

# Algorithm Optical Codes: An Alternative to Random Optical Codes in an Intra-Satellite Optical Wireless Network

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## ABSTRACT

We propose a novel family of optical codes to be used in intra-satellite optical wireless networks. They are Algorithm Optical Codes (AOCs). This family is related with the recently introduced family Random Optical Codes (ROCs). Both families of codes are designed to be used in optical Code-Division Multiple-Access (OCDMA) systems. Similarly than their predecessor, i.e. ROCs, AOCs are specially suitable for optical networks where the number of channels varies, ever large, low bit rate requirements, energy limitations and packed data are presented. For example sensor networks demand these requirements.

A detailed description of AOCs is shown in this paper. Furthermore a comparison between them and ROCs is presented. We find that AOCs provide less probability of error than ROCs, for a given length of the code and a number of users. Moreover a system using AOCs can change the length of the code easier than a system using ROCs. This fact permits a more efficient accommodation to the actual number of users.

Finally the implementation in an intra-spacecraft Telecommand and Telemetry (TC/TM) optical wireless network is also described. We compare the families in the intra-spacecraft optical wireless network.

**Keywords:** intra-satellite optical wireless networks, algorithm optical codes (AOCs), random optical codes (ROCs), optical codes, generalized optical orthogonal codes (OOCs), optical code-division multiple-access (OCDMA).

## 1. INTRODUCTION

In this paper we propose an alternative to the Random Optical Codes (ROCs) for intra-satellite optical wireless networks introduced by the authors in [1], [2]. The proposed family is Algorithm Optical Codes (AOCs).

Both families are used in Optical Code-Division Multiple-Access (OCDMA) systems. In particular we focus on networks of sensors. The large number of users, simplicity of the implementation, number of channel adaptation and low duty cycle is a must in these applications.

Incoherent, intensity encoding/decoding techniques have been commonly implemented in Optical Code-Division Multiple-Access (OCDMA) systems. Signature sequences as optical orthogonal codes (OOCs) [3]-[6], prime sequences [7]-[9] or  $2^n$  prime codes [10]-[12] have been used in OCDMA systems. Recently, Salehi has revised many of these codes used in OCDMA systems in [13]. There, it is highlighted how the multiple-access interference (MAI) effect reduction has been deeply studied in the literature [14]-[18]. However an important step in OOCs evolution was the analysis of the performance for Generalized OOCs [19]. OOCs with any correlation value were considered. Furthermore the optimal parameters were found based on the exact probability of error.

Following the unrestricted correlation idea, ROCs [1], [20] present also several important features used in some OCDMA systems as sensor networks. Furthermore, the proposed family, i.e. AOCs, provide additional improvements keeping the advantages of ROCs. The most important is the dynamical accommodation of the length of the code.

The system model using AOCs is rigorously described in Sec. 2. The procedures to optimize the model are described in Sec. 3. The comparison between systems using respectively ROCs and AOCs is presented in Sec. 4. An example of a system where AOCs are used is shown in Sec. 5.

## 2. SYSTEM MODEL

The model is an on-off keying (OOK) OCDMA system. The phase synchronism between the emitter and the intended receiver is assumed. However others channels could be randomly delayed. Perfect chip synchronization is assumed, i.e. frequency locking. Thus, the channels are asynchronous incoherent and additive. The number of users is  $N+1$ .

An algorithm builds the codes as follows. Let  $\{\xi_i^{(k)}\}_{i=1}^{\infty}$  be an infinite pseudo-random sequence of Bernoulli random variables (r.v.), for each channel. It is identically repeated by encoder and decoder starting in a common seed.

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$$\xi_i^{(k)} = \begin{cases} \text{Yes} \\ \text{No} \end{cases} \quad i \in \mathbb{N}, k = \{1, \dots, N + 1\}. \quad (1)$$

Let  $p$  defined by the probability of a success. Hence, the probability mass function (PMF) of  $\{\xi_i^{(k)}\}_{i=1}^{\infty}$  is written as,

$$PMF = \begin{cases} P(\xi_i^{(k)} = \text{Yes}) = p \\ P(\xi_i^{(k)} = \text{No}) = 1 - p \end{cases} \quad (2)$$

We denote by marked the chip  $i$ , if  $\xi_i^{(k)} = \text{Yes}$ . Thus,  $p$  is the frequency of these marked chips.

