*J. Meseguer and J. M. Perales*

# Non Steady Phenomena in the Vibration of Viscous, Cylindrical, Long Liquid Bridges[1]

*This paper deals with the dynamic response of long cylindrical viscous liquid bridges subjected to an oscillatory microgravity field whose frequency varies linearly with time. The problem has been solved by using a one-dimensional model for the dynamics, derived from Cosserat theory for continuum, in which the axial velocity is considered to be constant over each cross-section of the liquid column. The dynamic response of the liquid bridge has been obtained by applying the Laplace transform to the problem formulation. The results obtained show that a variable-frequency excitation could give rise to erroneous measurements of the resonance frequencies of viscous liquid bridges.*

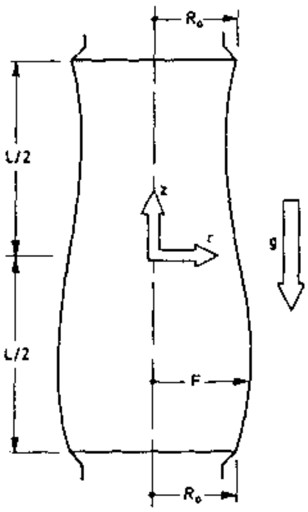


*Fig. 1. Geometry and coordinate system for the liquid bridge problem*

## 1 Introduction

The fluid configuration analyzed in this paper consists of a column of liquid spanning between two coaxial solid disks of the same radius, $R_o$, placed a distance $L$ apart, as sketched in fig. 1. The liquid bridge is assumed to be isothermal and the properties of both the liquid (density, $\varrho$, and viscosity, $\nu$) and the interface (surface tension, $\sigma$) are assumed to be uniform and constant, and the effects of the gas surrounding the liquid bridge negligible. Additional characteristics of the fluid configuration here analyzed are that the interface must remain anchored to the edges of the disks and that the volume of liquid equals that of a cylinder of radius $R_o$ and length $L$.

In the last decade a large number of papers dealing with the dynamics of liquid bridges have been published, most of them being devoted to the analysis of resonance phenomena of such fluid configurations. Early studies [1, 2] were related to the analysis of the frequencies of resonance (eigenfrequencies) whereas in the last years the attention has been focused on the analysis of the response of the liquid bridge when subjected to harmonic perturbations [3–8], and only a few attempts have been made to analyze non-harmonic perturbations [9–11]. In one of these papers [10] the dynamic response of long cylindrical liquid bridges when subjected to a small change in the value of the axial acceleration acting on the liquid bridge was calculated by using a one-dimensional Cosserat model, widely used in capillary jet problems as well as in liquid bridge problems.

Such a solution of the dynamics of the liquid bridge is used here to analyze several aspects of the liquid bridge dynamics which are of importance from the experimental point of view, namely the onset of the vibration of the liquid bridge and the forced vibration of the liquid bridge when the frequency of the perturbation varies with time.

In the following all physical quantities are made dimensionless using the characteristic length $R_o$ and the characteristic time, $t_c = (\varrho R_o^3/\sigma)^{1/2}$. The set of dimensionless parameters defining the fluid configuration are the slenderness, $\Lambda = L/(2R_o)$, the capillary number $C = \nu(\varrho/\sigma R_o)^{1/2}$, the Bond number $B = \varrho g R_o^2/\sigma$, $g$ being the axial acceleration, and the dimensionless volume of liquid, $V = 2\pi\Lambda$, which has been made dimensionless with $R_o^3$.

## 2 Theoretical Background

Under the assumptions done, the set of nondimensional differential equations and boundary conditions for the axisymmetric, non rotating, viscous flow, according to the Cosserat model are [10]:

$$S_t + Q_z = 0, \tag{1}$$

$$DS - \frac{1}{8}\left\{S^2\left[D_z - \frac{3}{2}(Q/S)_z^2\right]\right\}_z = -SP_z - \frac{1}{8}C[S^2(Q/S)_{zz}]_{zz} + 3C[S(Q/S)_z]_z, \tag{2}$$

where

$$D = \frac{Q_t + (Q^2/S)_z}{S}, \tag{3}$$

$$P = 4(2S + S_z^2 - SS_{zz})(4S + S_z^2)^{-3/2} + B(t)z. \tag{4}$$

In these expressions $S = F^2$ and $Q = SW$, $F(z, t)$ being the dimensionless equation of the liquid gas interface and $W(z, t)$ the dimensionless axial velocity which, in this model, is uniform in each plane parallel to the disks. Boundary conditions state that the interface must remain anchored to the disk edges, $S(\pm A, t) = 1$, and that the axial velocity is zero at each one of the disks, $Q(\pm A, t) = 0$, whereas the initial conditions are that the liquid bridge is at rest at $t = 0$, $Q(z, 0) = 0$, its shape being that of a cylinder, $S(z, 0) = 1$, which implies $B(t) = 0$ for $t \le 0$.

