Application of Fractal and Wavelets in Microcalcification Detection

M.G. Cortina-Januchs, J.Quintanilla-Dominguez, Aleksandar Jevtić, Diego Andina.
Departamento de Señales, Sistemas y Radiocomunicaciones.
Universidad Politécnica de Madrid.
Madrid, Spain.
Email: l.januchs@gc.ssr.upm.es, joelq@salamanca.ugto.mx,a.jevtic@gc.ssr.upm.es,andina@gc.ssr.upm.es.

Abstract— **Breast cancer has been recognized as one or the most frequent, malignant tumors in women, clustered microcalcifications in mammogram images has been widely recognized as an early sign of breast cancer. This work is devote to review the application of Fractal and Wavelets in microcalcifications detection.**

I. Introduction

Breast cancer is leading cause of mortality in women population. The early detection and diagnosis of breast microcalcifications (MC) are the key decrease death rate and provide prompt treatment. One of the significant signs of possible cancerous changes is an existence of small deposits in the breast tissue, usually referred as MC. MC appear in mammograms as small points (0.2 to 0.3 mm), localized or broadly diffused points along the breast, or even in clustered pattern. Now MC are detected and diagnosed by a combination of physical examination, imaging and biopsy. Up to date, to confirm whether a patient has breast cancer, it has to rely on biopsy. However, biopsy is one kind of invasive surgical operation and imposes both psychological and physical impacts on patients [1], [2]. Mammography, ultrasound and Thermal texture mapping system are the main image techniques for detect MC [3].

Mammograms are among difficult radiological images to interpret by radiologists, in general, its images are often very poor in contrast, can show different features and patterns depending on breast anatomy and tissue density, which vary with patient age and its hormonal or physical condition. In Figure 1 show an example of mammography image. Certainly, by digitizing radiology films and applying digital images processing algorithms, significant improvements of image analysis are posible. Designing an effective diagnosis system is a problem widely investigate now a day. The researchers have centred your attention on two principal problems: the first is to distinguish between the normal tissue and different types of tumor such as MC cluster, spiculated lesions, circumscribed masses, ill-defined lesion. The second function is to differentiate between benign and malignant tumors. The paper is organized as follow: section 2, basics principles of fractals, definition and characteristics, box counting method; section 3, a review of application of fractal in MC detection; section 4, basics principles of Wavelet transform; section 5, a review of application Wavelet transform in MC detection; section 6, one presents a brief description of the classifier types due to the importance that they have in the selection and classification of MC characteristics; section 7, contains a brief discussion; section 8, present conclusions.

[1]This research has been partially supported by the National Spanish Institution "Ministerio de Educación Y Ciencia" as part of the project AGL2006-12689/AGR. and Consejo Nacional de Ciencia y Tecnología (CONACYT) of Mexico

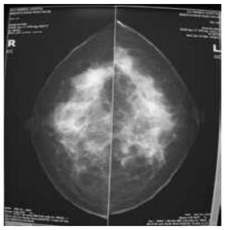

Fig. 1. Example mammography image.

II. Fractals

The concept fractal was introduced by Mandelbrot [4] and was derived from the Latin fractus meaning broken or fractured. Basically, a fractal is any pattern that reveals greater complexity as it is enlarged. A fractal often has the following features:

- It has a fine structure at arbitrarily small scales.
- It is too irregular to be easily described in traditional Euclidean geometric language.
- It is self-similar (at least approximately or stochastically).

Because they appear similar at all levels of magnification, fractals are often considered to be infinitely complex (in informal terms). Natural objects that approximate fractals to a degree include clouds, mountain ranges, plants, lightning bolts, coastlines, and snow flakes. However, not all self-similar objects are fractals. The fractal figures are useful in areas as botany, biology, physics, mathematics, economy, computation, linguistics and the art.

A. Multifractal analysis

The simple knowledge of the fractal dimension of an object is insufficient to characterize his geometry, as well as also any physical property inherent in the above mentioned object.

