Rashba spin–orbit coupling effect on the quantum Hall magnetoresistivity

M.A. Hidalgo [a], R. Cangas [b,*]

[a] Departamento de Física, Universidad de Alcalá, Alcalá de Henares (Madrid), Spain
[b] Departamento de Física, Escuela Universitaria de Ingeniería Técnica Industrial, Universidad Politécnica de Madrid, Spain.

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ABSTRACT

In this paper we analyze the influence of the Rashba spin-orbit coupling on the quantum Hall magnetoresistivity in a two-dimensional electron system (2DES). The study is based on an analytical model for the integer quantum Hall effect (IQHE) and the Shubnikov-de Haas (SdH) phenomena. This model shows the behaviour of the Hall magnetoresistivity when the Rashba parameter is varied, and reproduces Hall plateaux of a 2DES confined in a III–V heterostructure. We also discuss the Rashba and Zeeman competition and its effect on the width of the Hall magnetoresistivity plateaux.

The quantum Hall effect is one of the most amazing and interesting phenomena in the condensed matter physics discovered at the end of the past century. The integer quantum Hall effect (IQHE) is characterized by the appearance of quantized plateaux in multiples of $\nu e^2/h$ in the non-diagonal magnetoconductivity σ_{xy} (h is the Planck constant, e the electron charge and $\nu=1,2,3,\ldots$ an integer), and vanishing values in the diagonal magnetoconductivity σ_{xy} observed at the same magnetic field ranges [1]. The magnetoconductivities are measured on a two-dimensional electron system (2DES) at very low temperatures. Up to now the physical realizations of a 2DES are provided in semiconductor heterostructures, MOSFETs devices, and more recently in graphene layers [2]. From the theoretical point of view several attempts to understand the integer quantum Hall effect (IQHE) have been published. The most accepted is based on the 'gendanken' experiment thought up by Laughlin [3], where the 2DES localized states due to ionized impurities and defects play a crucial role to explain the plateaux. However, experimental evidences show that higher electron mobility (materials with less defects and impurities) provides better plateaux precision [4]. The appearance of even and odd plateaux (ν even or odd) in the IQHE is due to the breakdown of the spin degeneration of every Landau level (LL) as a consequence of the Zeeman effect, creating spin-polarized carriers with parallel and anti-parallel orientations with respect to the direction of the applied magnetic field and splitting the LLs. Even without any external magnetic field applied, the carriers of the 2DES confined in the inversion layer of an heterostructure can be spin-polarized by the spin–orbit magnetic field created by the internal built-in electric field, that is formed by the structure inversion asymmetry (SIA) of the heterostructure device. The carriers have parallel and anti-parallel orientations with respect to the spin–orbit magnetic field. In 1989 Das et al. [5] obtained an evidence of spin splitting at zero magnetic field in InGaAs/InAlAs heterostructures. The SIA electric field is normal to the 2DES confined in the inversion layer or quantum well, and the spin splitting provided by this is given by the expression [6]:

$$\Delta E_{so}^{SIA} = 2\,\alpha\,k \tag{1}$$

where $\vec{k}=(k_x,k_y)$ is the 2DES wave vector, and the Rashba parameter, that depends on the electric field asymmetry of the heterostructure, α has values that varies between 2×10^{-12} eV m and 5×10^{-11} eV m for InAs [7,8] and is of the order of 1.5×10^{-13} eV m for AlGaAs [9]. In 1990 Datta et al. [10] proposed a spin-polarized field effect transistor based in the interference of two spin-polarized currents, controlling the polarization of the currents with the gate electric field. Yang and Chang [11] have found that Rashba spin splitting energy has a nonmonotonic and anisotropic behaviour with the momentum, in contrast to the widely used isotropic linear model. In our model the 2DES is confined in the $In_{0.53}Ga_{0.47}As$ layer of the device used in Ref. [12], and we consider a linear behaviour with the momentum of the Rashba spin splitting.

