THE HYDRODYNAMIC EFFICIENCY OF LASER-TARGET ACCELERATION

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Abstract—The acceleration of a thin foil using a laser pulse is studied. It is shown that the acceleration efficiency η_H is heavily dependent on the behaviour of the corona ejected by the foil: there is no universal relation $\eta_H(\Delta M/M_0)$, M_0 and ΔM being initial foil mass and ablated mass, respectively. Known results on the coronal flow are used to check the theory against experimental data available in the literature; effects due to both a non-planar corona, and the time-dependence of the laser irradiance, are considered. The agreement with experiments is substantially better than that for previous analyses. Acceleration of thin spherical shells is also discussed.

1. INTRODUCTION

ONE OF the most important parameters in laser-fusion is the fraction η of absorbed laser energy E_a that goes into the target. In experiments on acceleration of thin foils (or thin spherical shells), most of the final energy within the remains of the target may be kinetic energy. Then η is the hydrodynamic efficiency, $\eta_H = Mu^2/2E_a$, where M is the unablated mass and u its velocity, assumed uniform throughout it. Usually (McCALL, 1983; RIPIN *et al.*, 1980; DUDERSTADT and MOSES, 1982) some simple rocket model is used to calculate η_H. The standard formula for η_H is

$$\eta_H = \frac{(\ln(1-\Delta M/M_0))^2(1-\Delta M/M_0)}{\Delta M/M_0} \equiv \bar{\eta}\left(\frac{\Delta M}{M_0}\right), \tag{1}$$

M_0 and ΔM being initial and ablated mass. Some corrections to (1) have been discussed in the past (FABBRO, 1982; MAX *et al.*, 1983).

Here we reconsider the calculation of η_H. We show that when known results on the expanding plasma (in the corona outside the ablation surface) are taken into account, η_H differs substantially from previously published values. In the paper we also discuss time-dependent and non-planar effects on the hydrodynamic efficiency. On the whole, the agreement with experimental data is greatly improved.

In Section 2 we review the analysis of planar coronae. In Section 3 both steady and unsteady planar-target acceleration are considered. Section 4 studies non-planar effects on acceleration, making use of known results on spherical coronae. Section 5 collects conclusions from all previous sections to compare theory with experimental data on η_H available in the literature. Spherical shells are discussed in the Appendix.

2. THE ANALYSIS OF A PLANAR CORONA

Let laser-light be incident on a solid foil on the left, and the beam cross-section be large enough to allow considering the problem as one-dimensional (Fig. 1). Under broad conditions the equations for the quasineutral plasma ablated from the target

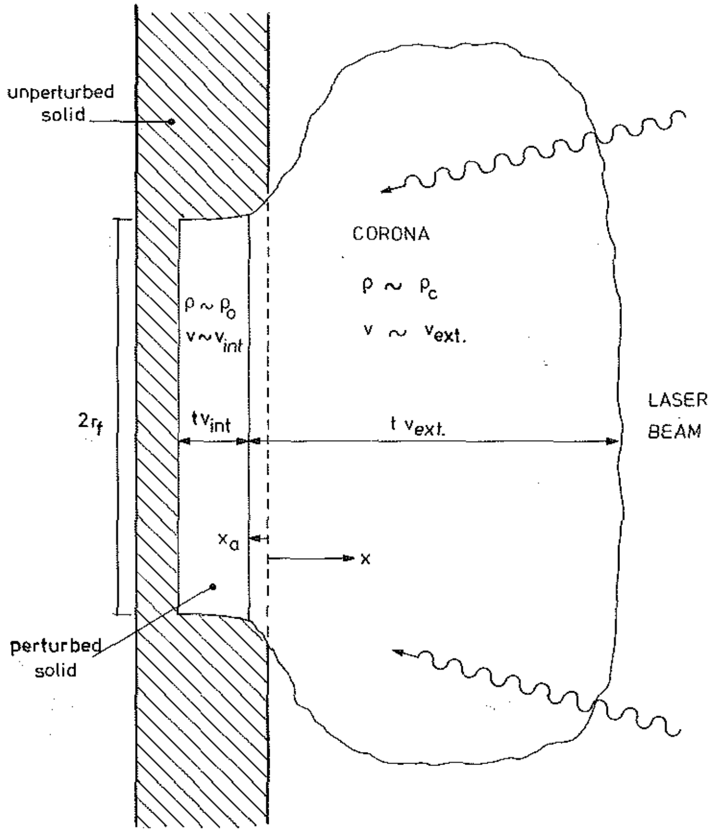

FIG. 1.—Interaction between a laser-beam incident from the right and a solid foil, before the perturbation reaches its back: r_f is the spot radius, ρ_c and ρ_0 are critical and unperturbed solid densities, respectively; x_a locates the ablation surface.

are conservation laws for mass, momentum, and energy, in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho v = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\rho v + \frac{\partial}{\partial x}(\rho v^2 + P) = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\rho\left(e_{in} + \frac{1}{2}v^2\right) + \frac{\partial}{\partial x}\left(\rho v\left(e_{in} + \frac{1}{2}v^2\right) + vP + q + I_+ - I_-\right) = 0, \tag{4}$$

and the ion entropy equation

$$\rho T_i\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)s_i = \frac{3}{2}n\frac{T_e - T_i}{t_{e_i}}. \tag{5}$$

