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Laminar Gas Jets in High-Temperature Atmospheres

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Abstract Numerical and asymptotic methods are used to describe the structure of low-temperature laminar gas jets discharging into a hot atmosphere of the same gas in the limit of small jet-to-ambient temperature ratios $\varepsilon = T_j/T_o \ll 1$. In the limit $\varepsilon \to 0$, heat conduction cannot modify significantly the temperature in the cold gas, leading to a two-region flow structure consisting of a neatly defined unperturbed cold jet for $r < r_f(x)$, where $T = T'/T_o = \varepsilon$ and $u = U'/U_j = 1$, surrounded by a hot gas. These two region are separated by a transition layer where $T - \varepsilon \sim \varepsilon$ and $1 - u \ll 1$. In planar jets the front thickens with distance achieving thickness of order unity at axial distances $x \sim \varepsilon^{-(1+\sigma)} Rea$ forcing the near-axis fluid to change slowly its velocity and temperature, being necessary distances $x \sim \varepsilon^{-2} Rea$ to reach values of T and $1 - u$ of order unity. In round jets the front remains at $r = a$ up to distances $x \sim \varepsilon^{-(1-\sigma)} (\log \varepsilon^{-1})^2$ where the front is forced to move radially towards the axis of the jet, reaching $r = 0$ at $x \sim \varepsilon^{-1} \log \varepsilon^{-1} Rea$. The arrival of the front forces the change of the velocity and temperature in the near-axis region, reaching values of order unity in a far field region of characteristic length $x \sim \varepsilon^{-1} \log \varepsilon^{-1} Rea$, distance comparable to that needed by the front to achieve the axis. In both geometries, the distance necessary for the fully development of the cold jet is considerably longer than that required by the isothermal jet $x \sim Rea$.

Keywords: laminar, cold gas jet, front, compressible, asymptotic

1 Introduction and formulation

This paper describes the structure of plane and round laminar gas jets discharging into a hot atmosphere of the same gas in the limit of small jet-to-ambient temperature ratios $\varepsilon = T_j/T_o \ll 1$. The boundary-layer approximation is used to describe the slender steady solution that emerges for moderately large values of Reynolds number $Re = \rho_o U_j a/\mu_o$, where U_j represents the exit jet velocity, a is its characteristic transverse dimension (the initial radius for the round jet and the initial half-width for the planar jet) and ρ_o and μ_o are the ambient values of the density and viscosity.

The convection-diffusion balance indicates that at downstream distances from the jet exit of order Rea the acceleration due to the shear stresses and the cooling due to heat conduction have reached transverse distances of order a in the ambient fluid, generating a coflowing stream surrounding the cold jet where the velocity is of order U_j. In formulating the problem in dimensionless form, we choose to use the scales of this initial development region. Thus, the density and viscosity ρ and μ are scaled with their ambient values, while the streamwise and transverse coordinates x and r are scaled with $Re\, a$ and a, respectively, and the axial and radial velocity components u and v are scaled with U_j and $\mu_o/(a\rho_o)$, leading to a formulation independent of the Reynolds number.

In terms of the dimensionless variables selected, the problem reduces to that of integrating

$$\frac{\partial}{\partial x}(\rho u)+\frac{1}{r^i}\frac{\partial}{\partial r}(\rho r^i v)=0 \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=\frac{1}{r^i}\frac{\partial}{\partial r}\left(r^i\mu\frac{\partial u}{\partial r}\right) \tag{2}$$

$$\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial r}=\frac{1}{Pr}\frac{1}{r^i}\frac{\partial}{\partial r}\left(r^i\mu\frac{\partial T}{\partial r}\right) \tag{3}$$

with initial conditions at $x=0$

$$\begin{cases} 0\leq r\leq 1: & u-1=T-\varepsilon=0\\ r>1: & u-u_c=T-1=0\end{cases} \tag{4}$$

and with boundary conditions for $x>0$

$$\begin{cases} r=0: & \partial u/\partial r=v=\partial T/\partial r=0\\ r\rightarrow\infty: & u-u_c=T-1=0.\end{cases} \tag{5}$$