In order to track the chips where the marked chips are, the information is stored in numerical variables  $\{\theta_n^{(k)}\}_{n=1}^{\infty}$ . Any  $\theta_n^{(k)}$  is defined as the  $n^{\text{th}}$  marked chip for the user ( $k$ ), i.e.

$$\begin{aligned} \theta_n^{(k)} = j, n = 1, & \text{ when } \xi_1^{(k)} = \text{No} \cap \dots \cap \xi_{j-1}^{(k)} = \text{No} \cap \xi_j^{(k)} = \text{Yes}, \\ \theta_n^{(k)} = j, n = \{2, \dots, \infty\}, & \text{ when } \xi_{\theta_{(n-1)}^{(k)}+1}^{(k)} = \text{No} \cap \dots \cap \xi_{j-1}^{(k)} = \text{No} \cap \xi_j^{(k)} = \text{Yes}. \end{aligned} \quad (3)$$

The weight ( $w$ ) of the code is the number of marked chips used to transmit a bit. The length ( $L$ ) is the number of chips that the sequence has to wait before to obtain  $w$  marked chips. Furthermore,  $L$  is the position of the  $w^{\text{th}}$  marked chip, i.e.  $L = \theta_w$ . Notice that each bit could use sequences with different values of  $L$ . Moreover  $L$  is a random variable. It measures the number of trials until the  $w^{\text{th}}$  success of a Bernoulli random variable. Thus,  $L$  have Negative Binomial distribution with parameters  $w$  and  $p$ , thus  $E[L] = \frac{w}{p}$ , where  $E[L]$  is the expectation of  $L$ . This fact allows us to give the expectation of the bit rate of the model.

The model is easily interpreted if we consider the expectation of  $L$  as a parameter of the system. Notice that  $p$  can be tailored in order to get the desirable  $E[L]$  for our system, i.e.  $p = \frac{w}{E[L]}$ .

A scheme of the model is shown in Fig. 1.

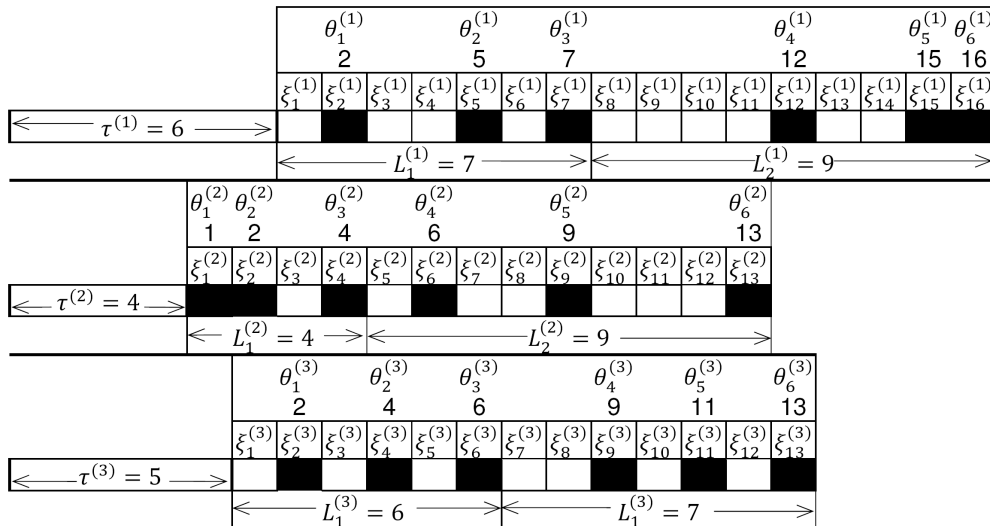


Fig. 1. Scheme of the model.

### 3. OPTIMIZATION

In this section the parameters of the model are optimized for systems using ROCs and AOCs. In both cases the parameters of the model are the length of the code ( $L$ ), the weight ( $w$ ) and the number of users in the model ( $N + 1$ ).

The performance of the system is described by the following expressions for the probability of error ( $Pe$ ). It is a function of the parameters aforementioned.

- Probability of error for ROCs ( $P_e^{ROCs}$ ), by [2]:

$$P_e^{ROCs} = \frac{1}{2^{N+1}} \sum_{i=0}^N \binom{N}{i} \left( 1 - \left( 1 - \frac{w}{L} \right)^i \right)^w, \quad (4)$$

- Probability of error for AOCs ( $P_e^{AOCs}$ ), by [21]:

$$P_e^{AOCs} = \frac{1}{2} \left( 1 - \left( 1 - \frac{w}{2E[L]} \right)^N \right)^w. \quad (5)$$

The optimization of the parameters in both cases is a numerical search of the lower  $w$  reaching the minimum probability of error for a given  $L$  and  $N$ .

#### 4. COMPARISON

In this section the performance of two on-off keying (OOK) OCDMA systems with asynchronous incoherent additive channels are compared. One of them is widely described by the authors in [1] and [20]. The other one is analyzed in [21].