The mass density above is $\rho = \bar{m}n \equiv m_i n/Z_i$, where n is electron density. The pressure is $P = nT_e + nT_i/Z_i$; the specific internal energy and ion entropy are $e_{in} = 3T_e/2\bar{m} + 3T_i/2\bar{m}Z_i$, and $s_i = m_i^{-1}\ln(T_i^{3/2}/n) + \text{const}$. The heat flux is $q = \min(\bar{K}T_e^{5/2}|\mathrm{d}T_e/\mathrm{d}x|, fnT_e^{3/2}m_e^{-1/2}) \times \operatorname{sign}(-\mathrm{d}T_e/\mathrm{d}x)$, $\bar{K}T_e^{5/2}$ being Spitzer's conductivity (SPITZER, 1967) and f a flux-saturation factor (DUDERSTADT and MOSES, 1982); the ion-electron energy relaxation time $t_{e_i} \propto m_i\bar{K}T_e^{3/2}/n$ is also given by Spitzer. The incident and reflected irradiances I_- and I_+ change according to $\partial I_\mp/\partial x = \pm\kappa I_\mp(x > x_c)$ where κ is the absorption coefficient due to inverse bremsstrahlung (JOHNSTON and DAWSON, 1973); also $I_+(x_c^+) = (1-\alpha)I_-(x_c^+)$, α being an anomalous absorption fraction (FORSLUND *et al.*, 1977; ESTABROOK and KRUER, 1978) and x_c such that $n(x_c) = n_c \equiv$ critical density. As $x \to +\infty$, one has $I_- \to I_L(t)$ (the laser-pulse irradiance, characterized by a peak value I_m and a full-width at half-maximum τ).

Two facts simplify the problem of solving analytically system (2)–(5) to yield n, v, T_e, and T_i as functions of x and t. Usually the density in the perturbed solid (a few times ρ_0) is large compared with the characteristic density (ρ_c) in the corona, which is the region to the right of the (ablation) surface $x = x_a(t)$ where $v \simeq 0$, $\partial P/\partial x \simeq 0$. For instance, $\rho_0/\rho_c \simeq 50$ for solid D–T and 1.06 μm light; ρ_0/ρ_c is much larger for targets of high atomic number, for which $Z_i \gg 1$. In analysing the corona we might tentatively set $\rho_0/\rho_c \to \infty$. This would require that $\rho \to \infty$ as $x \to x_a(t)$, leading to T_e, $T_i \to 0$ there, because mass and momentum within the corona, and thus surface mass ablation rate $\dot{m}_s$ and ablation pressure P_a, must remain finite. The fact that makes possible such behaviour (v, T_e, T_i vanishing at a surface which is at finite distance from a region where energy is deposited) is the nonlinear character of plasma heat conduction. Nonlinear heat waves in the absence of motion have been studied extensively (ZEL'DOVICH and RAIZER, 1967).

For a thick foil or a short pulse, the perturbed mass per unit surface in (the irradiated part of) the foil is of order $tv_{\text{int}}\rho_0$ (Fig. 1); v_{int} and v_{ext} are characteristic velocities for corona and perturbed solid. Conservation of momentum yields

$$v_{\text{int}} \times tv_{\text{int}}\rho_0 \sim v_{\text{ext}} \times tv_{\text{ext}}\rho_c \sim v_{\text{ext}} \times \Delta M_s, \tag{6}$$

where ΔM_s is the surface ablated mass. Then

$$|\dot{x}_a| \sim v_{\text{int}} \ll v_{\text{ext}} \to x_a(t) \simeq x_a(0) = 0. \tag{7}$$

Equation (7) completes the uncoupling of corona and target; all is needed to analyse the corona is that it starts at $x = 0$, and that is hot and rarefied when compared with the target.

For the thin foils of interest here we have $tv_{\text{int}}\rho_0 > M_s$ (surface mass in the target), so that (6) should read

$$v_{\text{int}} \times M_s \sim v_{\text{ext}} \times \Delta M_s; \tag{8}$$

thus condition (7) requires $\Delta M_s/M_s$ to be small (Section 3).

In writing down dimensionless results from the analysis, it proves convenient to introduce a characteristic speed

$$U \equiv (n_c \tau / \bar{m}^{5/2} \bar{K})^{1/3}$$

$$\simeq 0.915 \times 10^7 \frac{\text{cm}}{\text{s}} \left(\frac{2Z_i}{A_i}\right)^{5/6} \left(\frac{1.06\,\mu\text{m}}{\lambda}\right)^{2/3} \left(\frac{\tau}{1\text{ ns}} \frac{Z_i \ln \Lambda}{100\,\varepsilon \delta_T}\right)^{1/3} \qquad (9)$$

