

# IGNITION AND FLAME SPREAD IN LAMINAR MIXING LAYERS

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## ABSTRACT

In order to identify some of the mathematical problems encountered in analyzing the ignition and flame spread in mixing layers, we shall describe the structure of the laminar mixing layer between two parallel streams of a fuel and air, initially separated by a splitter plate, undergoing an Arrhenius reaction.

If the activation energy of the reaction is lower than a critical value, there is only one steady solution of the problem, showing a transition from nearly frozen mixing to diffusion controlled combustion downstream of the plate.

For higher, typical, values of the activation energy we may find a multiplicity of solutions, depending on the value of the Damköhler number  $D$ , characterized by the temperature of the hotter of the two streams. For values of the Damköhler number lower than a critical value  $D_c$ , there is a solution where a thermal runaway is found to occur, after an induction length, at a point that serves as the origin of premixed flames that do not propagate upstream. For values of  $D$  larger than a critical lift-off value  $D_l (< D_c)$  we find a solution with diffusion controlled combustion in a diffusion flame. This flame is anchored, with a triple-flame structure, in the near wake of the splitter plate, where upstream heat conduction to the plate plays a dominant role.

In the interval  $D_l < D < D_c$  there is a third, unstable, solution. This solution determines where an external ignition source should be placed so that, by means of upstream triple-flame propagation to the splitter plate, transition to diffusion controlled combustion can take place.

## 1. Introduction

The chemical reaction in the mixing layer between two streams of fuel and oxidizer will take place in a thin diffusion flame, without

significant effects of the finite rate of the chemical reaction, only after spontaneous or artificially triggered ignition and only if the mixing layer is not subject to an excessive strain that may extinguish the flame.

Spontaneous ignition occurs, for one-step irreversible Arrhenius reactions, in the form of a thermal runaway due to the enhancement of the reaction rate by the heat release in the nearly frozen region of the mixing layer downstream of the splitter plate. A hot spot created by this process serves as the steady origin of premixed flames, if they are not fast enough as to propagate upstream to the nearly frozen region. The premixed flames separate the upstream nearly frozen region from two downstream regions of near-equilibrium, one without fuel and the other without oxidizer that are separated by a diffusion flame. An analysis of this spontaneous ignition process, and of the triple flame system which is set up in this way, was carried out by Liñán and Crespo (1976) for the unsteady mixing layer and extended later by Jackson and Hussaini (1988) and Grosch and Jackson (1991) to the analysis of compressible supersonic mixing layers. See also the review paper by Jackson (1992).

Although the spontaneous ignition may be expected to be the mechanism characterizing the transition from nearly frozen mixing to diffusion controlled combustion in the mixing layers encountered in supersonic combustion, this is not always the case in combustion in unpremixed systems. If we ignite locally the mixing layer, by means of a spark or a hot body, we create a premixed flame front that moves in the mixing layer, relative to the fluid, both upstream and downstream, leaving behind a diffusion flame. The premixed flame front, which together with the diffusion flame forms a triple flame, may propagate all the way up to the splitter plate, leaving the diffusion flame attached to the plate. If propagation up to the plate is not possible, then, the flame remains lifted-off or is blown away. See Liñán (1988) and the early and the more recent work of Dold (1989) and coworkers on triple-flame propagation. See also the numerical work of Prasad and Price (1992) and the review of Takahashi and Schinoll (1990).

One could expect that the lift-off distance should be determined as the distance from the injector where the triple flame front velocity equals the local flow velocity. However this configuration is stable only if the flow velocity is decreasing downstream from the lift-off distance. In addition, the flame front velocity in the partially mixed,

typically turbulent, mixing layers is different from the laminar premixed flame velocity for a stoichiometric mixture. The triple flame system must move along very thin mixing layers strongly distorted and strained by turbulence. Propagation along these layers is only possible if there is a large enough fraction of thin mixing layer regions not excessively strained to extinguish the diffusion flames. Peters and Williams (1983) used ideas of percolation theory to estimate the lift-off height in turbulent jet diffusion flames, where the evaluation of flame front propagation is difficult.

The lift-off height may, however, be determined in some cases by a spontaneous ignition process, as indicated before. The main purpose of this work is to show, analyzing as an example the low Mach number laminar mixing layer downstream of a thin splitter plate, how to identify the mechanism that determines the flame lift-off height and when the flame is attached to the splitter plate.