In previous equation $i=0$ for the planar jet and $i=1$ in the round jet. As can be seen, the formulation considers the presence of a coflowing outer stream, with u_c representing the coflow-to-jet velocity ratio. The Prandtl number, Pr, is assumed to be constant in the analysis. The continuity, momentum and energy conservation equations need to be supplemented with

$$\rho T=1 \quad \text{and} \quad \mu=T^\sigma \tag{6}$$

corresponding, respectively, to the equation of state written in the near isobaric approximation and the presumed power-law dependence for the temperature variation of the viscosity and heat conductivity, where the exponent will be taken equal to $\sigma=0.70$. Note that the solution of the previous problem must satisfy $2\varepsilon\int_0^\infty r^i u(u-u_c)/T dr=1-u_c$ and $2\varepsilon\int_0^\infty r^i u(1-T)/T dr=(1-\varepsilon)$, obtained from the radial integration of (2) and (3) once written in conservative form.

The solution to (1)–(5) can be simplified in the case $u_c=1$, when $u=1$ everywhere in the flowfield. Integration of (3) with the boundary condition at $r=0$ then provides $v=Pr^{-1}T^\sigma\partial T/\partial r$, which can be substituted into (1) to give

$$\frac{\partial T}{\partial t}-\frac{T^2}{r^i}\frac{\partial}{\partial r}\left(r^i T^{(\sigma-1)}\frac{\partial T}{\partial r}\right)=0. \tag{7}$$

This equation, to be integrated with the initial and boundary conditions indicated in (4) and (5), describes the time evolution of a symmetric pocket of cold gas in a hot atmosphere, with $t=x/Pr$ representing the dimensionless time. Note that with $i=2$ the equation describes the evolution of a spherical gas pocket, a problem previously investigated by Tarifa, Crespo & Fraga [2]
They noticed that, in the limit $\varepsilon\ll 1$, heat conduction cannot modify significantly the temperature in the cold gas, leading for $t\gg 1$ to a two-region flow structure consisting of a neatly defined unperturbed cold pocket for $r<r_f(t)$, where $T=\varepsilon$ and $v=0$, surrounded by a nearly steady region where the temperature is given in the first approximation by $T^\sigma=1-r_f/r$, as corresponds to a uniform mass flux $Pr\rho v r^2=r_f/\sigma$ with negligible mass accumulation for $r>r_f$. These two regions, of comparable size, are separated by a thin transition region where $T-\varepsilon\sim\varepsilon$, whose inner structure is also quasisteady when described in terms of the local coordinate $(r-r_f)/\varepsilon^\sigma$.