The fractal dimension is a non integer number and describes uniform objects or homogeneous systems, but it does not offer any information to near the low or high irregular distributions inside the system.The most popular way for calculating the fractal dimension is a box counting method [5].

The concept of multifractal contemplates an infinite number of fractal dimensions and therefore it can be adapted for the description of physical properties. A multifractal process is characterized by events extreme and more or less isolated, associated with an average that represents the "matter" contained in every pixel of the image. The use of multifractals in images is an excellent tool to characterize irregularities of a curve or a surface.

B. Box-Counting Method

The quantitative description of multifractal property can be derived in several ways. Usually, the procedure stars with finding the noninteger exponent (α), known as the Hölder exponent, describing the pointwise singularity of the object, and then deriving the distribution of this quantity, known as multifractal spectrum $f(\alpha)$.

The structure S be divided into nonoverlapping boxes S_i (Figure 2) of size ε. Each box S_i is characterized by some amount of measure, $\mu(S_i)$, and boxes may be assumed as measure domains. An equivalent parameter suggested to the multifractal analysis is defined by Eq. 1

$$\alpha_i = \frac{-ln(\mu(S_i))}{ln(\varepsilon)} \tag{1}$$

Parameter α depends on the actual position on the fractal end describes local regularity of the structure. In whole structure there usually are many boxes with the same parameter α. The multifractal spectrum describes the global regularity of observed structure. The multifractal spectrum can be assumed as the fractal dimension over the subset characterized by α (Eq. 2).

$$f(\alpha_i) = \frac{-ln(N_\varepsilon(\alpha_i))}{ln(\varepsilon)} \tag{2}$$

where $N_\varepsilon(\alpha_i)$ is the number of boxes S_i containing α_i. The multifractal spectrum is also known as the Hausdorff dimension. To extend the information about box-counting method consult Tomislav [5].

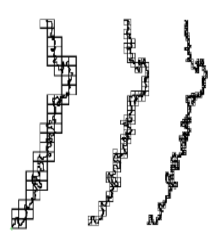

Fig. 2. Box-Counting method to different scales

The multifractal approximation has been used to characterize variations in natural phenomena, including the spatial distribution of the rains, characteristics of the minerals and fractures of surfaces, spatial distribution of earthquakes, pattern analysis of vegetation, properties of the soil [6], [7], river networks [8]. The use of the fractal theory in the analysis of medical images has received importance in the recent years [9].

III. Application of fractal in microcalcification detection

The multifractal analysis is a powerful mathematical tool that allows to characterize complex objects. It separates structures and discovers the relations that they support among different components, at the same time as allows to plan possible evolutions or behaviors of these structures. It is for this reason that nowadays they are used for the analysis of edges, segmentation, description of irregularities and textures tissue.

V. Velanovich et al. [1] propose a method to discriminate the malignant and benign lesions, using the fractal dimension calculated with the box-counting method, with different sizes of box (2mm, 4mm, 8mm and 16mm) in a the portion of image where one was finding a MC, 75 patients were analyzed, obtaining that the fractal dimension of malignant lesions was higher than benign lesions. The average ($\pm$ the standard deviation) of the malignant lesions were 2.545$\pm$0.067 compared with the benign lesions were 1.936$\pm$0.114.

Dar-Ren et al. [2] use fractal techniques to quantify the information of the texture in ultrasound images of breast, taking into account that a benign lesions are classically described as regular masses with homogenous internal echoes, but malignant lesions are described as masses with fuzzy border and heterogenous internal echoes. They propose fractal characteristics to differentiate the benign and malignant breast lesion, realizing this analysis using a Region Of Interes (ROI) of data bases with histologically confirmed cases: 110 malignant tumors and 140 benign tumors. The ROIs containing the tumors are very dissimilar in image size and different in gray levels, for improve the images they used morphological operations (erosion and dilatation) as an image filter to eliminate de noise and histogram equalization. After they calculate the fractal Brownian motion to describe texture features to be classified later in malignant or benign lesions. Finally the k-means classification method was adopted for classifying the fractal features in two classes (benign and malignant lesions). From the experimental results they concluded that the fractal analysis is useful to represent the texture information of breast lesions. The accuracy rate of proposed system is up to 88.80%

