DEPENDABILITY ESTIMATION FOR NON-MARKOV CONSECUTIVE-K-OUT-OF-N: F REPAIRABLE SYSTEMS BY RESTART SIMULATION

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1. INTRODUCTION

The reliability of consecutive-*k*-out-of-*n*: F system (or C(*k*, *n*: F) system) has aroused great interest since it was first studied by Kontoleon in 1980 [1]. The system consists of a sequence of n ordered components along a line such that the system fails if and only if at least k consecutive components in the system have failed. A list of typical applications of C(*k*, *n*: F) system was given by Yam et al. [2]. A research book by Chang et al. [3] provide rich information about C(*k*, *n*: F) system.

Although in most related research work, all the components of the system are assumed to have an equal failure rate, this is not always the case. Fu provided a convincing example [4]. Suppose that we want to transport oil from place A to place B by an oil pipeline and that there are *n* pressure pumps equally spaced between A and B. Each pump can transport the oil no more than a distance of *k* pumps. This is obviously a C(k, n: F) system. If pumps (*i*-*l*) to (*i*-1) have failed but the system still works (*l*<*k*), then pump *i* must work very hard to raise the pressure so that the oil can pass a distance of *l* pumps. Therefore, pump *i* will have a higher probability of failure, and the failure rate of a pump should depend on the states of the preceding (*k*-1) pumps. This dependence is called the (*k*-1)-step Markov dependence.

On the other hand, there has been increasing interest in the study of C(*k*, *n*: F) repairable systems. In 2001, Lam and Ng studied a model for a C(*k*, *n*: F) repairable system with (*k*-1)-step Markov dependence [5]. The lifetime of components and repair times are exponential random variables. A priority repair rule based on system failure risk was adopted. Some dependability measures were evaluated by a numerical method. For this model with a large *n*, the method would be intricate. Moreover, repair time usually does not follow exponential distribution. Xiao et al. [6] revised the model by assuming that repair time is a random variable following a general distribution. Then in this situation, the system is a non-Markov C(*k*, *n*:F) system with (*k*-1)-step Markov dependence. They used Monte Carlo simulation to estimate the dependability (including reliability, transient availability, MTTF and MTBF) of the new model. Since crude simulation is inefficient for highly dependable systems, fast simulation methods for rare event simulations as importance sampling and conditional expectation were used.

In this paper, we extend the model in Ref. [6] by assuming that not only repair time but also the lifetime distribution of components are random variables following a general distribution. Moreover, we estimate a dependability measure of great interest, the steady-state availability, which was not estimated in [6]. We use the rare event simulation method RESTART for estimating all the measures. This method has a precedent, of much more limited scope [7], in the splitting method described in [8]. M. and J. Villén-Altamirano coined the name RESTART in [9] and made a theoretical analysis that yields the variance of the estimator and the gain obtained with one threshold. A detailed analysis with multiple thresholds is made in [10].

A limitation of the RESTART methodology for simulating highly-reliable systems is the difficulty to define enough thresholds. For this reason, L'ecuyer et al. [11] pointed out that this methodology is not appropriate for this type of systems and Xiao et al. [6] pointed out that "importance splitting is hard to be adopted for dependability estimation of non-Markov systems, because thresholds function is hard to be presented under this situation". However, as it will be shown in the paper, probabilities up to the order of 10^{-12} can be accurately estimated within a reasonable computational effort.

2. CONTENT

RESTART has been described with detail in several papers, e.g., [7, 10]. Nevertheless it is briefly described here. In the RESTART method a more frequent occurrence of a formerly rare event is achieved by performing a number of simulation retrials when the process enters regions of the state space where the importance is greater, i.e., regions where the chance of occurrence of the rare event is higher. These importance regions are defined by comparing the value taken by a function of the system state, called importance function, with certain thresholds. Optimal values for thresholds and the number of retrials that maximize the gain obtained with RESTART were derived in [10].

The application of this method to particular models requires the choice of a suitable importance function. An inefficiency factor related to the importance function was analysed in [9] and guidelines for selecting heuristically such a function were provided. In this paper the following importance function (at an instant t) is defined: $\Phi(t) = cl - oc(t)$, where cl is the cardinality of the minimal cutset with lowest cardinality and $oc(t)$ is the number of components that are operational at time t in the cutset with lowest number of operational components. Thresholds T_i are 1, 2, … , cl-1.

A measure of the efficiency for computing $\hat{P}$ is given by RCNC, the relative confidence-normalized cost, which is defined as $CV(\hat{P})/P^2$, where C is the computer cost. The gain G obtained with RESTART can be defined as the ratio of the RCNC with crude simulation to the RCNC with RESTART. In [10] it is shown that G is given by:

$$G = \frac{1}{f_V\, f_O\, f_R\, f_T} \frac{1}{P\left(-\ln P + 1\right)^2} \qquad (1)$$

The term $1/\left(P\left(-\ln P+1\right)^2\right)$ can be considered the ideal gain. Factors f_V, f_O, f_R and f_T, all of them equal to or greater than 1 can be considered inefficiency factors that reduce the actual gain with respect to the ideal gain. Each factor reflects:

- f_R: inefficiency due to the non-optimal choice of the number of trials.
- f_T: inefficiency due to the non-optimal choice of the thresholds.
- f_V: inefficiency due to the non-optimal choice of the importance function.
- f_O: inefficiency due to the computer overhead produced by the implementation of RESTART.

We have studied C(k, n: F) systems for n = 60, and for k = 4 and k = 6. For k = 4, we had 57 minimal cutsets, cl = 4 and we defined 3 thresholds associated with values of Φ equal to 1, 2 and 3 respectively. For k = 6 we defined 2 thresholds more and thus the inefficiency factor f_T was lower than for k = 4.

The system unreliability was estimated for different small values of intervals (0, t_e). Simulations were made assuming first that component lifetimes are exponentially distributed (model Exp), and second assuming that component lifetime distributions are Raleigh (Weibull distribution with shape parameter equal to 2). The repair time distributions are Lognormal in all the cases. In all the runs, the simulation length was adjusted to have a relative half width of the 95% confidence interval (relative error) equal to 10%. The results are given in Table I.

Accurate results were obtained within short computational time. To evaluate the gain in time with respect to a crude simulation, the computational time for achieving a relative error of 10% with crude simulations were measured for the Exp model with $t_e = 5$ (58.4 hours), and for the Weibull model with $t_e = 10$ (16.1 hours). The measured values were extrapolated for the other cases of each model. The factor f_T is estimated in the simulation and the product of the factors f_V x f_O is obtained comparing the measured gain with the theoretical one given by equation (1).

Simulation results for the estimation of the steady-state unavailability of the system C(4, 60: F) and for the system C(6, 60: F) will be provided in the conference.

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Table I: Unreliability estimates for network in Fig.2. 95% conf. interval $= \pm 10\%$

Model	Interval (hours)	$\hat{P}$	Run-time (minutes)	Gain in time	Factor f_T	Factors f_V x f_O
Exp	(0, 25)	2.5×10^{-10}	0.4	274039	10.1	2.7
Exp	(0, 5)	3.6×10^{-12}	1.0	6162037	21.7	2.8
Exp	(0, 1)	1.1×10^{-13}	4.5	42994924	50.1	4.4
Weibull	(0, 50)	1.1×10^{-6}	0.5	729	3.1	1.9
Weibull	(0, 15)	5.1×10^{-10}	3.2	199501	9.0	2.2
Weibull	(0, 5)	2.6×10^{-13}	23.7	5.2×10^{7}	32.4	2.5