which substitutes the awkward factor $\bar{K}$ in the set of dimensional parameters available. Quantities of interest, such as peak values of $P_a/\rho_c U^2$, $\dot{m}_s/\rho_c U$, are found to be functions of five dimensionless numbers: α, Z_i, $f(\bar{m}/m_e)^{1/2}$, $\hat{I}_m \equiv I_m/\rho_c U^3$, and $\hat{U} \equiv \bar{m}U/m_e c$. [$\hat{I}_m$ characterizes the steepness of the pulse, $\hat{U}$ bremsstrahlung absorption; note that $\kappa \propto n^2/\bar{K} m_e c n_c (1 - n/n_c)^{1/2} T_e^{3/2}$.] Fluid variables depend, in addition, on t/τ and $x/U\tau$. For specific power-law pulses, $I_L \propto t$ (ANISIMOV, 1970; BARRERO and SANMARTIN, 1977), $I_L \propto t^{3/2}$ (NICOLAS, 1984), the flow may be self-similar and has been analysed in detail in the past (SANMARTIN and BARRERO, 1978a, b; BARRERO and SANMARTIN, 1980; RAMIS and SANMARTIN, 1983; NICOLAS, 1984).

3. PLANAR TARGET ACCELERATION

If M_s is the mass per unit area remaining in the target at any given time, and u its velocity, then (see Appendix A)

$$\mathrm{d}M_s/\mathrm{d}t = -\dot{m}_s \qquad (10)$$

$$M_s \mathrm{d}u/\mathrm{d}t = -P_a. \qquad (11)$$

The target experiences a leftward acceleration. To obtain $\eta_H \equiv M_s u^2 / 2\int_0^t I_a(t')\mathrm{d}t'$ we solve (10) and (11) for $t(\Delta M_s)$, $u(\Delta M_s)$ using coronal results for $\dot{m}_s$ and P_a; here I_a is absorbed intensity, and $\Delta M_s \equiv M_{s0} - M_s$.

First neglect inverse bremsstrahlung ($\hat{U} \to 0$) and consider the linear pulse $I_a = \alpha I_L \equiv I_{am} t/\tau$. In the resulting self-similar motion one gets

$$\dot{m}_s = \dot{m}_{sm}(t/\tau)^{1/3},\; P_a = P_{am}(t/\tau)^{2/3}.$$

One may then integrate (10) and (11) to obtain

$$\eta_H = \frac{P_{am}^2}{2\dot{m}_{sm} I_{am}} \times \bar{\eta}_1(\Delta M_s/M_{s0}) \qquad (12)$$

where

$$\bar{\eta}_1(\Delta) \equiv \frac{3}{2} \frac{1-\Delta}{\Delta^{3/2}} \left(2 \tan^{-1} \Delta^{1/4} + \ln \frac{1+\Delta^{1/4}}{1-\Delta^{1/4}} - 4\Delta^{1/4}\right)^2.$$

The function $\bar{\eta}_1(\Delta)$ is close to that appearing in (1)

$$\bar{\eta}(\Delta) \equiv \frac{1-\Delta}{\Delta} (\ln(1-\Delta))^2;$$

the ratio $\bar{\eta}_1/\bar{\eta}$ is 0.96 for $\Delta \to 0$, 1 at $\Delta \simeq 0.3$ and 1.17 at $\Delta = 0.9$.

Note that $P_a^2(t)/2\dot{m}_s(t)I_a(t) = \text{const} = P_{am}^2/2\dot{m}_{sm}I_{am}$. The ratio $P_a^2/2\dot{m}_sI_a$ depends on Z_i, $f(\bar{m}/m_e)^{1/2}$, and $\hat{I}_{am}$, though not on α. For $\hat{I}_{am}$ small the corona separates into a quasisteady, thin (deflagration) layer where absorption and conduction occur, and a large region of isentropic expansion. For classical conduction, i.e. $f(\bar{m}/m_e)^{1/2}$ large, one gets $P_a^2/2\dot{m}_sI_a = 16/25 = 0.64$ independently of Z_i (though both P_a and $\dot{m}_s$ *do* depend on Z_i) (SANMARTIN and BARRERO, 1978a). For $Z_i \gg 1$, flux-saturation does not modify this result: $P_a^2/2\dot{m}_sI_a \simeq 0.64$ for $f > 0.03$ (RAMIS and SANMARTIN, 1983).

Results for $\hat{I}_{am} \gg 1$ and Z_i arbitrary, and $Z_i \gg 1$ and $\hat{I}_{am}$ arbitrary, with classical conduction, are known (SANMARTIN and BARRERO, 1978b; BARRERO and SANMARTIN, 1980). For $\hat{I}_{am}$ and Z_i large we have $P_a^2/2\dot{m}_sI_a \simeq 0.265$. Flux-saturation yields a similar result if $f \simeq 0.6$, and $P_a^2/2\dot{m}_sI_a \simeq 0.09$ if $f \simeq 0.03$.