Tomislav et.al [5] propose the adaptation of multifractal analysis to segmentation of MC, using 25 cases of the MIAS database, applying variations in the image processing analyzing the texture. The images contrast were improve to use a negative image before obtain its fractal dimension. They analyzed images in two domain, the first domain is the original image and the second correspond to multifractal transform domain (α and f(α)). The box-counting method was use to find the fractal dimension, using box sizes of 1 to 16 pixels. They obtained that fractal dimension (α) in edge points is high and multifractal spectrum (f(α)) is low, these values represent a sharp change or rare event in images. The characteristics extracted were segment by means of umbralization. In all case the method successfully found declared MC even in dense images where they are highly invisible within the background tissue.

M.E. Mavroforakis et al. [10] propose a method to extraction of MC texture characteristics. Each images was first segmented into mass and non-mass according to the tumor's boundary description provided by the experienced physician. The identified mass tissue areas were further divided according two configurations, into continuos sub-regions of sampling boxes and sizes of 20 and 50 pixels. The specific sampling box sizes correspond to spatial resolution of 1.27 and 3.175 mm of mass tissue, which were asserted by the expert as segments of adequate to capture significant textural information content in mammography. The result was the creation of two compound texture datasets, namely for sampling boxes of sizes 20 and 50 pixels, each contenting a complete set of textural features over three pixel-neighboring configuration. The total number of features extracted for each sampled sub-region of the images was 120. The MANOVA (Multivariate Analysis Of Variance) method was employed to select the most prominent features. From the resulting ranked sets, subset of the best 10-20 features were evaluated in real classification cases against verified diagnostic, using the linear and non-linear classifiers (K-nearest neighbors, radial basis function, multi-layer perceptron, support vector machine [25]). According with the results, they concluded that the sampling box size and pixel neighboring scheme proved to be very important factor in the optimization and final selection of the various textural features functions. The best results in the classification was obtain with support vector machine.

Fredrik Georgsson et al. [11], analyze the difference in local fractal dimension between glandular tissue, supporting tissue and muscle tissue based on assessment from a mammpgraphy. The mammography used in this study are randomly selected from MIAS databases,in total they have use 142 different mammograms. The fractal Brownian motion was compute, obtaining the tissue local dimensions and the tissue density. These characteristics were classify by means of umbralization knowing the a priori probability of each tissue. They conclude that there is a difference in tissue local fractal dimension but the high variance of the local density function of local dimension makes it difficult to make use of this difference in tissue segmentation.

D.A. Cristian et al [12], characterize the mammographic lesion using the fractal property. The analysis was base in the contour shape, because a regular anomaly is associate to a benign case, while an irregular shape characterizes a malign lesion. On the other hand, the fractal dimension grows with the irregularity of the shape. They used a system proposed by the American college of radiology in order of classification lesions. The scope BI-RADS (Breast Imaging Reporting and data systems). Experiment was developed on a lot of 30 cases, 18 benign cases an 12 malign case. The study area was marked by the radiologist (64 x 64, 128 x 128, 256 x 256 or 512 x 512 pixels), after the images were binarize using a threshold between 1-255 gray level; all pixels whose gray level is greater or equal to the threshold will be transformed in white, the rest will become black. The contour is automatically traced and the fractal dimension is computed using the box counting method. The results show that the benign lesions have lower fractal dimension, between 1-1.50, while malign lesions have higher dimension, between 1.35-2. They conclude that the fractal dimension grows as the irregularity of the object grows and a regular outline is associated to a malign lesion.

