

# $w$ -singularities in cosmological models

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# Sketch

- 1 Introduction
- 2  $w$ -singularities
- 3 Conclusions



# Motivation

- Astronomical observations of luminosity distances derived from Type Ia supernovae, CMB spectrum and global matter distribution provide evidence of cosmic speed up of the Universe.
- One approach to deal with this riddle is to consider modifications of general relativity.
- Alternatively, cosmic acceleration might be due to an exotic fluid filling the Universe, known as dark energy.
- These have given rise to a collection of new cosmological evolutions, future singularities being the most perplexing ones (“big rip”, “sudden singularities”...).



# Classification of singularities

- Big Bang / Crunch: zero  $a$ , divergent  $H$ , density and pressure. Strong.
- Type I: “Big Rip”: divergent  $a$ . Strong.
- Type II: “Sudden”: finite  $a$ ,  $H$ , density, divergent  $\dot{H}$  and pressure. Weak
- Type III: “Big Freeze”: finite  $a$ , divergent  $H$ , density and pressure. Weak/Strong
- Type IV: “Big Brake”: finite  $a$ ,  $H$ ,  $\dot{H}$ , density and pressure but divergent higher derivatives. Weak.



Dabrowski & Denkiewicz introduced a cosmological model,

$$a(t) = \frac{a_s}{1 - \frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}} + \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{na_s}{1 - \frac{2}{3\gamma} \left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}} \left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}}$$

$$+ \frac{a_s}{\frac{3\gamma}{2} \left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1} - 1} \left(1 - \frac{1 - \frac{2}{3\gamma}}{n - \frac{2}{3\gamma}} \frac{t}{t_s}\right)^n, \quad \gamma > 0, \quad n \neq 1.$$

- The scale factor  $a(t_s) = a_s$  is finite.
- Density and pressure vanish at  $t_s$ ,  $\rho(t_s) = 0 = p(t_s)$ .
- If  $n$  is natural, the derivatives of  $a$  and the Hubble parameter are regular.
- Just the barotropic index  $w$  ( $p = w\rho$ ) is singular at  $t = t_s$ .



# Barotropic index expansions

The barotropic index  $w$  may be written in terms of the scale factor,

$$w = -\frac{1}{3} - \frac{2}{3} \frac{a\ddot{a}}{\dot{a}^2}.$$

Assuming that the scale factor admits a generalized power expansion with real exponents,

$$a(t) = c_0(t_s - t)^{\eta_0} + c_1(t_s - t)^{\eta_1} + \dots, \quad \eta_0 < \eta_1 < \dots, \quad c_0 > 0,$$

$\eta_0$	$\eta_1$	$\eta_2$	$w_s$
$\neq 0$	$(\eta_0, \infty)$	$(\eta_1, \infty)$	Finite
0	$(0, 1)$	$(\eta_1, \infty)$	Infinite
	1	$(1, 2)$	Infinite
	1	$[2, \infty)$	Finite
	$(1, \infty)$	$[\eta_1, \infty)$	Infinite



# Density and pressure

Besides a diverging barotropic index, we need vanishing density and pressure.

For  $\eta_0 = 0$ ,  $\eta_1 \neq 1$ ,

$$\rho = \frac{3c_1^2\eta_1^2}{c_0^2}(t_s - t)^{2(\eta_1-1)} + \dots,$$

$$p = -\frac{2c_1\eta_1(\eta_1 - 1)}{c_0}(t_s - t)^{\eta_1-2} + \dots,$$

$\eta_1 > 2$  is required.



# Results

- A FLRW cosmological model has a singular barotropic index  $w$  with vanishing pressure and density at  $t_s$  if and only if the generalized power expansion of  $a(t)$  is of the form

$$a(t) = c_0 + c_1(t_s - t)^{\eta_1} + \dots,$$

with  $\eta_1 > 2$ .





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- A FLRW cosmological model has a  $w$ -singularity at a finite time  $t_s$  if and only if the scale factor  $a(t)$  admits a Taylor series at  $t_s$  with vanishing linear and quadratic terms,

$$a(t) = c_0 + O((t - t_s)^3).$$







# Scale factor vs. singularities

$\eta_0$	$\eta_1$	$\eta_2$	Tipler	Królak	N.O.T.
$(-s, 0)$	$(\eta_0, s)$	$(\eta_1, s)$	Strong	Strong	I
0	$(0, 1)$	$(\eta_1, s)$	Weak	Strong	III
	$(1, 2)$	$(\eta_1, s)$	Weak	Weak	II
	1	$(1, 2)$	Weak	Weak	II
		$[2, s)$	Weak	Weak	IV
	$[2, s)$	$(\eta_1, s)$	<b>Weak</b>	<b>Weak</b>	<b>IV</b>
$(0, s)$	$(\eta_0, s)$	$(\eta_1, s)$	Strong	Strong	Crunch

Hence w-singularities are weak singularities.



## Further reading

-  L. Fernández-Jambrina,  
On  $w$ -cosmological singularities  
(preprint)
-  L. Fernández-Jambrina, R. Lazkoz,  
Singular fate of the universe in modified theories of gravity  
*Phys. Lett. B* **670** 254 (2009) [arXiv:0805.2284]
-  L. Fernández-Jambrina, R. Lazkoz,  
Classification of cosmological milestones  
*Phys. Rev. D* **74** 064030 (2006) [arXiv:gr-qc/0607073]
-  L. Fernández-Jambrina, R. Lazkoz,  
Geodesic behaviour of sudden future singularities  
*Phys. Rev. D* **70** 121503 (2004) [arXiv:gr-qc/0410124]



Thank you all!