## 2. Characteristic Scales in the Near Region of the Mixing Layer

We shall analyze the structure of the steady laminar mixing layer between two parallel streams of fuel and oxidizer, separated by a thin plate. The Reynolds number  $R_B$  of the flow, based on the higher free stream flow velocity  $U_o$ , kinematic viscosity  $\nu_o$ , and the thickness  $\delta_B$  of the corresponding boundary layer just upstream of the end of the splitter plate, is considered to be large. Therefore, the boundary layer approximation can be used for the description of the mixing process between the two streams, thus neglecting the effects of upstream heat conduction and diffusion.

Jackson (1992) has called our attention to the role that the wake of the splitter plate and the initial boundary layer thickness may have in determining the ignition length and in the stability of the diffusion flame. The triple deck theory, introduced by Stewartson (1969) and Messiter (1970), for the analysis of the trailing edge region in boundary layer flows should be used, as done by Daniels (1977), to describe the viscous mixing in the trailing edge.

According to this theory, the effects of upstream heat conduction and diffusion always play a role in a small Navier-Stokes region embedded in the larger lower deck of the much larger triple deck region. Transverse heat conduction, diffusion and viscous effects are confined to the lower deck; but the pressure gradients, produced by

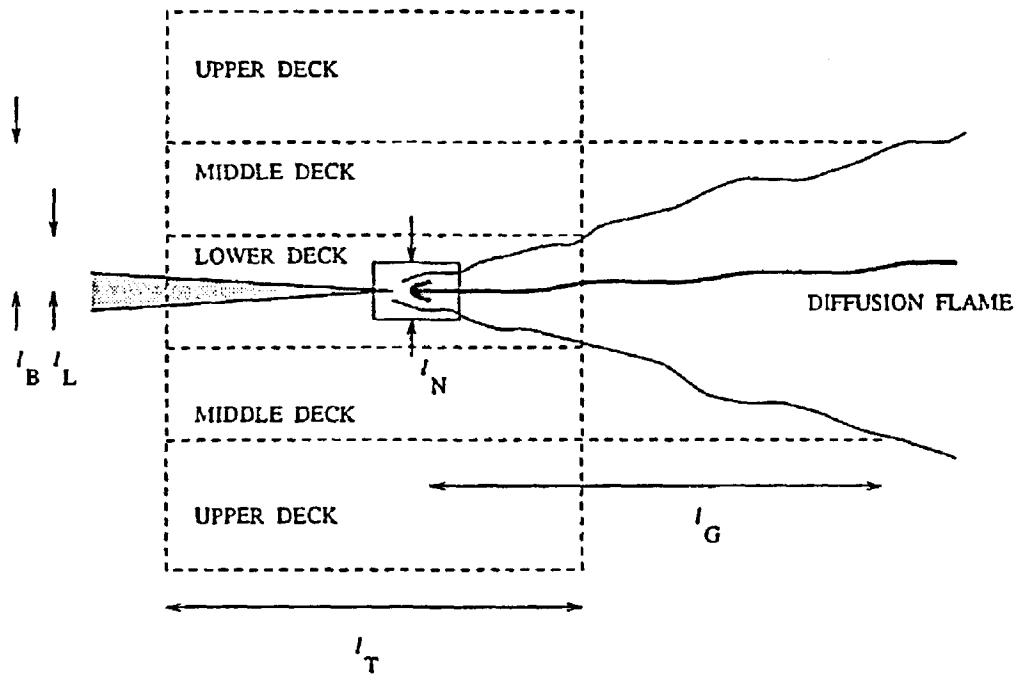


Figure 1. Schematic showing the Triple-deck, Navier-Stokes, and the Goldstein regions.

the displacement induced in the outer decks by the lower deck, also determine the flow velocity in the lower deck.

The analysis of the viscous lower deck determines the air and fuel stream velocity gradients  $A_o$  and  $A_F$  at the wall, at the edge of the splitter plate. These gradients determine the boundary conditions and, therefore, the flow structure in the Navier-Stokes region.

The characteristic sizes of the triple decks, sketched in Fig. 1, are given in terms of the Reynolds number  $R_B$  based on velocity  $U_o$ , boundary layer thickness  $l_B$ , and free stream kinematic viscosity  $\nu_o$  by the relations

$$l_T/l_B = R_B^{1/4}, \quad l_L/l_B = R_B^{-1/4} \quad (1)$$

determining the characteristic values of the size and streamwise extent  $l_T$  of the upper deck and of the thickness  $l_L$  of the lower deck. Let the wall velocity gradient on the air side just upstream of the Navier-Stokes region be  $A_o$ , of order  $l_B/U_o$ . Then, the size  $l_N$  of the Navier-Stokes region and the characteristic velocity  $U_N$  in this region are given by the relations

$$U_N/l_N = A_o, \quad U_N l_N / \nu_o = 1 \quad (2)$$

so that

$$l_N/l_B = U_N/U_o = R_B^{-1/2}. \quad (3)$$

Notice that  $1/A_o$  is the residence time of the fluid particles in the Navier-Stokes region.