IV. Wavelets

A. Wavelet Transform (WT)

The wavelet is basis functions of the wavelet transform, are generated from a basic wavelet function, by means of translation and dilatation. The wavelet transform is not only local in time, but in frequency as well. These functions allow to rebuilt the original signal using the inverse wavelet transform (IWT). Ψ is a function to referred to as mother wavelet or basic wavelet:

$$\int_{-\infty}^{+\infty} \Psi(t)dt \tag{3}$$

The wavelets are generated using translation and changing the scale of the same basic function:

$$\Psi_{u,s}(t) = \frac{1}{\sqrt{s}} \Psi \left(\frac{t-u}{s} \right) \tag{4}$$

where $s \in R^{+}$ is the scale or dilation parameter and $u \in R$ is the translation parameter. The values of s and u can be calculate using $s = 2^{-j}$ and $u = k^{2-j}$ while j and k are integers.

The wavelet transform of a function f with parameters s and u is calculated as follows:

$$Wf(u,s) = \int_{-\infty}^{+\infty} f(t)\Psi(t)^{*}_{u,s}(t)dt \tag{5}$$

Where $\Psi(t)^{*}$ denotes the complex conjugate of the $\Psi(t)$ [14].

Wavelets decomposition applied to 2D produces four different coefficients for each level of decomposition. The coefficients are: low frequency coefficients (A), vertical high frequency coefficients (V), horizontal high frequency coefficients (H), diagonal high frequency coefficients (D).

The Figure 3 show an example of the 2D wavelet decomposition [16].

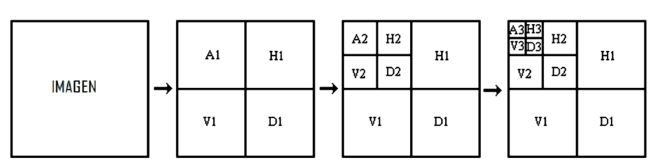

Fig. 3. Wavelets multiresolution decomposition [18].

V. Application of wavelet in microcalcification detection

Wavelet transform is widely applied, i.e. to the analysis of non stationary signals, analysis of electrocardiogram signals, mammography signals, sound signals, radar signals, image processing and patter recognitions. In mammography images, the fundamental operation needed to facilitate the MC detection is contrast enhancement. This is specially important in dense breast images where contrast between malignant tissues and normal tissues may be bellow human visual perception. There are two possible methods to enhance image features. One is the removal of background noise, the other is to increase the contrast on suspicious regions using linear or non-linear operations. The multiscale representation is frequently used improve the image contrast. The main objective is to make high frequency features more visible, thus facilitating image interpretation. The wavelet analysis allows decomposition the high frequencies the localization of the important features, allowing the decomposition of images into different levels.

J. Salavado et al. [14] use the wavelet transform for noise removal and in improve the image contrast, for facilitating MC detection. The authors applied the wavelet transform using Daubechies 6 coefficients with 10 decomposition levels. The MC became more visible for a better diagnosis. The MC in the mammographies generally appear as small groups of few pixels with relatively high intensity. From this aspect the wavelet transform represents an excellent analysis method using small windows and large windows, to separate the high frequencies of the MC from the low frequencies of the tissue in the mammographies.

Vega-Corona et al. [15], [16], implemented features extraction in mammographies using 50 samples from the Digital Database for Screening Mammography (DDMS) of the University of Florida. Initially, they selected a region of interest (ROI) where MC were diagnosed. Using adaptive histogram equalization technique to improve the contrast, they extracted the required features applying Daubechies 4 coefficients wavelet in 4 decomposition levels. They use a four decomposition levels to build four features by pixel. Additionally, use a method to found two features the most relevant in gray level structure, as the local contrast (LC) and normalized local contrast (NLC). In your experiment choose the window LC size as 9 x 9 and the window NLC size as 3 x 3. In the stage of classification they applied an unsupervised statistical method base on improved K-means algorithm. Later a GRNN neural network and Multilayer Perceptron neural network uses as selector of features obtaining better results with GRNN neural network.