The comparison between the performance of these systems is based on the optimizations described in Sec. 3. The optimizations is computed for codes shorter than 1000 and system supporting up to 100 users. Notice that  $P_e$  is an  $L$  and  $N$  function. This fact allows the plotting of the contour levels of  $P_e$  for ROCs (solid lines) and for AOCs (dashed lines) in Fig. 2.

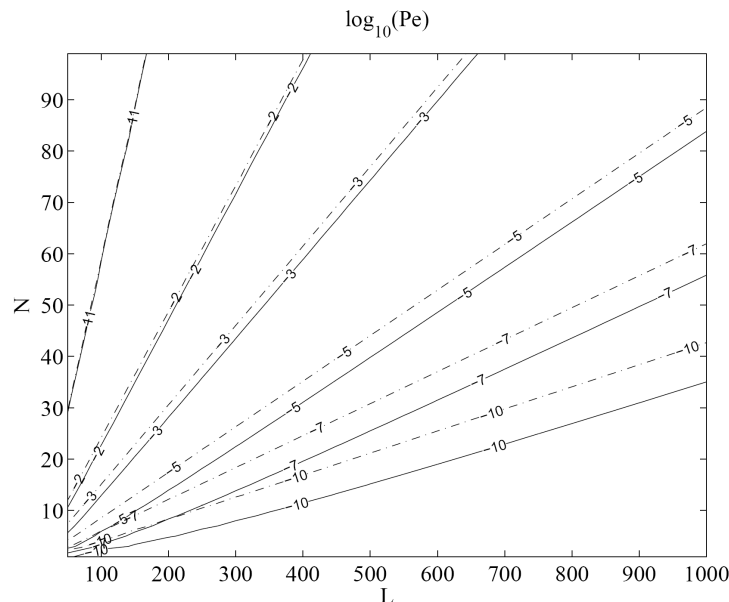


Fig. 2. Contour levels of  $\log_{10}(P_e)$ . They are function of  $L$  and  $N$ . Solid lines represent ROCs and dashes lines represent AOCs.

Results observed in Fig. 1 are the key to compare the performance of both families of codes. Comparison is shown for six different values of  $P_e$ , i.e.  $\{10^{-1}, 10^{-2}, 10^{-4}, 10^{-5}, 10^{-7}, 10^{-10}\}$ . The best method is the one which allows a larger number of users for a given  $L$  and  $N$ .

For a given  $L$ , AOCs accommodate more users than ROCs achieving the same value of probability of error. For example given  $L = 1000$ , the number of users is 42 for AOCs and 35 for ROCs when  $P_e = 10^{-10}$ .

However the performance of the system is not the only advantage of AOCs versus ROCs. The dynamical accommodation to the actual number of users introduced by the authors in [2] using ROCs, can be easier implemented using AOCs. Notice that modifying only the parameter  $p$ , i.e. the frequency of the marked chips, the length of the code can be tuned. Based on this fact the system can be designed to adapt automatically  $p$  to the actual number of users. This adaptation is identically predefined in the emitter and the received.

#### 5. APPLICATIONS AND EXAMPLES

Based on the optimization found in Sec. 3, we can select the parameters for any application.

Following the same scenario introduced in [2], we design an intra-spacecraft Telecommand and Telemetry (TC/TM) optical wireless network using AOCs.

In this system the On Board Computer (OBC) communicates with 25 terminals. The terminals include sensors and/or actuators thus the bit rate transmission could be low; actually 1 kb/s per channel is the requirement. We use two channels per station (for upward and downward links), hence  $N = 49$ .

Other constraining parameter is the probability of error. In this system it should be  $Pe \leq 10^{-7}$ . In Fig. 2  $L = 900$  is found for ROCs and  $L = 800$  for AOCs. Hence, it is shown than in this example, AOCs provide a shorter code than ROCs. In order to get the bit rate requested i.e. 1 kb/s, a chip period of 1.25  $\mu$ s is assigned.

## 6. CONCLUSION

We have shown the application of optical codes in an intra-satellite optical wireless network. In particular the use of a novel family of codes (AOCs) in this scenario is presented. The construction of this family is based in an algorithm and a common seed for the emitter and the receiver.

Moreover, the comparison between ROCs and AOCs used in OCDMA has been developed. AOCs present less probability of error than ROCs for any length and number of users.

We find AOCs as an alternative family of optical codes to be used in optical wireless networks. They keep the main features of ROCs: huge cardinality, adaptability to the system requirements, flexible duty cycle. Moreover they provide better performance and more capability of adaptation to the actual number of users.

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