Mixing between the fuel and the oxygen of the air begins in the Navier-Stokes region and continues downstream, first in the lower of the decks and then in the viscous sublayer of the Goldstein region.

In the Goldstein region the velocity  $u_G$  along the dividing streamline and the thickness  $l_M$  of the mixing layer vary initially with the distance  $x$  from the splitter plate as

$$u_G/U_o = l_M/l_B = (x/l_G)^{1/3} \quad (4)$$

where  $l_G$  is the extent of the Goldstein region, given by the relation  $l_B = \sqrt{\nu_o l_G / U_o}$  or

$$l_G/l_B = R_B. \quad (5)$$

### 3. Analysis of the Navier-Stokes Region

If the diffusion flame is attached to the splitter plate, the flame attachment region is in the Navier-Stokes region. There, upstream heat conduction losses from the flame to the colder splitter plate play an important role.

We shall, for simplicity in the presentation, consider that the thickness of the plate  $l_P$  is small compared with the size  $l_N$  of the Navier-Stokes region. Otherwise, the ratio  $l_P/l_N$  will affect the flame attachment process. We shall also consider that the plate conductivity is high enough so that its temperature will be equal to  $T_o$ , the temperature of the ambient air and fuel streams.

We shall write the conservation equations in non-dimensional form, using as scales:  $l_N$  for the spatial coordinates,  $U_N$  for the velocities, the free stream air values  $\rho_o$ ,  $T_o$  and  $\alpha_o$  for the density, temperature, and thermal diffusivity. We shall measure with  $\rho_o U_N^2$  the spatial pressure variations  $p'$  from the trailing edge value given by the outer triple-deck structure. Here we redefine  $l_N$  and  $U_N$  in terms of  $\alpha_o$  and the wall velocity gradient  $A_o$  by the relations

$$l_N = U_N/A_o = \sqrt{\alpha_o/A_o}. \quad (6)$$

We shall consider that the reaction between the oxygen and the fuel is an Arrhenius reaction consuming a mass  $s$  of oxygen and releasing an energy  $q$  per unit mass of fuel. The mass rate of consumption of the fuel per unit volume is

$$w_F = \rho B Y_F Y_o \exp(-E/RT) \quad (7)$$

in terms of the frequency factor  $B$  and activation energy  $E$ .

We shall measure  $Y_F$  and  $Y_o$  in terms of their free-stream values  $Y_{F0}$  and  $Y_{o0}$  to obtain  $Y_1$  and  $Y_2$ .

Then the conservation equations take the form

$$\nabla \cdot (\rho \vec{v}) = 0 \quad (8)$$

$$\rho \vec{v} \cdot \nabla \vec{v} = -\nabla p' + P_r \nabla \cdot \tau \quad (9)$$

$$\rho \vec{v} \cdot \nabla Y_1 - \frac{1}{L_1} \nabla \cdot (\rho \alpha \nabla Y_1) = -\rho D Y_1 Y_2 e^{-\beta/T} \quad (10)$$

$$\rho \vec{v} \cdot \nabla Y_2 - \frac{1}{L_2} \nabla \cdot (\rho \alpha \nabla Y_2) = -S \rho D Y_1 Y_2 e^{-\beta/T} \quad (11)$$

$$\rho \vec{v} \cdot \nabla T - \nabla \cdot (\rho \alpha \nabla T) = \gamma(S+1)\rho D Y_1 Y_2 e^{-\beta/T} \quad (12)$$

and the equation of state

$$\rho T = 1 \quad (13)$$

if the mean molecular mass is constant.

In these equations,  $\tau$  is the non-dimensional viscous stress tensor, and the main parameters are the Lewis Numbers,  $L_1$  and  $L_2$ , of the fuel and of the oxygen, the Prandtl number  $P_r$ , together with the Damköhler number, or non-dimensional value of the frequency factor,  $D = Y_{oo}B/A_o$ , the activation energy  $\beta = E/RT_o$ , the heat release  $\gamma(S+1) = Y_{FO}q/c_p T_o$ , and the stoichiometric ratio  $S = sY_{FO}/Y_{oo}$ .

