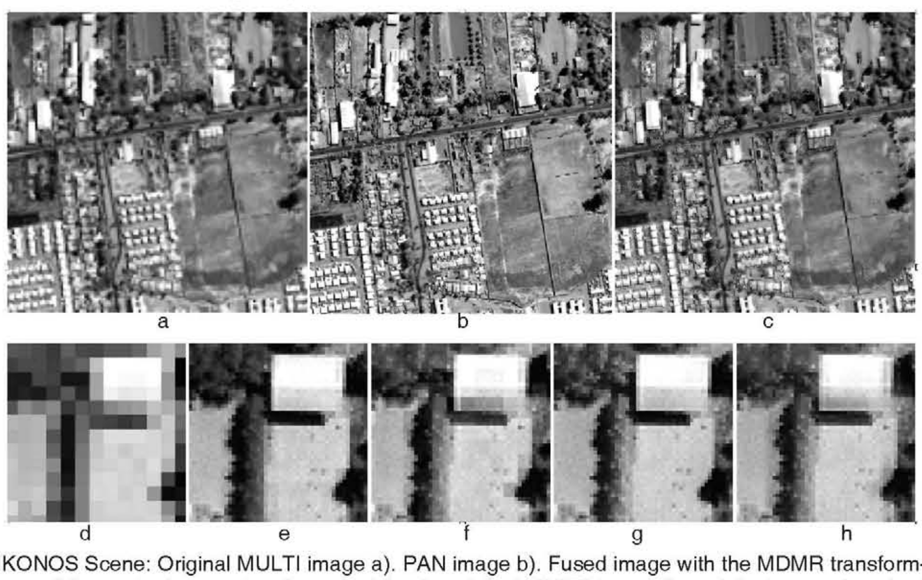

Figure 5. IKONOS Scene: Original MULTI image a). PAN image b). Fused image with the MDMR transform-based method c). Zooms of the particular area into figure 5 a) for the original MULTI image d), and the images fused using the methods based in the transform IHS e), WMT f), WAT g) and MDMR h)

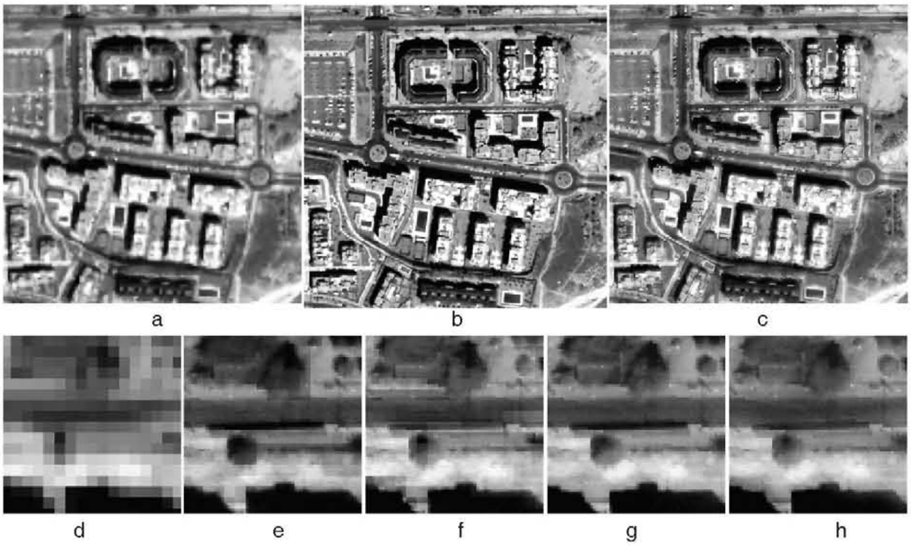

Figure 6. QUICKBIRD Scene: Original MULTI image a). PAN image b). Image fused with the method based in the MDMR transform c). Zooms of the particular area into figure 6 a) for the original MULTI image d), and the images fused using methods based in the transforms IHS e), WMT f), WAT g) and MDMR h)

The search method proposed to determine filter parameters a and b has been applied to the two scenes described in the previous section. Considering the results obtained in the study of the method behaviour, the search space has been divided in k=2^3 directions and the filter size (m) set at 5 samples.