On the other hand, zinc-blend semiconductors have bulk inversion asymmetry (BIA), varying their lattice potential with the crystal directions and therefore the local electric field. Due to this there exist an intrinsic BIA spin–orbit effect. The BIA effect is stronger than the SIA effect in the GaAs/AlGaAs heterostructure

2DES, and the values measured of spin–split energy in this alloy are of the order of 20 μeV at the Fermi level [13]. In quantum well and heterostructure devices made with InGaAs/InAlAs systems the SIA effect has a dominant effect on BIA effect , being SIA spin–orbit split energy at least two times greater than BIA spin–orbit split energy [14]. Measured spin–split energies are of the order of meV at Fermi level [15]. Ref. [16] shows a theoretical analysis of the interplay between Zeeman spin splitting, Rashba and Dresselhaus spin–orbit interaction.

In a previous paper [17] we have presented an analysis of the diagonal magnetoresistivity in a 2DES affected only by Rashba spin–orbit coupling, and this paper treats its effect on the Hall magnetoresistivity at larger magnetic fields. The model used is a simple semi-classical theory of a 2DES confined in a quantum well under the application of external electric and magnetic fields [18], and with the presence of Rashba spin–orbit coupling [17]. In this theory we assume that variations in the external magnetic field produces a fluctuation in the electron density, keeping constant the chemical potential, which is fixed by the environment. In all realization of a confined 2DES there is a reservoir of charge surrounding the system that provides electrons to it. Then the chemical potential must be established by the whole 3D structure. A similar scenario was intended at the beginning of the discovery of the quantum Hall effect [19], although considering a capture mechanism of electrons by the impurities.

At zero external magnetic field we consider the 2DES as a two-dimensional non-interacting electron gas under the effective mass approximation, perturbed by impurities, defects and spin–orbit coupling. Then, the energy of each electron can be approached by $E(k) = \hbar^2 k^2/(2m+U) \pm \Delta E_{SO}/2$, where m is the electron effective mass, U takes into account the electrostatic interaction with impurities and defects, and ΔE_{SO} is the spin–orbit split energy caused by the SIA effect (Eq. (1)). On the other hand, when an external magnetic field B is applied normal to the 2DES, and assuming no spin–orbit coupling effects, the energy of the system is discretized in Landau levels (LL) with values $E^{\pm}_{N_{LL}} = (N_{N_{LL}} + 1/2)\,\hbar\omega \pm 1/2\, g\mu\, B$, where the last term (Zeeman term) correspond to spin ↑↓ orientations, $N_{LL} = 0, 1, 2, 3\ldots$, $\omega = e\,B/m$ the cyclotron frequency, μ the Bohr magneton and g is the effective g-factor, that can be as high as 15 in InGaAs alloys [20]. In a 2DES confined in a heterostructure device immersed in an external magnetic field, and taking into account the SIA and Zeeman effects, the energy of the carriers is determined by the expression [6]

$$E^s_{N_L} = \hbar\omega \left[N_L + s\frac{1}{2}\sqrt{\left(1-|g|\frac{m}{2m_0}\right)^2 + \frac{\gamma}{B}\,N_L} \right] \quad (2)$$

with $s = \pm 1$ for $N_L = 1, 2, 3, \ldots$, and $s = +1$ for $N_L = 0$ (the index s refers to spin ↑↓ orientations), where $\gamma = 8\alpha^2 m^2/\hbar^3 e$, and m_0 is the free electron mass. Rashba and Zeeman effects break the spin degeneration. We analyze the entire two-dimensional electron gas as two 2DES with different spin orientations (parallel and anti-parallel to the magnetic field) with quantized energies $E^s_{N_L}$. When the applied magnetic field increases, the energy levels $E^s_{N_L}$ move to the Fermi level (E_F), and the conduction occurs when each energy level crossesE_F, providing beating patterns in the Subnikov-de Haas oscillations and in the Hall magnetoresitivity, with nodes at the same values of magnetic field (Fig. 1). Fig. 1 is reproduced with Eqs. (5a) and (5b) and the values showed are obtained from the experimental data given in Ref. [12], with a carrier concentration of $2.0 \times 10^{16}\ \mathrm{m}^{-2}$ at zero magnetic field, an effective mass of $0.05m_0$, $\alpha = 0.72 \times 10^{-11}$ eV m, $g = -4$, and a relaxation time of $\tau = 1.0 \times 10^{-12}$ s [21]. The maximum/minimum (nodes) oscillations are due to the coincidence/no coincidence of different energy levels $E^s_{N_L}$ at the Fermi level. The nodes occur with no coincidence condition of $E^+_{N_L}$ and $E^-_{N_L}$ ($N_L \neq N_L'$) levels. The oscillations of the magnetoresistivity at low magnetic field can be used to determine the spin–split energy, and hence the Rashba parameter [12,17] assuming no BIA spin–orbit effect.

