

Equation of state and singularities in FLRW cosmological models

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Abstract. We consider FLRW cosmological models with standard Friedmann equations, but leaving free the equation of state. We assume that the dark energy content of the universe is encoded in an equation of state $p = f(\rho)$, which is expressed with most generality in the form of a power expansion. The inclusion of this expansion in Friedmann equations allows us to construct a perturbative solution and to relate the coefficients of the equation of state with the formation of singularities of different types.

1. Introduction

Since the end of the 20th century there has been an increase of evidence for an accelerated expansion of the universe [1]. These puzzling observations do not fully fit in our present theoretical framework and therefore there are two trends of development attempting to amend it.

One trend is grounded on modifications of general relativity as the correct theory of gravity [2]. The other major trend assumes the validity of general relativity and postulates the existence of an exotic component in the content of the universe known as dark energy [3].

So far the only final state for our universe could be either infinite expansion or a final collapse, the big crunch. But these new scenarios open the possibility of a different final fate, since both violate some of the classical energy conditions [4]. New candidates for a final fate of our universe are the big rip [5], sudden singularities [6], big brake [7], big freeze [8], inaccessible singularities [9], directional singularities [10], w -singularities [11], in braneworld models [12], among others.

In [13] we dealt with such singular scenarios from the point of view of geodesic completeness in modified theories of gravity. In this talk we do it following the approach of a dark energy fluid. More details can be found in [14].

2. Equations of state

Instead of restricting to a specific model, we propose a framework which may comprise most of them. With this aim in mind, we assume the validity of standard Friedmann equations,

$$H^2 = \rho, \quad \dot{\rho} + 3H(\rho + p) = 0, \quad (1)$$

assuming that pressure p has a power series in density ρ of the matter content around a value ρ_* , for which a qualitative change of behaviour is expected,

$$p = p_0(\rho - \rho_*)^{\alpha_0} + p_1(\rho - \rho_*)^{\alpha_1} + \dots, \quad (2)$$

where the exponents are real and ordered, $\alpha_0 < \alpha_1 < \dots$.

Combining cosmological equations with our power expansion, we get

$$\dot{\rho} = -3\sqrt{\rho_*} \left(\rho_* + \frac{3}{2}(\rho - \rho_*) + \frac{3}{8} \frac{(\rho - \rho_*)^2}{\rho_*} + \dots + p_0(\rho - \rho_*)^{\alpha_0} + p_1(\rho - \rho_*)^{\alpha_1} + \dots \right). \quad (3)$$

We may integrate it for ρ in powers of $(t_0 - t)$, where t_0 satisfies $\rho_* = \rho(t_0)$, if it exists,

$$\rho(t) = \rho_* + \rho_1(t_0 - t)^{\beta_1} + \rho_2(t_0 - t)^{\beta_2} + \dots.$$

In order to compare our results with previous ones in terms of expansions of the scale factor of the universe [15],

$$a(t) = a(t) = c_0|t - t_0|^{\eta_0} + c_1|t - t_1|^{\eta_1} + \dots, \quad (4)$$

we are to relate both series through Friedmann equation,

$$\rho = \frac{\eta_0^2}{(t - t_0)^2} + \frac{2c_1}{c_0} \eta_0(\eta_1 - \eta_0)(t_0 - t)^{\eta_1 - \eta_0 - 2} + \frac{2c_2}{c_0} \eta_0(\eta_2 - \eta_0)(t_0 - t)^{\eta_2 - \eta_0 - 2} + \dots,$$

These expansions cope with almost every cosmological model, except those with oscillatory behaviour as the ones in [16]. The results are consigned in table 1.

Table 1. Expansions of density and scale factor.

ρ_*	β_1	β_2	η_0	η_1	η_2
0	$(-\infty, -2)$	(β_1, ∞)	-	-	-
0	-2	$(-2, \infty)$	$\pm\sqrt{\rho_1}$	(η_0, ∞)	(η_1, ∞)
0	$(-2, 0) \cup (0, \infty)$	(β_1, ∞)	0	$\frac{\beta_1 + 2}{2}$	(η_1, ∞)
$\neq 0$	$(0, 1)$	(β_1, ∞)	0	1	$(1, 2)$
$\neq 0$	1	$(1, \infty)$	0	1	$[2, \infty)$
$\neq 0$	$(1, \infty)$	(β_1, ∞)	0	1	2

3. Classification of singularities

We follow the popular classification of singularities in [4]:

- Big bang / crunch: zero a , divergent H , density and pressure.
- Type I: “Big rip”: divergent a . [5]
- Type II: “Sudden”: finite a , H , density, divergent \dot{H} and pressure. [6]
- Type III: “Big freeze”: finite a , divergent H , density and pressure. [8]
- Type IV: “Big brake”: finite a , H , \dot{H} , density, pressure, divergent higher derivatives. [7]

In [17] a detailed analysis of the strength of these singularities is performed, making use of the concepts of geodesic completeness [18] and the definitions of strong singularities due to Ellis, Schmidt, Tipler and Królak [19]. Sudden and big brake singularities cannot be considered as such [20], since the cosmological spacetime may be continued beyond the singular event. And particles travelling at the speed of light do not experience the big rip, since they need infinite proper time to reach it [17]. These results may be checked in table 2 in terms of the exponents of the expansion of the density. Conformal diagrams for these models can be found in [21].

Table 2. Singularities and density expansions.

ρ_*	β_1	Tipler	Królak	N.O.T.
No	$(-\infty, -2)$	Strong	Strong	Big crunch/rip
No	$(-2, 0)$	Weak	Strong	III
No	$(0, 2)$	Weak	Weak	II
No	$[2, \infty)$	Weak	Weak	IV
Yes	$(0, 1)$	Weak	Weak	II
Yes	$[1, \infty)$	Weak	Weak	IV

4. Singularities and equations of state

Now we have all the ingredients for checking the formation of singularities in cosmological models with an equation of state of the form (2). We may integrate (3) in terms of power series of time and get the relevant exponents and coefficients. Comparing them with the results in table 2, we are ready to obtain the types of singularities which may appear in these models. The results may be found in table 3.

Table 3. Singularities and equation of state.

ρ_*	p_0	α_0	β_1	N.O.T.
0	Any	$(-\infty, 0)$	$2/(1 - 2\alpha_0)$	II
0	Any	$[0, 1/2)$	$2/(1 - 2\alpha_0)$	IV
Yes	Any	$(-\infty, 0)$	$(1 - \alpha_0)^{-1}$	II
Yes	$\neq -\rho_*$	0	1	IV
Yes	$-\rho_*$	0	$1/(1 - \alpha_1)$	IV
Yes	Any	$(0, \infty)$	1	IV

5. Conclusions

In this talk a detailed classification of the future behaviour of FLRW cosmologies in terms of the equation of state is provided. It depends at most on the exponents of the first two terms of a power expansion of the equation of state. The scheme provides an easy route to conclude the sort of singular behaviour.

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