To determine the δ values that provide the best compromise between speed of convergence of algorithm and search efficiency in terms of fused image quality, a series of experiments have been performed. Specifically, different pairs of the parameters (a and b) have been determined for the δ decrease values equal to 0.4, 0.6 and 0.8. The results indicated that δ values greater or equal to 0.8 have provided the best results, being this value used in all experiments carried out in this study.

Table 1 includes, for each scene and for each spectral bands, the values of a and b, tuned-up for k=2^3, m=5 and δ=0.8.

The NGB compositions of the fused images based on MDMR, using the values of Table 1, are presented in Figs. 5 (c) and 6 (c).

In comparison with the multispectral images (5 and 6 (a)), an important improvement in spatial quality, while maintaining spectral quality, it can be visually observed.

The two scenes considered have been fused using transform-based methods: *IHS*, Wavelet-Mallat (*WMT*) and Wavelet-à trous (*WAT*) (Mallat 1999, Wald 2002). Fig. 5 (d) and 6 (d) present a zoom of the areas framed in Fig. 5 and 6 (a), while Figs. 5 and 6 (e), (f), (g) and (h) present the corresponding zooms for the fused images using the previously indicated methods and the proposed method (*MDMR*). A comparative visual analysis between the zooms indicates that the fusion methods based on *WMT* and based on *MDMR* conserve more faithfully the original image's spectral content for the two cases con-

SCENE	B1		B2		B3		B4	
	a	*b*	*a*	*b*	*a*	*b*	*a*	*b*
IKONOS	0.7035	1.4081	0.8848	1.9519	0.8833	1.9199	0.838	1.8354
QUICKBIRD	0.567	1.7205	0.7973	1.8117	0.824	2.0493	0.7014	1.5462

Table 1. Filter parameters determined using the search algorithm for *MDMR* transform-based fusion method

sidered. Moreover, the presence of artefacts that worsen spatial quality it can be observed in Figs. 5 (e) and (f) and 6 (e) and (f), while they are not present in Figs. 5 (g) and (h) and 6 (g) and (h).

In order to quantify the results discussed in the previous paragraph, the *ERGAS* (spatial and spectral) index values as well as its average and standard deviation have been calculated. The two last are interpreted as measures of global quality and trade-off between spatial and spectral quality, respectively.

The indexes' values for the two scenes are included in Tables 2 and 3. In these tables, it can be observed that the lowest $ERGAS_{Spatial}$ is produced by the *WAT* method, although it does not result in equilibrium between spatial and spectral quality as reflected in the value of standard deviation. On the other hand, the *MDMR* method provides a total equilibrium between spatial and spectral quality. Additionally, this method gives a lower $ERGAS_{Spectral}$ value than the other evaluated methodologies.

METHOD	$ERGAS_{spatial}$	$ERGAS_{spectral}$	$ERGAS_{average}$	*St. Dev.*
IHS	1.9931	2.6574	2.3252	0.6643
WMT	2.079	2.2083	2.1436	0.1293
WAT	**1.7067**	2.3029	2.0048	0.5962
MDMR	1.9226	**1.9226**	**1.9226**	**0**

Table 2. ERGAS values for the fused IKONOS scene

METHOD	$ERGAS_{spatial}$	$ERGAS_{spectral}$	$ERGAS_{average}$	*St. Dev.*
IHS	1.886	2.5938	2.2399	0.5004
WMT	2.1334	1.7731	1.9533	0.2548
WAT	**1.7079**	1.8822	1.7951	0.1233
MDMR	1.7627	**1.7627**	**1.7627**	**0**

Table 3. ERGAS values for the fused QUICKBIRD scene

CONCLUSIONS

The results obtained in this study have verified that the search methodology based in the SA algorithm tunes-up the scale (a) and the elongation (b) parameters used to design the filter bank involved in MDMR fusion strategy, providing an equalized trade-off between the spatial and spectral qualities of the fused image, independently of the source images characteristics.

Both qualitative (visual comparison) and quantitative (*ERGAS* indexes) studies have shown that an adequate *OF* definition joint to a parameter-directed search provide fused images with superior spectral quality than the fused images by the other algorithms evaluated; being their spatial quality comparable to the quality provided by the *WAT* method. Still, the most notable characteristic of the proposed methodology is its capabilty to provide an equalized trade-off between both qualities.

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