If we compare Eq. (2) with the conventional spin-up and spin-down energy states associated with N_L Landau level number, this corresponds to $E^+_{N_L}$ and $E^-_{N_L+1}$ states, i.e. $\Delta E_{spin} = \left| E^+_{N_L} - E^-_{N_L+1} \right|$ [22]. In the absence of Rashba effect, Eq. (2) reproduces well-known LL energy spectrum. In the limit of large magnetic fields the Zeeman term dominates the spin splitting obtaining $\Delta E_{spin} = g\mu\, B$. In the opposite limit when $B \to 0$, we obtain $\Delta E_{spin} = 2\alpha\, k_F$ at the Fermi energy, where $k_F = \sqrt{2\pi\, n}$ is the Fermi wave vector and n the equilibrium carrier concentration. At intermediate magnetic fields also exists a competition between Rashba and Zeeman effects which occurs by the coincidence of levels $E^+_{N_L}$ and $E^-_{N_L+1}$, i.e. when $E^+_{N_L} = E^-_{N_L+1}$ at a fixed value of the magnetic field. In this point $\Delta E_{spin} = 0$. At magnetic fields below this field the Rashba effect dominates, and for fields above, the Zeeman effect does. When the coincidence occurs at Fermi level, in very clean samples, a maximum in magnetoresitivity (and also in magnetoconductivity) should be observed [12,23]. Fig. 2 shows a fan of energy levels of the 2DES. Continuous line corresponds to conventional Landau levels with spin degeneration, and dash lines correspond to energy levels obtained from Eq. (2). In the figure, we have used higher values of Rashba parameter ($\alpha = 3 \times 10^{-11}$ eV m) and the gyromagnetic factor ($g = -8$) in order to show the Rashba–Zeeman competition $E^+_{N_L} = E^-_{N_L+1} \approx E_F$ with more detail, that occurs near to 14.5 T ($E_F = 0.096$ eV).

The magnetoconductivities are obtained relating the current carrier density for a 2DES to the applied electric and magnetic fields, taking into account the presence of two populations of electrons in the 2DEG with two different spin orientations. Then, the general expression for the current density is given by

$$\vec{j}(\vec{r}, t) = \vec{j}_+ + \vec{j}_- = e\sum_s \int_{-\infty}^{\infty} \vec{v}_s\, f(E)\, D(E)_s\, dE \quad (3)$$

where $\vec{v}_s$ is the carrier velocity, E the electron energy, $D(E)_s$ the density of states for each spin orientation [12,24], and f is the modified distribution function due to the electric and magnetic fields [25]. We assume that the density of states is the *ad hoc* function $D(E)_s = (eB/h)\sum_s \sum_{N_L} \left[(\pi/2)\,\Gamma^2_{N_L s}\right]^{-1/2} \exp\left\{-2\,(E - E^2_{N_L s})/\Gamma^2_{N_L s}\right\}$, and with gaussian level broadening $\Gamma_{N_L s}$ of the energy