When both conduction and inverse bremsstrahlung are considered the flow can not be self-similar. However for small $\hat{I}_{am}$ conduction only counts in the deflagration layer, which is quasisteady; to make the outside flow self-similar with bremsstrahlung absorption included [$\hat{U} = 0(1)$], we take $I_L = I_m(t/\tau)^{3/2}$ (NICOLAS, 1984). Then one has

$$\dot{m}_s = \dot{m}_{sm}(t/\tau)^{1/2}, \quad P_a = P_{am}t/\tau,$$

and integration of (10) and (11) yields

$$\eta_H = \frac{P_{am}^2}{2\dot{m}_{sm}I_{am}} \times \bar{\eta}_2(\Delta M_s/M_{s0}) \tag{13}$$

where $P_{am}^2/2\dot{m}_{sm}I_{am} = P_a^2(t)/2\dot{m}_s(t)I_a(t)$, and

$$\bar{\eta}_2(\Delta) \equiv \frac{5}{3}\frac{1-\Delta}{\Delta^{5/3}}\left(3^{1/2}\tan^{-1}\frac{3^{1/2}\Delta^{1/3}}{2+\Delta^{1/3}} + \frac{\ln(1+\Delta^{1/3}+\Delta^{2/3})^{1/2}}{1-\Delta^{1/3}} - 3\Delta^{1/3}\right)^2$$

and $I_{am} = A_TI_m$, A_T being total absorption; $\bar{\eta}_2$ is always very close to $\bar{\eta}_1$: $\bar{\eta}_2/\bar{\eta}_1 \simeq 0.98$ for $\Delta \to 0$, and 1.03 at $\Delta = 0.9$. For Z_i and $f(\bar{m}/m_e)^{1/2}$ large, ($\hat{I}_{am}$ small), and $\hat{U} = 0(1)$ we have $P_a^2/2\dot{m}_sI_a \simeq 0.314$ independently of α; for $\hat{U} \to 0$ we again have $P_a^2/2\dot{m}_sI_a =$ 0.64. Note that inverse bremsstrahlung is negligible for $\hat{I}_{am}$ large.

In the past, authors always considered $I_a = \text{const}$, assuming P_a and $\dot{m}_s$ constant too. Then equations (10) and (11) give

$$\eta_H = \frac{P_a^2}{2\dot{m}_sI_a} \times \bar{\eta}\left(\frac{\Delta M_s}{M_{s0}}\right). \tag{14}$$

Either a cold-rocket model ($P_a^2/2\dot{m}_sI_a = 1$), or a coronal model consisting of a steady deflagration layer and a self-similar expansion, was used. However, a steady deflagration will only exist if (a) $\hat{I}_a(t) \ll 1$, i.e. $t \gg I_a\bar{m}^{5/2}\bar{K}\rho_cn_c$; and if (b) $\hat{U}(t) \ll 1$, i.e. $t \ll (m_ec/\bar{m})^3\bar{m}^{5/2}\bar{K}/n_c$. In the above we used $U(t) \equiv (n_ct/\bar{m}^{5/2}\bar{K})^{1/3}$. Condition (a) marks the time required to set up the deflagration; condition (b) means that bremsstrahlung absorption outside the layer will prevail in the long run because the self-similar expansion is growing all the time. These conditions are rarely satisfied simultaneously. If they are satisfied, nonetheless, the expansion will be definitely isentropic, and *not* isothermal (SANMARTIN *et al.*, 1983); this point has produced some confusion in the past.

The result $\bar{\eta} \simeq \bar{\eta}_1$ ($\simeq \bar{\eta}_2$) shows that the shape of the pulse has a negligible effect on the dependence of η_H on $\Delta M_s/M_{s0}$ (specially so if $\Delta M_s/M_{s0} < 0.5$). Thus we can use equation (14) for η_H for a real pulse (which only in its rising-half can be reasonably approximated by a linear pulse). The factor $P_a^2/2\dot{m}_s I_a$ in equation (14) is to be taken from the above results for the different regimes [depending on the values of $\hat{I}_m$, $f(\bar{m}/m_e)^{1/2}$, and $\hat{U}$, mainly]. Table 1 resumes those results.

TABLE 1.—VALUES OF $P_a^2/2\dot{M}_s I_a$ IN EQUATION (14) FOR DIFFERENT REGIMES OF A PLANAR CORONA

Regime	$\frac{I_m}{\rho_c U^3}$	$\frac{\bar{m}U}{m_e c}$	$f\left(\frac{\bar{m}}{m_e}\right)^{1/2}$	Z_i	$\frac{P_a^2}{2\dot{m}I_a}$
I	Small	Small	Large/any	Any/large	0.64
II	Small	~ 1	Large	Large	0.31
III	Large	Small	Large	Large	0.27
IV	Large	Small	$\simeq 1.8$	Large	0.09

The uncoupled analysis of the corona was partly based on the approximation $x_a(t) \simeq 0$, and this required $\Delta M_s/M_{s0}$ to be small. For this range we may set $\bar{\eta} \simeq \Delta M_s/M_{s0}$. The analysis should be actually valid up to about $\Delta M_s/M_{s0} \simeq 0.2$ or 0.3, because (i) the ablated mass is growing during the entire pulse, ΔM_s being its final value, and (ii) $x_a(t)$ and $u(t)$ are found from time-integrations.