In [10] it was assumed that the Bond number changes at $t = 0$ from the initial value $B = 0$ to a new one $B = \varepsilon$, $\varepsilon \ll 1$, that is $B(t) = \varepsilon \mathscr{H}(t)$, $\mathscr{H}(t)$ being the Heaviside function ($\mathscr{H} = 0$ for $t \le 0$ and $\mathscr{H} = 1$ for $t > 0$). In view of such perturbation the variables were expanded as $S = 1 + \varepsilon s_o(z, t) + O(\varepsilon)$, $Q = \varepsilon q(z, t) + O(\varepsilon)$. The introduction of these expressions in the problem formulation, neglecting second order terms, and the elimination of the variable $s_o(z, t)$ from the momentum equation by using the continuity equation allows to formulate the problem in terms only of the variable $q(z, t)$:

$$q_{tt} - \frac{1}{8} q_{ttzz} + \frac{1}{2} q_{zz} + \frac{1}{2} q_{zzzz} + \frac{1}{8} C q_{tzzzz} - 3C q_{tzz} = -\mathscr{H}_t \tag{5}$$

boundary conditions being $q(\pm A, t) = 0$, $q_z(\pm A, t) = 0$, and initial conditions $q(z, 0) = 0$, $q_t(z, 0) = 0$.

The application of the Laplace transform to the last equation allows to calculate the liquid bridge response in the Laplace domain as well as in the time domain by using the Inversion Theorem, in such a way that the dependence of the liquid bridge interface on the dimensionless time is given by

$$s_o(z, t) = 2\left(z - \frac{A}{\sin A} \sin z\right) - \sum_{n=1}^{\infty} \frac{D_z(z, h_n)}{h_n^3 D_h(A, h_n)} \exp(h_n t), \tag{6}$$

where

$$D(z, h) = \theta_2 \sinh \theta_2 A \cosh \theta_1 z - \theta_1 \sinh \theta_1 A \cosh \theta_2 z, \tag{7}$$

$$D_z(z, h) = \theta_1 \theta_2 (\sinh \theta_2 A \sinh \theta_1 z - \sinh \theta_1 A \sinh \theta_2 z), \tag{8}$$

and

$$D_h(A, h_n) = \left.\frac{\mathrm{d}D(A, h)}{\mathrm{d}h}\right|_{h = h_n} \tag{9}$$

In these expressions $h_n$ are the roots of $D(A, h) = 0$, which in general are complex, $h_n = \gamma_n + i\omega_n$; $\theta_1$ and $\theta_2$ are related to $h$ and $C$ through the characteristic equation

$$\left(1 + \frac{1}{4} hC\right)\theta^4 + \left(1 - \frac{1}{4} h^2 - 6hC\right)\theta^2 + 2h^2 = 0. \tag{10}$$

Once the liquid bridge response to a perturbation consisting of a sudden change in the value of Bond number ($B = \varepsilon \mathscr{H}$) is calculated, $s_o(z, t)$, the response $s(z, t)$ to any variation of Bond number $B(t) = \varepsilon b(t)$ can be calculated by applying the Duhamel's theorem

$$s(z, t) = b(t) s_o(z, t) + \int_0^t b(\tau) s_{o_t}(t - \tau)\, \mathrm{d}\tau \tag{11}$$

and taking into account that $s_o(z, 0) = 0$ finally results

$$s(z, t) = \sum_1^{\infty} \frac{D_z(z, h_n)}{h_n^2 D_h(A, h_n)} \int_0^t b(\tau) \exp[h_n(t - \tau)]\, \mathrm{d}\tau. \tag{12}$$

## 3. Results

The considered perturbation has been $b(t) = \sin at^2$, that is, the axial microgravity varies sinusoidally with a frequency that increases linearly with the time (note that the instantaneous frequency is defined as $\omega(t) = 2at$). Such a kind of perturbation could be considered as an idealization of the one available in the Fluid Physics Module, one of the multiuser experimental facilities provided by the European Space Agency for fluid experimentation aboard space platforms like *Spacelab* (this equipment has been flown in past *Spacelab* missions and it will be flown in the next *Spacelab-D2* mission). In the Fluid Physics Module one of the disks supporting the liquid bridge can be vibrated at constant amplitude with frequencies that change linearly with time, the minimum rate being 1/90 Hz $s^{-1}$. Of course this kind of perturbation is not exactly equal to a time variation of the Bond number, the main difference being that, as stated elsewhere [6] an oscillatory Bond number only excites non-symmetric oscillation modes (in respect to the middle plane parallel to the supporting disks) whereas the vibration of one of the disks excites both non-symmetric and symmetric oscillation modes. However this difference is not relevant if the interest is mainly focused on the measurement of the frequency of resonance corresponding to the first oscillation mode, so that in the following the analysis is kept within this boundary.