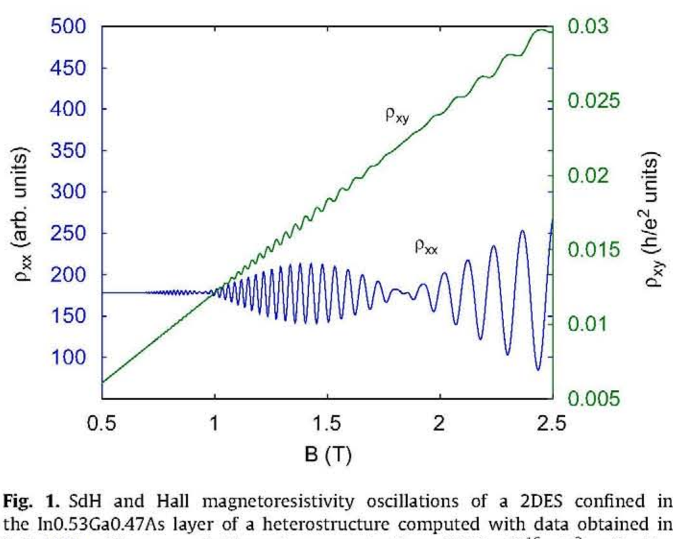

Fig. 1. SdH and Hall magnetoresistivity oscillations of a 2DES confined in the In0.53Ga0.47As layer of a heterostructure computed with data obtained in Ref. [12], with a zero field carrier concentration of $2.0 \times 10^{16}\ \mathrm{m}^{-2}$, a Rashba parameter $\alpha = 0.72 \times 10^{-11}$ eV m and a g-factor of -4.

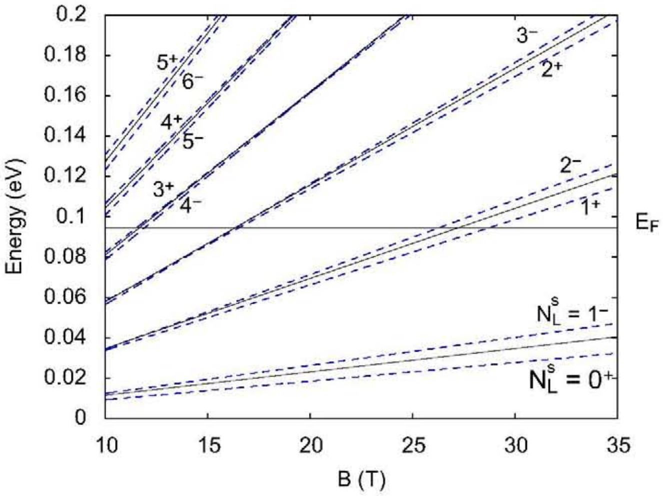

Fig. 2. Plot of the energy levels (from N_L=0–6) of the 2DES system. Continuous line are the conventional Landau levels without spin split. Dash line corresponds to $E^{\pm}_{N_L}$ levels. It shows the Rashba–Zeeman competition where the adjoining levels intersect. The used values of Rashba parameter and g-factor are $\alpha = 3 \times 10^{-11}$ eV m and $g = -8$, respectively, and $m = 0.05m_0$. The Rashba–Zeeman competition occurs near 14.5 T at Fermi level.

levels that strongly depends on the range of the scattering potentials [24]. For short range scatters $\Gamma^2_{N_L s}$ depends on the strength of the magnetic field. The broadening due to long range potentials is proportional to fluctuations of the local potential energy $(V(r) - \langle V(r) \rangle)^2$, and can be considered negligible in δ-doped samples where the impurities are far from the 2DES. Then, we use the expression given by Ando et al. [24] for short range scatters $\Gamma_{N_L s} = \Gamma_0 + \kappa\sqrt{(2\hbar^2/\pi)\,(\omega/\tau)}$, where Γ_0 and κ are fitting parameters, and τ is the relaxation time without applied magnetic field.

Hence, we assume two currents with different spins (parallel and anti-parallel to the magnetic field). Taking into account the relationship $\vec{j} = [\sigma]\,\vec{E}$, where $\vec{E}$ is the electric field and $[\sigma]$ the magnetoconductivity tensor, with terms $\sigma_{xx} = \sigma_{yy} = (e^2 N\tau/m)/(1+(\omega\tau)^2)$, $\sigma_{xy} = -\sigma_{yx} = (e^2 n\omega\tau^2/m)/(1+(\omega\tau)^2)$. n is the whole equilibrium carrier concentration, and N the carrier concentration at Fermi level, given by

$$n = n_+ + n_- = \sum_s \int_{-\infty}^{\infty} f_0(E) D(E)_s \, dE \tag{4a}$$

$$N = N_+ + N_- = \sum_s \int_{-\infty}^{\infty} D(E)_s E\left(-\frac{\partial f_0}{\partial E}\right) dE \tag{4b}$$