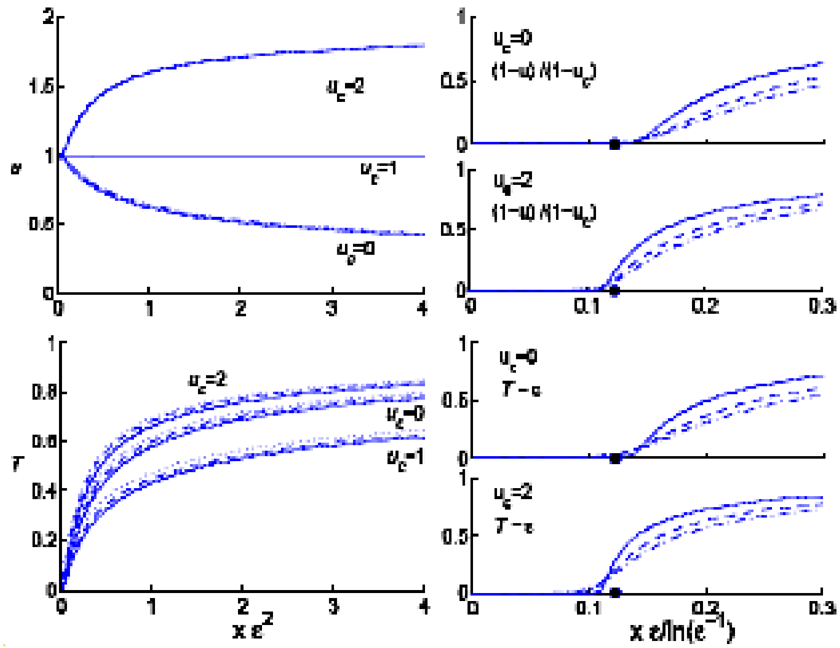

Fig. 1: Velocity and temperature evolution at the axis in plane (left plots) and round jets (right plots) in the far field. For plane jets we compare the evolution of velocity and temperature in the limit $\varepsilon = 0$ (solid line) with calculations made with $\varepsilon = 0.1$ (dotted lines) and $\varepsilon = 0.05$ (dashed lines). For round jets (right plots) we represent the velocity deficit and temperature at the axis for different coflow velocities u_c and $\varepsilon = 0.01$ (solid lines), $\varepsilon = 0.03$ (dot-dashed lines) and $\varepsilon = 0.05$ (dashed lines). The dot placed at the horizontal axis marks the point, predicted from the asymptotic analysis carried out in the limit $\varepsilon \to 0$, where the front achieves the axis $x_f \varepsilon / \log \varepsilon^{-1} = Pr\sigma/4$.

The solution for cold jets downstream from the jet exit shows also an unperturbed cold core where $T = \varepsilon$ and $u = 1$ surrounded by an outer region where the temperature and the velocity are seen to evolve to reach its boundary values $T = 1$ and $u = u_c$ as $r \to \infty$. Significant differences with the evolution of a cold spherical pocket are however encountered.

2 The plane jet

In contrast to what happens in the spherical case, the surrounding outer gas never reaches a quasi-steady balance. Instead, axial convection and radial transport are seen to balance everywhere in the outer region, whose transverse extent increases continuously with downstream distance according to $(r - 1) \sim \sqrt{x}$. The same balance is found in the thickening transition layer separating the outer region from the unperturbed core, which remains centered at $r = 1$, with a thickness that increases continuously with distance according to $(r-1) \sim \varepsilon^{(1+\sigma)/2}\sqrt{x}$. Both regions admit self-similar solutions, that remain valid until the unperturbed core and the transition layer merge at distances $x \sim \varepsilon^{-(1+\sigma)}$, where we find $T - \varepsilon \sim 1 - u \sim \varepsilon$ at the jet axis. In this far field, the outer region extends over large radial distances of order $r \sim \varepsilon^{-(1+\sigma)/2}$, resulting in small transverse gradients of temperature and velocity and, consequently, reduced rates of heating and deceleration of the near-axis fluid. As a result, the final development of the jet to attain axial values of T and $1-u$ of order unity is postponed to large downstream distances of order $x \sim \varepsilon^{-2}$, giving the evolution plotted in figure .

3 The round jet

Initially, the heat coming from the hot ambient does not suffice to warm up the cold fluid emerging from the jet. As a result, a thickening layer of very low density gas surrounds initially the jet front, which remains at $r = 1$ for $x \sim O(1)$, much as in the case of the planar jet. However, a quasi-steady balance with negligible axial convection eventually settles outside the jet for $x \gg 1$ as the mass flux emerging from the jet decreases. At distances $x \sim \varepsilon^{-(1-\sigma)}(\ln \varepsilon^{-1})^2$, the front starts moving radially towards the axis as the cold gas is ablated from the cold jet surface, much as in the case of a spherical gas pocket [2]. The front, with thickness $r - r_f \sim \varepsilon^{\sigma} \log(\sqrt{x})$, moves slowly reaching the center of the jet at distances that can be seen to be $x_f = \sigma Pr/4\varepsilon^{-1} \ln \varepsilon^{-1}$. Unlike the spherical case, the arrival of the front does not mark the end of the jet evolution. The temperature and the velocity near the axis, of order $T \sim 1 - u \sim \varepsilon$ when the front arrives, evolve slowly downstream to reach values of order unity in a far field region of characteristic length $\varepsilon^{-1} \ln \varepsilon$ comparable to the length needed for front propagation to the axis.

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