Sung-Nien et al. [17], develop a CAD (Computer Aided Diagnostic) system for the detection of MC. For the experiments use mammographies with the MC area marked by the radiologist. In agreement the information provided by MIAS the MC grey levels varies in the range from 175 to 226 . They decompose the mammographies into a two-levels wavelet representation with Daubechies 4 coefficients, and reconstruct the mammographies by eliminating low frequency subbands in the second level. Subsequently Markov random field parameters based on the Derin-Elliot model are extracted from the neighborhood of every suspicious MC as the primary features. The features vectors (with wavelet filters and MRFP) are segmented by Bayes classifier and backpropagation neural network, for both cases 25 images with MC tissue and 46 with normal tissue use for training set. The performance is valued with two factors: sensitivity and specificity. Where sensitivity is true positives rate and specificity is false positives rate for normal tissue. The test set was of 20 mammographies, each one with 25 zones of MC cluster, obtaining 89% of false positives removed and sensitivity of 92%.

Essam et al. [18] proposes a supervised classifier for digital mammographies using the wavelet transform decomposition. A set of images that are provided by data base MIAS (Mammographic Image Analysis Society). They use Daubechies 4, 8 y 16 wavelet using four different decomposition levels. The Euclidean distance is used to design the classifier that is based on calculating the distance between the feature vectors and class core vector. The core vector represents each class (normal tissue, clusters of mc, circumscribed masses, spiculated lesions). They achieve interesting results classifying the lesion types and distinguish between benign and malignant lesions. They obtain the best results with wavelet coefficients Daubechies 8, with 91.7% of cases benign detecting and 100% cases malignant detecting.

Sepehr et al. [19] achieve a decomposition wavelet to estimate MC presence. At first, a mammography pre-processing is achieve to locate the breast region, that image is passed by a median filter. At the image obtaining applying the wavelet transform with wavelet coefficients of Daubechies 2 to 10, Symlet 2 to 8, Coiflet 1 to 4 and Biorthogonal 1.3 to 6.8, obtaining 32 features of each MC. This features fall into three categories related with the intensity, shape and texture properties of each object; the features selection is achieve, applying principal component analysis method for to eliminate the features that contribute less than 3%. The best feature were 7, this features are classified by Feed Forward Neural Network that contains one input, two hidden and one output layers. They obtain the best performance for the coefficients Coiflet, concluding that the combination of intelligent methods and processing image technique can contribute to the improvement the diagnosis and reduction of biopsies of breast.

Juárez et al. [20], they use 30 mammographies of the database MIAS (15 images with MC and 15 with glandular tissue) for detecting MC. At first, they obtain the negative image, after applying the wavelet transform using two decomposition levels, for each image they use coefficients wavelet Daubechies 2, 4, 8 and 18, the image resulting is applying a threshold for obtain the area with MC, they obtain a 80% of success in the detection of the MC.

Retico et al. [21] use the databases CALMA INFN collected in the Italian National screening program and the database MIAS, they develop a CAD system based en WT and ANNs.

The breast region is decompose in the overlap partially images of 5 mm.The feature obtained are classify under two class (MC and normal tissue). The system provides in the exit an image with the contours drawn in the suspicious area of the original image. Applying a wavelet decomposition of 4 levels and 5 coefficients Daubechies. After the features obtained are processing by self-organizing neural network for obtain the better features. With the better features the classification is applying by feed forward neural network with 3 layers. This methodology appears as an option of unification given the different methods and resolutions of acquisition of these images, obtaining a standard form of mammographies that allows the classification of the different databases.

L. Song et al. [22],propose a method for the detection of MC using the database MIAS, applying morphology technic to heighten MC contrast. The wavelet transform is applied to 3 decomposition levels with coefficients Daubechies 4. The image reconstruction using a global umbralización, this images is multiply by the original image(AND operation), obtaining a final image with the MC enhancement and the background tissue eliminated. Obtaining as result 85.37 % of TPR for the images obtained of the morphologic treatment, 91.84 % for the images obtained with the wavelet transform and 80.2% with the multiplication of both features, with a cost of the FP (false positive) of 7.9%, 10% and 2.5% respectively. Concluding the methods of morphology and DWT presents True Positive (TP) high range, but also FP's high range, on the other hand the results obtained by means of the operation AND it guarantees TP's high range and reduces the FP.