where f_0 is the Fermi–Dirac distribution function. Fig. 3(a) and (b) shows the oscillations of each equilibrium concentration of the two carrier's populations as a function of the magnetic field, and the sum of both, Eq. (4a), with the data used in Ref. [12], and as obtained from the model. Fig. 3(a) represents n in all the magnetic field range and Fig. 3(b) at low magnetic fields. As it is seen, in the limit of zero magnetic field, different values for n_+ and n_- appears as a consequence of the Rashba spin–orbit effect. Therefore, this difference of the carriers concentration also allows to determine the Rashba parameter. Of course, the two kind of spin carriers are presented in samples where more than one sub-band is occupied [26,27] and with non-parabolic energy dispersion. Taking into account that tensor of the resistivity is obtained from $[\rho] = [\sigma]^{-1}$,

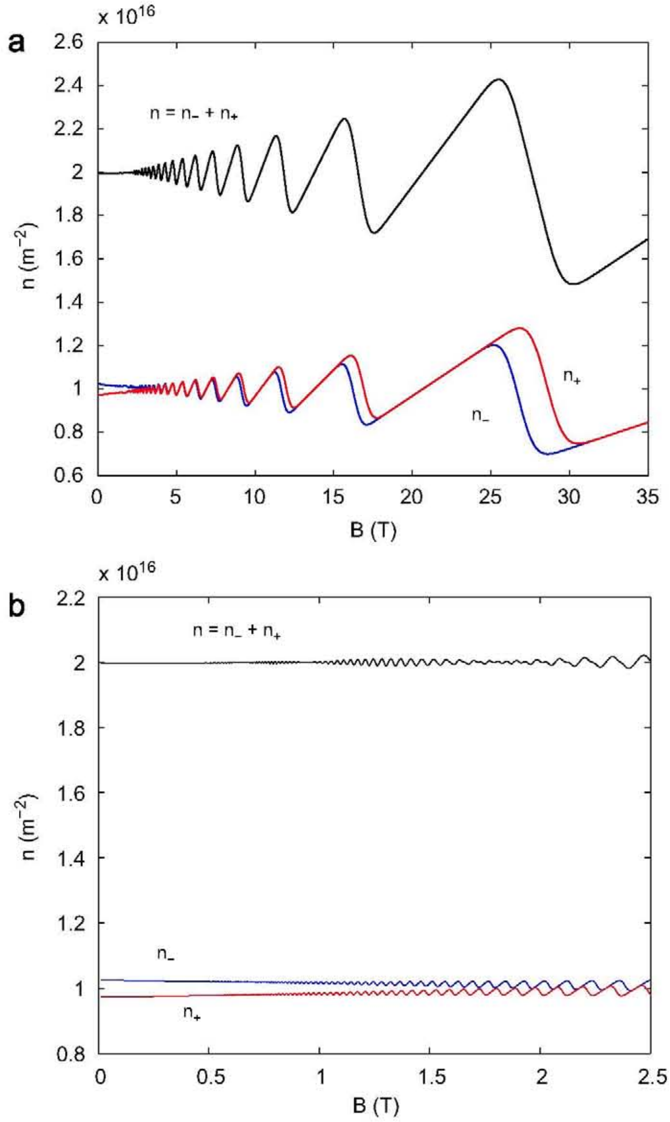

Fig. 3. (a) Oscillations of the bulk carrier concentrations n_- and n_+, and the sum of both (Eq. (5)), as a function of the magnetic field. The figure was obtained with the data used in Fig. 1[12]. (b) Detail of the carrier concentrations of Fig. 3a at low magnetic field.

the magnetoresistivities diagonal and Hall are

$$\rho_{xx} = \rho_{yy} = \sigma_{xx}/(\sigma^2_{xx} + \sigma^2_{xy}) \tag{5a}$$

$$\rho_{xy} = \rho_{Hall} = -\rho_{yx} = -\sigma_{xy}/(\sigma^2_{xx} + \sigma^2_{xy}) \tag{5b}$$