If $\hat{I}_{am}$ is small, results may be extended to values $\Delta M_s/M_{s0} = 0(1)$. Use of new variables $x' \equiv x - x_a$, $v' \equiv v - \dot{x}_a$, may always take back the problem to the $x_a = 0$ case, but equations (3) and (4) must then include an inertial force per unit volume, $-\rho\ddot{x}_a$, which in general destroys the self-similar character of the flow. This however is unimportant for $\hat{I}_{am} \ll 1$ because the relation between $\dot{m}_s$, or P_a, and I_a is then obtained from an analysis of the deflagration layer, which is steady or quasisteady. Inertial forces have been often considered in the past in the analysis of deflatrations (FELBER, 1977; FABBRO, 1982). In fact, under deflagration conditions the inertial force may be ignored altogether because $-\rho\ddot{x}_a \sim \rho P_a/M_s \sim (\Delta M_s/M_s)P_a/$(characteristic length of corona), which is small compared with the sharp-pressure gradient within the deflagration layer (unless $M_s/M_{s0} \ll 1$).

4. TARGET ACCELERATION FOR A NON-PLANAR CORONA

The assumption of planar geometry for the corona (Section 2) is valid if $\tau v_{\text{ext}} \ll r_f$ where r_f is the focal spot radius. This condition applies when the pulse is short or the beam is wide. When $\tau v_{\text{ext}}/r_f$ is of order unity the coronal flow depends clearly on Z_i, $f(\bar{m}/m_e)^{1/2}$, α, $\hat{I}_m$, and $\hat{U}$, and *on* $U\tau/r_f$. For long pulses or narrow beams ($\tau v_{\text{ext}} \gg r_f$) the flow approaches the conditions of spherical geometry. To take this into account we use known results about spherical coronae. In the past the divergence in the flow was dealt with by introducing some mean divergence angle into the cold-rocket model (RIPIN *et al.*, 1980).

A discussion similar to that in Section 2 shows that in spherical geometry there exists an ablation surface at some radius $r_a(t)$, and that in analysing the corona one may set $v = T_e = T_i = 0$, $n \to \infty$, at $r = r_a$, and $\dot{r}_a \simeq 0$. If, in addition, $W_L/\dot{W}_L \sim \tau \gg$

r_a/v_{ext}, the corona may be studied by using a quasisteady approximation (AFANAS'EV *et al.*, 1977; GITOMER *et al.*, 1977; MAX *et al.*, 1980; SANZ *et al.*, 1981); τ is then an ignorable parameter. One may introduce a characteristic speed that does not involve τ,

$$V \equiv \left(\frac{n_c r_a}{\bar{m}^{5/2}\bar{K}}\right)^{1/4} \simeq 1.40\times 10^7\,\frac{\text{cm}}{\text{s}}\left(\frac{2Z_i}{A_i}\right)^{5/8}\left(\frac{1.06\,\mu\text{m}}{\lambda}\right)^{1/2}\left(\frac{2r_a}{1\text{ mm}}\frac{Z_i\ln\Lambda}{100\varepsilon\delta_T}\right)^{1/4}; \quad (15)$$

note that $V = U(r_a/U\tau)^{1/4}$. Quantities such as $P_a/\rho_c V^2$, $\dot{m}/4\pi r_a^2\rho_c V$ are then functions of Z_i, $f(\bar{m}/m_e)^{1/2}$, α, $\hat{V} \equiv \bar{m}V/m_e c$, and $\hat{W} \equiv W_L/r_a^2\rho_c V^3$.

To use those results in equation (14) giving the efficiency η_H for our foil problem, we introduce an equivalent spherical target with equal values for ablation pressure, and mass ablation rate and absorbed power per unit area of ablation surface,

$$P_a^* = P_a, \quad \dot{m}^*/4\pi r_a^2 = \dot{m}_s, \quad W_a^*/4\pi r_a^2 = I_a, \quad (16)$$

quantities marked * corresponding to the spherical problem. If a value for r_a is needed we equate the one-sided spherical ablation rate $\dot{m}^*/2$ to the foil rate $\pi r_f^2\dot{m}_s$, so that $r_a = 2^{-1/2}r_f$. (The one-sided spherical absorbed power $W_a^*/2$ is then equal to the power in the foil problem, $\pi r_f^2 I_a$.)

For $\hat{W}_a^*$ small a deflagration regime exists in spherical geometry (SANZ *et al.*, 1981; NICOLAS and SANMARTIN, 1985). We have

$$\eta_H = 0.64\bar{\eta}(\Delta M_s/M_{s0}), \quad \hat{V}\to 0, \quad (17a)$$

$$= 0.34\bar{\eta}(\Delta M_s/M_{s0}), \quad \hat{V} = 0(1), \quad (17b)$$

independently of r_a.

For $\hat{W}_a^*$ large within the range $10^2 < \hat{W}_a^* < 10^5$, Z_i large and f not too low, results for $P_a^*(W_a^*)$, $\dot{m}^*(W_a^*)$ are particularly simple. For $f > 0.05$ roughly, one has $\dot{m}^*/4\pi r_a^2 = \beta_l^{-1/6}\rho_c V(P_a^*/\rho_c V^2)^{5/6}$ independently of $\hat{V}$ and α; $\beta_l \simeq 11.3$. Also $P_a^*/\rho_c V^2 = \phi\hat{W}_a^{*2/3}$ where ϕ is a weak function of $\hat{W}_a^*$ and $\hat{V}$, and nearly independent of α and $f(\bar{m}/m_e)^{1/2}$ for $f > 0.08$ roughly. For $Z_i = 0(1)$ changes are weak (SANZ *et al.*, 1981; SANZ and SANMARTIN, 1983; NICOLAS and SANMARTIN, 1985). Experiments with spherical targets suggest that appropriate f-values to use in that geometry are sensibly larger than 0.03 (GOLDSACK *et al.*, 1982). For the above conditions we get from (14) with $\Delta M_s/M_{s0} < 0.3$,

