Cosmological singularities and modified theories of gravity

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Abstract. We consider perturbative modifications of the Friedmann equations in terms of density corresponding to modified theories of gravity proposed as an alternative route to comply with the observed accelerated expansion of the universe. Assuming that the present matter content of the universe is a pressureless fluid, the possible singularities that may arise as the final state of the universe are surveyed. It is shown that, at most, two coefficients of the perturbative expansion of the Friedman equations are relevant for the analysis. Some examples of application of the perturbative scheme are included.

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INTRODUCTION

Recent astronomical data from Type Ia supernovae [1] as well as from the CMB spectrum [2] confirm that our universe is undergoing an accelerated expansion period. In order to comply with this feature, one may resort to postulating a dark energy content for the universe [3], with undesired properties, such as violation of some energy conditions, or going beyond general relativity in the quest for another theory of gravity [4, 5].

With either approaches for dealing with the observed accelerated expansion, cosmology is much richer than it was thought in the previous century. According to classical cosmologies, the universe started at an initial singularity, the Big Bang, and it was doomed to expand forever, since the matter content was not dense enough to stop expansion and collapse into a final singularity.

MODIFIED GRAVITY

Instead of dealing with a full theory of modified gravity, we focus on the consequences at the level of cosmological equations, namely Friedmann equation, just requiring that it admits a generalised power expansion on the density ρ around a value ρ_*,

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = h_0(\rho-\rho_*)^{\xi_0} + h_1(\rho-\rho_*)^{\xi_1} + \cdots, \quad \xi_0 < \xi_1 < \cdots. \tag{1}$$

TABLE 1. Singularities in FLRW cosmological models

η_0	η_1	η_2	**Tipler**	**Królak**
$(-\infty,0)$	(η_0,∞)	(η_1,∞)	Strong	Strong
0	$(0,1)$	(η_1,∞)	Weak	Strong
	1	$(1,2)$	Weak	Weak
		$[2,\infty)$	Weak	Weak
	$(1,2)$	(η_1,∞)	Weak	Weak
	$[2,\infty)$	(η_1,∞)	Weak	Weak
$(0,\infty)$	(η_0,∞)	(η_1,∞)	Strong	Strong

The standard density term arises as the lineal term with an exponent equal to one and the cosmological constant appears with null exponent in this expansion. Further terms are interpreted as modifications of the theory.

On the other hand, the energy conservation law implies

$$\dot{\rho}+3H(\rho+p)=0, \tag{2}$$

but assuming that the accelerated expansion of the universe is solely due to the modification of the theory of gravity, we choose a pressureless dust as matter content of the universe, so that the scale factor of the universe and the density are related by $\rho a^3=K$.

Thereby we may get rid of the scale factor and write down a modified Friedmann equation in terms of density,

$$\frac{\dot{\rho}}{\rho} \;=\; -3\sqrt{h_0}(\rho-\rho_*)^{\xi_0/2}-\frac{3}{2}\frac{h_1}{\sqrt{h_0}}(\rho-\rho_*)^{\xi_1-\xi_0/2}+\cdots \quad . \tag{3}$$

Solving this equation provides a perturbative expansion of the density in coordinate time, which we want to compare with a similar expansion for the scale factor,

$$a(t)=c_0|t-t_0|^{\eta_0}+c_1|t-t_1|^{\eta_1}+\cdots, \quad \eta_0<\eta_1<\cdots,$$

in terms of coordinate time. This is useful, since in [6] we have related the exponents η_i with the strength of singularities [7, 8], as we see in Table 1.

Inserting the modified Friedmann equation into the conservation equation one gets:

Our purpose it to integrate the latter by considering all the possibilities which arise from different values of the parameters, and then use the aforementioned map between the energy density and the scale factor so that we can finally obtain asymptotic expressions for the expansionary behaviour of the models. Then, we will identify the specific late-time behaviour of the models, focusing on the existence of future singularities of various types. This classification resorts to earlier works by ourselves.

<A subsubsection>

Et iam nox umida caelo praecipitat J_{ion} suadentque cadentia sidera somnos. Sed si tantus amor casus cognoscere nostros et breviter Troiae J_{ion} supremum audire laborem:

$$J_{ion} = A \frac{exp\left[-\frac{E_a}{kT}\right]}{kT} \alpha \tag{4}$$

lamentabile regnum cruerint Danai; quaeque ipse miserrima vidi, et quorum pars magna fui. A talia fando, E_a iam nox umida, k caelo praecipitat, suadentque cadentia sidera somnos. See (4).

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TABLE 2. Average turnover per shop: by type of retail organisation

	Single outlet	**Small*** **multiple**	**Large multiple**	**Total**
1982	98	129	620	847
1987	138	176	1000	1314
1991	173	248	1230	1651
1998†	200	300	1500	2000

* 2-9 retail outlets
† predicted

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