As we mentioned before, the whole 2DES can be viewed like two 2DES with different spin orientation, separated in energy by $\Delta E^{SIA}_{so} = 2\alpha k_F$. Then, from a naïve point of view assuming a parabolic energy dispersion, and taking into account the density of states of a 2DES at zero magnetic field, $D(E) = m/2\pi\hbar^2$ for each electron spin system, the Rashba parameter can be deduced from the expression (4a)

$$\alpha = \frac{\Delta n_0}{2D(E)k_F} \tag{6}$$

where $\Delta n_0 = n_{0-} - n_{0+}$ is the difference in carrier concentrations at zero field between both possible spin orientations. Thus, for example, the value obtained with this procedure with the data used in Fig. 3b is $\alpha = 0.77 \times 10^{-12}$ eV m.

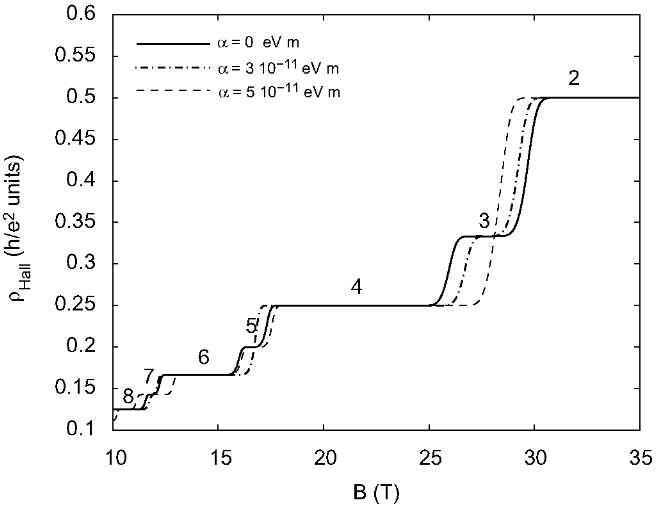

Fig. 4. Quantum Hall magnetoresistivity at three different Rashba parameters and an effective g-factor of -8. Continuous line correspond to α=0. For $\alpha=3\times10^{-11}$ eV m (dash-dot line) and $\alpha=5\times10^{-11}$ eV m (dot line) the ν=5 and ν=3 plateaux vanishes, respectively. This occurs in both cases due to the Rashba–Zeeman competition at these values of the magnetic field.

Fig. 4 shows the Hall magnetoresistivity at three different values of Rashba parameter and considers an effective g-factor of -8. In order to resolve even and odd plateaux (even and odd ν, respectively) we use small gaussian width of energy levels [17]. When α=0 the spin degeneration is broken only by the Zeeman effect, and the width of the plateaux grows with the magnetic field. The odd plateaux became wider when B grows. When $\alpha\neq0$ the width of the plateaux varies due to the effect of the Rashba spin–orbit, producing the disappearance of the odd plateaux in the magnetic field regions where the Rashba–Zeeman competition occurs, i.e. when $E^{+}_{N_L}=E^{-}_{N_L+1}$ near to Fermi level. For $\alpha=3.0\times10^{-11}$ eV m this happens near to 14.5 T, vanishing the ν=5 plateaux, and for $\alpha=5.0\times10^{-11}$ eV m Rashba–Zeeman competition occurs at values close to 25 T, vanishing the ν=3 plateaux. As expected, the value of the Hall magnetoresistivity is not affected by the SIA spin–orbit effect ($\rho_{xy}=h/(\nu e^2)=25812.807/\nu\ \Omega$, ν=1,2,3, …). In fact, the quantum Hall effect is reproduced in dirty samples where local high electric fields due to impurities exist.

In summary, we have studied the Hall magnetoresistivity ρ_{xy} of a 2DES with a Rashba spin–orbit coupling in a perpendicular magnetic field, where we have assumed the system composed by two kinds of carriers (electrons with parallel and anti-parallel spins). At low magnetic fields ρ_{xy} also reflects the beating pattern with nodes at values of magnetic fields where the nodes of the SdH appear. At higher fields ρ_{xy} have values of $h/(\nu e^2)$, and not depends on the Rashba parameter. The whole spin degeneration only affects the width of the plateaux, vanishing the odd plateaux in the region of the magnetic field where Rashba effect is almost equal to Zeeman effect.

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