$$\eta_H \simeq \frac{P_a^2}{2\dot{m}_s I_a}\times\frac{\Delta M_s}{M_{s0}} = \frac{(4\pi)^{7/9}\beta_l^{1/6}\phi^{7/6}}{2(I_a/\rho_c V^3)^{2/9}}\frac{\Delta M_s}{M_{s0}}$$

$$= \frac{9.4\phi^{7/6}}{\hat{W}_a^{*2/9}}\frac{\Delta M_s}{M_{s0}}, \; (\hat{W}_a^* = 2^{3/8}\times 4\pi(U\tau/r_f)^{3/4}\hat{I}_a). \quad (18)$$

Note that $\eta_H \propto r_f^{1/6} \propto r_a^{1/6}$, is weakly dependent on r_a. We then find (NICOLAS and SANMARTIN, 1985)

$$\frac{\eta_H}{\Delta M_s/M_{s0}} \simeq \frac{P_a^2}{2\dot{m}_s I_a} \simeq \begin{cases} 0.30\ (\hat{V} \to 0) \\ 0.20\ (\hat{V} = 0(1)) \end{cases} \quad \text{for } \hat{W}_a^* = 10^2;$$

$$\simeq \begin{cases} 0.06\ (\hat{V} \to 0) \\ 0.05\ (\hat{V} = 0(1)) \end{cases} \quad \text{for } \hat{W}_a^* = 10^5. \tag{19}$$

The acceleration of thin spherical shells is studied in Appendix B.

5. DISCUSSION OF RESULTS

The analysis of Sections 2 and 3 showed that the efficiency η_H is heavily dependent on the behaviour of the corona, which is parametrized by several dimensionless numbers; mainly $f(\bar{m}/m_e)^{1/2}$, $\hat{I}_m \equiv I_m/\rho_c U^3$, and $\hat{U} \equiv \bar{m}U/m_e c$, where f is a heat-flux limit factor, I_m peak intensity, and U a speed defined by equation (9). This is in agreement with available data, (Fig. 2) from a variety of experiments with thin foils (McCall, 1983): it is clear that the data cannot be represented by an universal relation $\eta_H(\Delta M/M_0)$. Note also that equation (1), drawn in the figure for comparison, overestimates the efficiency substantially.

Figure 2 shows, in addition, theoretical curves taken from equation (14), using $P_a^2/2\dot{m}_s I_a$ as given in Table 1 for limit values of the dimensionless parameters (regimes I–IV). For the regimes III and IV the range of validity of the theory is roughly $\Delta M/M_0 < 0.3$ (Section 3); curves have been drawn accordingly. The overall agreement with the data is good. A more detailed comparison would require additional information on each particular experiment.

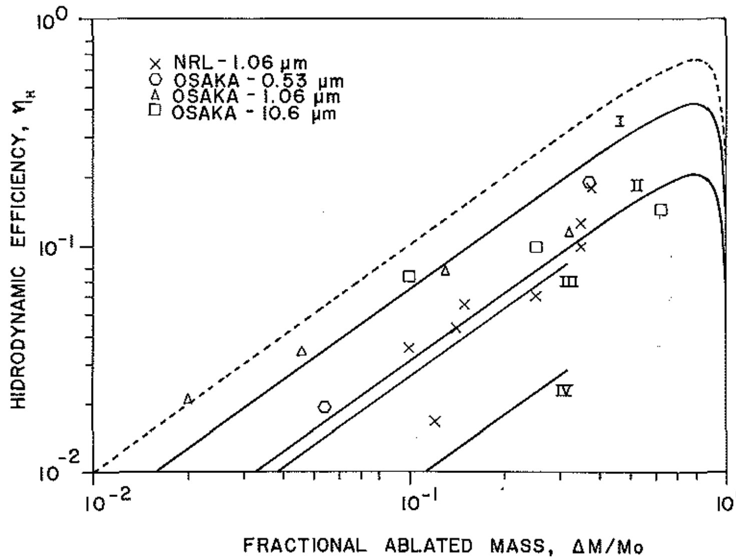

Fig. 2.—Hydrodynamic efficiency η_H vs ablated mass ΔM (M_0 = initial foil mass), from both experiments (data points) and theory: the dotted line represents equation (1); lines I–IV represent equation (14) with values from Table 1.

Table 1 corresponds to a planar corona, or equivalently $U\tau/r_f$ small; τ is the pulse full-width at half-maximum, and r_f the spot radius. In the opposite limit, studied in Section 4 in a spherical approximation, a speed $V \equiv U(r_f/2^{1/2}U\tau)^{1/4}$ replaces U in dimensionless relations. Results for low intensities, given in equation (17), are similar to those in Table 1 (regimes I and II). Results for high intensities, given in equations (18) and (19), decrease continuously with increasing intensity; they are practically independent of f for the not too low f-values suggested by experiments (GOLDSACK *et al.*, 1982).