VI. Classification Methods

The methods of classification are an important factor in the features recognition that represent an image. For this reason some classification methods are mentioned, knowing that perfect classifier does not exist, the final result always depends on the problem and the data used. The classifiers of agreement to your type of learning can divide in two groups: supervised and non-supervised. The supervised classifier is that one in which a knowledge is had a priori of the problem, by training data set and test data set that fits to the classifier up to obtaining the wished result. The non-supervised classifier does not have knowledge a priori, using only of similarities between the data of entry for the classification.

Some methods of supervised classifications are:

- Based in distance (least distance classifier and K-nn algoritm)
- Statistical (Bayes classifier, Parzen)
- Based in Artificial Neural Networks (Multi-layer Perceptron neural network, backpropagation neural network, radial base neural network)
- Support vector machine

Later some methods of non-supervised classifications are:

- Hierarchical algorithms
- No-hierarchical algorithms (k-means, fuzzy k-means)
- Based in Artificial Neural Networks (self-organizing maps neural network)
- Genetic algorithms

for major reference and applications see [23]–[25].

VII. Discussion

The knowledge of the multifractal dimension of an image allows to find it characteristics , they represent to every type of object inside the image. In the study of the mammography images the use of the multifractal theory allows to characterize tissue type (glandular fabric, oily fabric, muscular fabric) [11], the type of MC edge (definite or diffuse) [5], [10], the type of lesion (malignant or benign lesions) [1], [2], [12].

In spite of the fact that the method more frequently used to calculate the fractal dimension is the box-counting method [5], there does not exist a convention or size established of boxes dimension to use, knowing that different size of the MC between 3-300 mm [10], V.Velanovich [1] proposes boxes of size 2, 4, 6, l6 mm, M.E. Mavroforakis [10] of 1.27 and 3.17 mm, other authors omit the box size used.

The use of a region of interest (ROI) like in case of V.Velanovich [1], Dar-Ren [2] and F.Georgson [11] allows to isolate the area with MC, simplifying this way the problem of size image and image processing, nevertheless to the being different the specialists entrusted to select the ROI's this decision becomes subjective, problem that is overcome by the use of the multifractal theory [2].

Dar-Ren [2] mentions the importance of the image pre-processing, for the improvement of the contrast between MC and normal tissue, the pre-processing methods change according to the authors, they emphasize the histogram equalization [2] and inversely of the image [5].

The evaluation of every methodology is realized by the a priori knowledge of the image diagnosis and the method of classification proposed depending of the author (supervised or not supervised). The used methods are: the umbralization, k-means, linear discriminant analysis, least-square minimum distance, k-nearest neighbor, radial basis function, multi-layer perceptron artificial neural network, support vector machines classifier.

The TWV is raised to improve the characteristics of mammogram image contrast, being the principal aim to do the characteristics of high frequency more visible (MC), to facilitate the image interpretation, obtaining images to different levels of detail [14].

The authors realize your experiments using the databases: DDSM, MIAS and CALMA INF. Selecting a ROI, that can change depending on the specialist who chooses them. The image is process in some cases to heighten the grey levels, the histogram equalization [16] or extracting the negative of the image [20], the majority of the authors does not mention this step.

The principal variation in the followed methodologies is in the number of wavelet decomposition level applied to the images and the type of used coefficients. The levels of decomposition change from 2 to 10, being used in the bibliography checked mainly the decomposition of 4 levels [15], [16], [18], [21], followed by 2 niveles [17], [20]. The coefficients change equally depending on the author (Daubechies, Symlet, Coiflet) being more used Daubechies of 4 coefficients.

The selection of characteristics is realized using generalized neural network [15], principal component method [19], and self organizing mapping neural network [22]. The classification between normal tissue and MC is realized with methods like k-means, feedforward neural network, and by means of Euclidean distance from established center vectors .