Since there are a large number of parameters involved in the theoretical model, it is convenient to fix the values of some of them in order to limit the boundaries of the study. Therefore, taking into account that silicone oils are normally used as working liquid for liquid bridge experimentation either on Earth or in space laboratories, we have selected the physical properties of these liquids ($\varrho \approx 10^3$ kg $m^{-3}$, $\sigma \approx 0.02$ N $m^{-1}$, and viscosities ranging from $10^{-6}$ $m^2 s^{-1}$ to $10^{-5}$ $m^2 s^{-1}$) to estimate the values of the parameters involved. Then, assuming that the radius of the disks is $R_o = 0.015$ m, the value of the characteristic time becomes $t_c \approx 0.4$ s whereas the parameter of viscosity ranges from $10^{-3}$ to $10^{-2}$. On the other hand, taking into account the above mentioned value of the frequency ramp in the Fluid Physics Module, one gets $a \approx 0.005$.

The introduction of the selected law for the time variation of Bond number ($b(t) = \sin at^2$) in expression (12) allows to calculated the time variation of the liquid bridge interface. Unfortunately, for the selected perturbation the integrals appearing in (12) must be computed numerically, and some care is needed during such calculations in order to avoid aliasing errors. To measure the response of the liquid bridge to the imposed perturbation the adopted criterion has been to calculate the variation with time of the diameter of the interface at the section situated at a quarter of the total length, $s(\Lambda/2, t)$; note that this section is close to the one where the deformation of the liquid bridge interface becomes maximum. As already stated, the attention has been mainly focused on the resonance corresponding to the first oscillation mode.

The response of a liquid bridge with $\Lambda = 2$ and $C = 0.05$ to a perturbation characterized by the value $a = 0.005$ is shown in fig. 2. In this plot, as well as in the following figures, the abscissae are the dimensionless pulsation, $\omega = 2at$ instead of the dimensionless time. As it can be observed the amplitude of the deformation of the liquid bridge interface increases as the frequency grows until a relative maximum appears (which roughly corresponds to the first resonance), then the amplitude decreases and it will increase again close to the next relative maximum (that corresponding to the third resonance, not shown in fig. 2, according to the type of perturbation here considered) and so on.

For frequencies below the maximum of the interface deformation, the liquid bridge response is a wave of monotonically increasing amplitude and frequency, but the behaviour is rather different for frequencies larger than the resonance. The reason for this difference is that once the resonance is excited the liquid bridge starts to oscillate with its natural frequency (although, because of viscosity, the amplitude of such oscillation decreases with time) and at the same time the liquid bridge is subjected to a forced oscillation whose frequency increases as the time grows (such frequencies are greater than the one corresponding to the resonance).

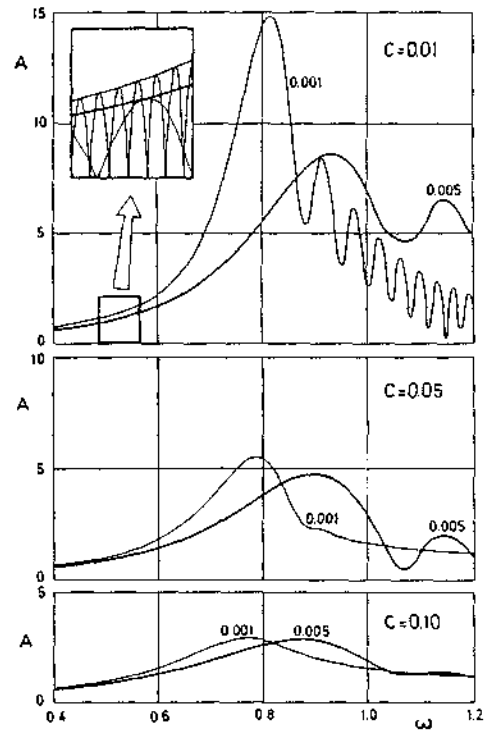


*Fig. 3. Variation with dimensionless pulsation, $\omega$ (or dimensionless time, $t = \omega/2a$) of the amplitude of the deformation of the liquid bridge interface at the section $z = \Lambda/2$, $A = \max|s(\Lambda/2, t)|$. The results correspond to liquid bridges with slenderness $\Lambda = 2$ and different values of the viscosity parameter C subjected to a perturbation whose frequency varies linearly with time with a sweep a. Numbers on the curves indicate the value of a*