When ΔM is not directly measured in an experiment (EIDMANN *et al.*, 1984), it must be calculated if a comparison with theory is to be made. This requires determining the mass ablation rate throughout the pulse. Results on mass ablation rate and ablation pressure will be discussed elsewhere.

The assumption $P_a^2/2\dot{m}_s I_a = 1$, underlying equation (1), is often used, independently of efficiency considerations, to calculate P_a from measurements of $\dot{m}_s$ and I_a. Table 1 shows that in this way the ablation pressure may be overestimated by a factor as high as 3.

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APPENDIX A

Integrating equation (2) between x_v, somewhere to the left of the foil where ρ and P vanish, and $x_a(t)$, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{x_v}^{x_a}\rho\mathrm{d}x - \dot{x}_a\rho_a = \int_{x_v}^{x_a}\frac{\partial\rho}{\partial t}\mathrm{d}x = -\int_{x_v}^{x_a}\mathrm{d}x\frac{\partial}{\partial x}\rho v = -\rho_a v_a$$

or

$$\frac{\mathrm{d}M_s}{\mathrm{d}t} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\int_{x_v}^{x_a}\rho\mathrm{d}x = -\rho_a(v_a - \dot{x}_a) \equiv -\dot{m}_s \tag{A.1}$$

which is equation (10).

Integrating equation (3) similarly, we get

$$M_s\frac{\mathrm{d}u}{\mathrm{d}t} = -P_a - \rho_a(v_a - \dot{x}_a)(v_a - u), \tag{A.2}$$

where $u = \int_{x_v}^{x_a}\rho v\,\mathrm{d}x/M_s$. Since $P_a \sim \rho_c v_{\mathrm{ext}}^2$, $\dot{m}_s \sim \rho_c v_{\mathrm{ext}}$, and $v_a \sim u \sim v_{\mathrm{int}} \ll v_{\mathrm{ext}}$, we may drop the last term in (A.2) to recover equation (11).

Finally integrating (4) with $q = 0$, $I_\mp = 0$, neglecting e_{in} against $\frac{1}{2}v^2$ (thin-foil or long-pulse approximation) we get

$$M_s\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}\overline{v^2} = -P_a v_a - \rho_a(v_a - \dot{x}_a)\left(\frac{1}{2}v_a^2 - \frac{1}{2}\overline{v^2}\right), \tag{A.3}$$

where $\overline{v^2} = \int_{x_v}^{x_a}\rho v^2\,\mathrm{d}x/M_s$. Neglecting the last term in (A.3), as in (A.2), we recover (A.3) from (A.2) if $v_a \simeq u$, $\overline{v^2} \simeq u^2$.

APPENDIX B

For completeness we consider the acceleration of thin spherical shells (MAX *et al.*, 1983; MAYER *et al.*, 1983). The equations equivalent to (10) and (11) are

$$dM/dt = -\dot{m}, \tag{B.1}$$

$$M du/dt = -4\pi r_a^2 P_a, \tag{B.2}$$

where M is the mass of the shell at time t, and u its outward velocity. For a spherical corona, $f > 0.08$, and $10^2 < \hat{W}_a \equiv W_a/r_a^2\rho_c V^3 < 10^5$, we have $P_a/\rho_c V^2 = \phi \hat{W}_a^{2/3}$, $\dot{m}/4\pi r_a^2 \rho_c V = \beta_l^{-1/6}(P_a/\rho_c V^2)^{5/6} = \beta_l^{-1/6}\phi^{5/6}\hat{W}_a^{5/9}$ (Section 4). Let $r_{a0} \equiv r_a(t=0)$ and consider a pulse of constant power and duration τ (Section 3). As the shell collapses, P_a and $\dot{m}$ change through their dependence on r_a; we have

$$\left(\frac{r_{a0}}{r_a}\right)^{13/18} \dot{m} = \dot{m}_0 \equiv 4\pi r_{a0}^2 \rho_c V_0 \beta_l^{-1/6} \phi^{5/6} \hat{W}_{a0}^{5/9}, \tag{B.3}$$

$$\left(\frac{r_a}{r_{a0}}\right)^{4/3} P_a = P_{a0} \equiv \rho_c V_0^2 \phi \hat{W}_{a0}^{2/3}. \tag{B.4}$$

From equations (B.1) to (B.4) we obtain

$$M\frac{du}{dM} = \frac{4\pi r_{a0}^2 P_{a0}}{\dot{m}_0}\left(\frac{r_{a0}}{r_a}\right)^{1/18}.$$

Neglecting the weak dependence on r_a/r_{a0} we get

$$u \simeq \frac{4\pi r_{a0}^2 P_{a0}}{\dot{m}_0}\ln\frac{M}{M_0}, \quad M_0 = M(t=0), \tag{B.5}$$

Using (B.1), (B.3), (B.5), and $u \simeq dr_a/dt$ we obtain

$$1 - \frac{M}{M_0}\left(1 - \ln\frac{M}{M_0}\right) \simeq \frac{\gamma^2}{2}\left(1 - \left(\frac{r_a}{r_{a0}}\right)^{31/18}\right), \tag{B.6}$$

where

$$\frac{\gamma^2}{2} = \frac{18}{31}\frac{\dot{m}_0^2}{4\pi r_{a0} P_{a0} M_0} = \frac{18}{31}\frac{\rho_c r_{a0}}{\rho_0 \Delta R}\frac{\phi^{2/3}}{\beta_l^{1/3}}\hat{W}_{a0}^{4/9}. \tag{B.7}$$