VIII. Conclusions

The multi-fractal approximation allow the analysis of the mammography image independent of size of the studied image. In general the multi-fractal approximation allow obtain information of the shape and texture from studied object, it is for this reason that has been widely used for the analysis of breast images. The processing of the images is very important due to the features of MCs, for contrast enhancement. In the calculation of the fractal dimension by box counting method is very important select correctly the box size, for avoid the redundancy of the information, select correctly the box size is it is possible to identify the zone of the image where the MC are located.

The procedure of pre-processing of the image improves the signals that reveal MCs presence, while suppress and noisy tissue non-pathological. A mammography usually dominated by information of low frequency, while the MC appears as contributions of high frequency. The wavelet use allow the separation of the high resolution components from mammography of the less important, low resolution components. The texture of the mammographies is irregular, the contrast and the homogeneity can be improved in the transformed domain multiresolution. The features of the mammography image obtained using the wavelet transform can be improved selecting the level of the decomposition and the kind of given coefficient.

References

[1] Vic Velanovich, "Fractal Analysis Of Mammographic Lesions: A prospective, blinded trail. Breast", Cancer Research and Treatment, 1998, Vol.49, No. 3, Pages 245-249, June, Springer Publisher.

[2] Dar-Ren Chen, Ruey-Feng Chang, Chii-Jen Che, Minng-Feng, Shou-Jen Kuo, Shou-Tung Che, Shin-Jer Hung, Woo Kyung Moon. "Classification of breast ultrasound image using fractal features", Journal of Clinical Imaging, 2005, Vol. 29, No. 4, Pages 235-245, Elsevier Publisher.

[3] Yune Yuan, Qing Wang, Yan Tan, Li Rong, Song San Tai, Li JI Ye, Zhongqi Lui. "Analysis of Breast Diseases Examination with Thermal Texture Mapping Mammography and Ultrasound". Proceeding of the 26th annual international conference of the IEEE EMBS, 2004, Pages 1166-1169, San Francisco, CA, USA.

[4] Mandelbrot B.B. "Fractal Geometry of Nature", New York: W.H.-Co, 1983.

[5] Tomislav Stojic, Irini Reljin, Branimir Reljin, "Adaptation of multifractal analysis to segmentation of microcalcifications in digital mammograms", Physica A: Statistical Mechanics and its Applications, 2006, Vol. 367, Pages 494-508,July, Elsevier Publisher.

[6] Alaxandra N. Kravchenko, Charles W.Boast, and Donald G.Bullock, "Multifractal Analysis of Soil Spatial Variability", Agronomy Journal, 1999, Vol. 91,Pag. 1033-1041.

[7] J.A.Piñuela, D.Andina, K.J. McInnes, A.M.Tarquis. "Wavelet Analysis in a Structured Clay Soil Using 2-D Image". Non Linear Processes in Geophys, 2007, Vol. 14, Pag. 425-234.

[8] Samuele G. De Bartolo, Primavera Leonardo, Gaudio Roberto, D Ippolito Antonio, "Fixed-mass multifractal analysis of river networks and braided channels". Physical Review American Physical Society E , 2006, Vol.74, No. 2.

[9] Li Tian-Gang, et.al, "Fractal research of pathological tissue image", Computerized Medical Imaging and graphics , 2007, Elsevier, Vol. 31, Pag. 665-671.

[10] Michael E. Mavroforakis, Harris V. Georgiou, Nikos Dimitropoulos, "Mammografic masses characterization base on localizad texture and dataset fractal analysis using linear neural and support vector machine classifiers", Artificial Intelligent medicine, Elsevier, 2006, Vol. 37, Pag.145-162l

[11] Fredrik Georgson, Stefan Jansson, and Christina Olsén, "Fractal analysis of mammograms", 15th Scandinavian Conference on Image Analysis, Springer, 2007, Vol. 4522, Pag. 92-101.