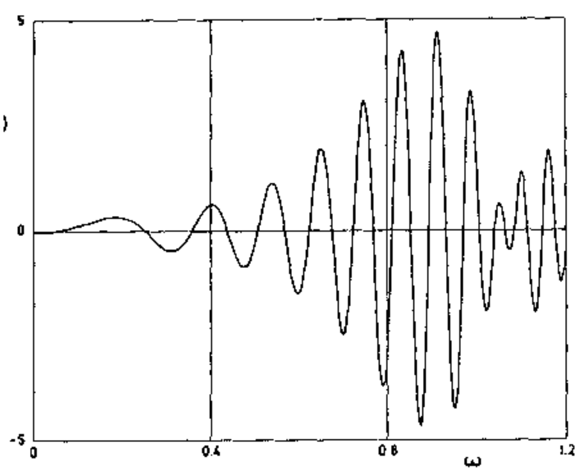


*Fig. 2. Variation with dimensionless pulsation, $\omega$ (or dimensionless time, $t = \omega/2a$) of the deformation of the liquid bridge interface at the section $z = \Lambda/2$, $s(\Lambda/2, t)$. The results correspond to a liquid bridge with slenderness $\Lambda = 2$ and viscosity parameter $C = 0.05$ subjected to a perturbation whose frequency varies linearly with time with a sweep $a = 0.005$*

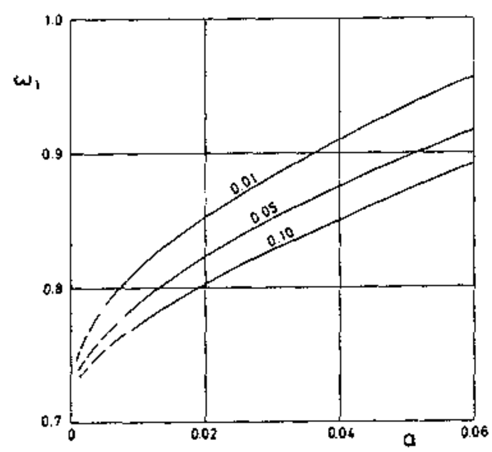


*Fig. 4. Variation with the frequency sweep a and the parameter of viscosity C of the dimensionless pulsation of resonance corresponding to the first oscillation mode $\omega_1$. The results correspond to a liquid bridge with slenderness $\Lambda = 2$*

Another characteristic of the liquid bridge response to be pointed out is that a criterion based on the maximum interface deformation for the measurement of resonances could lead to unacceptable errors in an experiment with a procedure similar to the one here analyzed. In effect, if this criterion were adopted to analyze the results shown in fig. 2, the measured dimensionless resonance pulsation corresponding to the first oscillation mode would be $\omega_1 \approx 0.90$ instead of the correct value $\omega_1 = 0.73$ which results from a harmonic excitation analysis [6]. This shift on the apparent frequency of resonance depends on the viscosity of the liquid as well as on the frequency sweep $a$, as shown in fig. 3, where the liquid bridge responses of the same fluid configuration ($\Lambda = 2$) with different viscosities and different frequency ramps have been represented. The curves shown in fig. 3 are the locii of the maxima of the absolute value of $s(\Lambda/2, t)$, so that they represent the variation with time of the amplitude of the deformation of the radius at the section $z = \Lambda/2$ (according to the definition $S = 1 + \varepsilon s = F^2 = (1 + \varepsilon f)^2 = 1 + 2\varepsilon f + O(\varepsilon^2)$, so that, within this approximation $s(z, t) = 2f(z, t)$, $f(z, t)$ being the expanded deformation of the radius of the interface at section $z$). Note that, in order to compare the effect of the frequency ramp of the excitation, in the abscissae frequency instead of time has been represented, that means that, since $t = \omega/2a$, in each of the plots each one of the curves has a different time scale (depending on the value of $a$) as illustrated in the insert.

Finally, the influence of both $a$ and $C$ on the pulsation of resonance, defined as the pulsation for which the interface deformation is maximum, is shown in fig. 4. Note that the differences between the measured frequencies and the real frequency of resonance increases as the frequency ramp increases, and that the contrary occurs with the viscosity.


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