One can show that velocities in the corona are of order $V_0 \hat{W}_{a0}^{1/9}$. Hence, the corona will be quasi-steady as long as $u \ll V_0 \hat{W}_{a0}^{1/9}$. According to (B.3)–(B.5), we have $u \simeq (\beta_l \phi)^{1/6} V_0 \hat{W}_{a0}^{1/9} \ln(M/M_0)$ so that the above inequality is equivalent to $\Delta M/M_0 \equiv 1 - M/M_0 \ll 1$, as in the planar case. Equation (B.6) shows that this condition is satisfied for the entire implosion ($0 < r_a < r_{a0}$) if $\gamma^2/2$ is small. This is fortunately the usual case in experiments: for instance, if $\Delta R = r_{a0}/20$, $\rho_c/\rho_0 = 1.3 \times 10^{-3}$ and $m_i/Z_i \simeq 2m_p$ (Al shell, 1.06 μm light, fully ionised corona), $r_{a0} = 200$ μm, and $W_a = 10^{12}$ W, we have $\gamma^2/2 \simeq 0.03$ ($\hat{W}_{a0} \simeq 2.9 \times 10^3$, $\phi \simeq 0.1$). (For $\gamma^2/2$ large, the shell would be ablated with almost no collapse.) Then equation (B.6) takes the form

$$(\Delta M/\gamma M_0)^2 = 1 - (r_a/r_{a0})^{31/18}. \tag{B.8}$$

Using (B.8) in (B.1) we get

$$\tau = \frac{\gamma M_0}{\dot{m}_0} I\left(\frac{\Delta M}{\gamma M_0}\right), \quad I(s) = \int_0^s \frac{ds'}{(1 - s'^2)^{13/31}}. \tag{B.9}$$

Equations (B.8) and (B.9) determine the values of r_a and ΔM at the end of the pulse, as long as (B.9) gives $\Delta M/\gamma M_0 < 1$; for longer pulses, equation (B.9) with $\Delta M/\gamma M_0 = 1$ gives the time for total collapse ($r_a = 0$).

Equations (B.5) and (B.9) lead to

$$\eta_H \equiv \frac{Mu^2}{2W_a \tau} = \frac{(4\pi r_{a0}^2 P_{a0})^2}{2W_a \dot{m}_0}\frac{\Delta M}{M_0}\frac{\Delta M/\gamma M_0}{I(\Delta M/\gamma M_0)} \tag{B.10}$$

where we set $\bar{\eta}(\Delta M/M_0) \simeq \Delta M/M_0$ (<0.2 or 0.3). When compared with

$$\eta_H = \frac{2\pi r_{a0}^2 P_{a0}^2}{\dot{m}_0 W_a/4\pi r_{a0}^2} \bar{\eta}\left(\frac{\Delta M}{M_0}\right) \tag{B.11}$$

(MAX *et al.*, 1983) three differences appear: (i) The full expression $\bar{\eta}(\Delta M/M_0)$ was retained in (B.11). Since a quasisteady corona was also assumed to get (B.11), it should not be valid above $\Delta M/M_0 = 0.3$, say, and thus $\bar{\eta} \simeq \Delta M/M_0$. Secondly, MAX *et al.* (1983) wrote $\tau = \Delta M/\dot{m}_0$, instead of (B.9), missing the factor $(\Delta M/\gamma M_0)/I(\Delta M/\gamma M_0)$. For $0 < s < 1$ we have $1 > s/I > 0.70$; hence (B.11) may overestimate the efficiency by as high as 42%.

Finally, approximate results for the corona for small f (MAX *et al.*, 1980; SANZ and SANMARTIN, 1983) were used in (B.11). For the not too low f suggested previously (SANZ and SANMARTIN, 1983) we have (B.3) and (B.4), and equation (B.10) becomes

$$\eta_H = \frac{2\pi\beta_l^{1/6}\phi^{7/6}}{\hat{W}_{a0}^{2/9}} \frac{\Delta M}{M_0} \frac{(\Delta M/\gamma M_0)}{I(\Delta M/\gamma M_0)},$$

which is just (18) $\times (\Delta M/\gamma M_0)/I(\Delta M/\gamma M_0)$. Using now (B.7) we finally get

$$\eta_H = \left(2\pi\left(\frac{36}{31}\frac{\rho_c r_{a0}}{\rho_0 \Delta R}\right)^{1/2} \phi^{3/2}\right) \times \frac{(\Delta M/\gamma M_0)^2}{I(\Delta M/\gamma M_0)}.$$

For $\rho_c/\rho_0 = 1.3 \times 10^{-3}$, $r_{a0}/\Delta R = 20$, $\phi = 0.1$, the expression within the bracket is 0.02; the function $s^2/I(s)$ has a maximum 0.76 at $s \simeq 0.94$. Thus η_H(maximum) $\simeq 0.015$.

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