[12] Daniela Alexandra Cristian, Radu Dobrescu, Peter Planinsic. "Mammographic Lesions Discrimination Based on Fractal Dimension as an Indicator", Systems Signals and Image Processing, and 6th EURASIP Conference focused on Speech and Image Processing, Multimedia Communications and Services, 14th International Workshop on, 2007, Pag. 74-77.

[13] Matteo Baldoni, Cristina Baroglio and Davine Cavagnino, "XFF: A Simple Method to eXtract Fractak Features for 2D Object Recognition", Lecture Notes in Computer Science, 1998, Springer Berlin , Vol.1451, Pag. 382-389

[14] José Salvado, Bruno Roque. "Detection of Calcification in Digital Mammograms using Wavelet Analysis and Contrast Enhancement". Intelligent Signal Processing, IEEE International Workshop On, 2005, Pag.200-205

[15] Antonio Vega-Corona, A. Alvarez-Vellisco, D.Andina. Feature Vectors Generation for Detection of Microcalcifications in Digitized Mammography using Neural Network, Lecture Notes in computer science, 2003, Springer, Vol. 2686, Pag. 1045

[16] Antonio Vega-Corona, Alexis Marcano, Diego Andina. "Computer-Aided Diagnosis Scheme Applying GRNN Neural Network", Proceeding of The 4th International Conference on Cybernetics and Information Technologies, 2007, Vol.3, No. 258, Pag.

[17] Sung-Nein Yu, Kuan-Yue Li, Yu-Kun Huang. "Detection of microcalcification in digital mammograms using wavelet filter and Markov field model", Computerized Medical Imaging and Graphics, 2006, Elsevier, Vol.30, Pag 163-173

[18] Essam A. Rashed, Ismail A. Ismail, Sherif I. Zaki, "Multiresolution mammogram analysis in multilevel descomposition". Pattern Recognition Letters, Science Direct, Elsevier, 2007, Vol. 28, Pag. 286-292

[19] A. Sepehr M.H. Jamarani, b. Gholamali Rezai-rad, Third c.Hamid Behnam, "A novel Method for Breast cancer prognosis using wavelet packet based neural network". Proceedings Engineering in Medicine and Biology 27th Annual Conference of the 2005, IEEE, pag. 3417-3417

[20] Cristina Juarez L., Volodymyr Ponomaryo, J. Luis Sanchez. "Detection of microcalcification in digital mammograms image using wavelet transform". Proceedings if Electronics, Robotics and Automotive Mechanics Conference (CERMA'06), 2006, IEEE Computer Society.

[21] A. Retico, P.Delogu, M.E. Fantacci, A. Preite Martinez, A.Stefanini, A.Tata. "A scalable computer-aided detection system for microcalcification cluster identification in a pan-European distributed database or mammograms". Neural Instruments and Methods in Physic Research A, 2006, Elsevier, Vol. 569, Pag. 601-605

[22] Lixin Song, Qi Wang, Jie Gao, "Microcalcification detection using combination of wavelet transform and morphology". Signal Processing 8th International Conference On (ICSP), 2006, IEEE, Vol. 4.

[23] J.M. Barrón-Adame, M.G.Cortina-Januchs, A.Vega-Corona. "A Neural Network to Solve Air Pollutant Level Estimation", 2007, Proceeding of The 4th International Conference on Cybernetics and Information Technologies 07, Vol.3,258, Pag. .ISBN-10 1-934272-10-8. ISBN-13. 978-1-934272-10-7

[24] Barrón-Adame JM, H. Delgado J, Cortina-Januchs MG, Andina D, Vega-Corona A. "Air Pollutant Level Estimation Applying a Self-organizing Neural Network" en Lecture Notes in Computer Science 4528, Editores: J. Mirá, Springer-Verlag, Berlin-Heidelberg, 2007, pp., ISBN 3-540-42237-4. ISSN 0302-9743. ISSN electrónico: 1611-3349.

[25] D. Andina, Duc Truong Pham (Eds). "Computational Intelligence for Engineering and Manufacturing". Springer, Alemania, marzo 2007, 219