These equations are to be solved with the boundary conditions

$$Y_1 = Y_2 - 1 = T - 1 = u - y = v = p' = 0 \quad (14)$$

for  $y \rightarrow \infty$  or  $y > 0$ ,  $x \rightarrow -\infty$

$$Y_2 = Y_1 - 1 = T - 1 = u = v = p' = 0 \quad (15)$$

for  $y \rightarrow -\infty$  or  $y < 0$ ,  $x \rightarrow -\infty$ .

At the splitter plate  $y = 0^+$  or  $y = 0^-$ , with  $x < 0$

$$T = 1, \quad \partial Y_1 / \partial y = \partial Y_2 / \partial y = 0. \quad (16)$$

We have written here the boundary conditions for the case where the velocity of one of the streams is negligible compared with the other. In this case, in first approximation, there is no upstream perturbation in the triple-deck region, and  $A_o$  is the trailing edge value of the unperturbed, boundary layer velocity gradient at the wall. If the two stream velocity gradients are comparable, we should replace the condition  $u = 0$  in Eq. (15) by  $u + ay = 0$ , and then an additional parameter  $a = A_F/A_o$  appears in the equations.

Downstream, the solution must evolve toward the boundary layer similarity solution associated with the near plate region of the lower deck, with the reaction taking place in a diffusion controlled way if  $D$  is large enough.

It should be noticed that the structure of the Navier-Stokes diffusion flame attachment region is locally two-dimensional, due to its small size, and given by the previous equations, even if the global flow is unsteady or turbulent as long as the lower of the characteristic times  $1/A_o$  or  $1/A_F$  associated with the wall velocity gradients are small compared with the time characterizing their time variations.

Notice that the two wall velocity gradients  $A_F$  and  $A_o$ , just upstream of the edge of the splitter plate, are determined by the outer triple-deck structure. If the diffusion flame is anchored at the Navier-Stokes region, the displacement velocities induced by the thermal expansion in the lower deck will produce overpressures in the trailing edge region. These overpressures decrease the values of the wall velocity gradients from their unperturbed values, if  $a$  is not negligible or large compared with 1. In fact, the displacement effects may even lead to boundary layer separation upstream of the splitter plate, and then, the character of the flow structure will change significantly.

The solution of the problem of Eqs. (8)-(16) should give the transition to the diffusion controlled combustion downstream, although this transition will take place for values of  $x$  of order unity only if  $D$  is large enough.

If the Lewis numbers can be considered as equal to 1, we can derive, from the conservation Eqs. (10)-(12), two conservation equations,

$$\rho \vec{v} \cdot \nabla Z - \nabla \cdot (\rho \alpha \nabla Z) = 0 \quad (17)$$

$$\rho \vec{v} \cdot \nabla H - \nabla \cdot (\rho \alpha \nabla H) = 0 \quad (18)$$

for the mixture fraction,

$$Z = (SY_1 + 1 - Y_2) / (S + 1), \quad (19)$$

and total enthalpy  $H$ ,

$$H = Y_1 + Y_2 + (T - T_e) / (T_e - 1), \quad (20)$$

where  $T_e$  is the non-dimensional adiabatic flame temperature,  $T_e = 1 + \gamma$ .  $H$  and  $Z$  satisfy the boundary conditions  $H = 0$ ,  $Z = 1$  on the fuel free stream and  $H = Z = 0$  in the air free stream.

When the reaction can be modelled by an Arrhenius reaction, the non-dimensional activation energy  $\beta$  is typically large compared with 1, but the value of  $D$  will also be very large. We can take advantage of the large values of  $\beta$  and  $D$  to carry out an asymptotic description of the problem.

Let us define a non-dimensional characteristic temperature  $T_c$ , by the relation

$$(T_c/\beta)^3 D \exp(-\beta/T_c) = 1. \quad (21)$$

$T_c$  is such that at this temperature the reaction term in Eqs. (10)-(12) is of the same order as the diffusion terms. The factor  $(T_c/\beta)^3$  has



been introduced to account, partially, for the small transverse size of the reaction zone and of the values of the reactant concentrations there.

We can now re-write, for the large activation energy analysis, the reaction term in Eqs. (10)-(12) in the form

$$DY_1Y_2\exp(-\beta/T_c) = (\beta/T_c)^3 Y_1Y_2\exp\{\beta(T - T_c)/TT_c\}$$

and then use the fact that  $\beta/T_c \gg 1$  to describe the solution of the problem in terms of the infinite activation energy solution  $\beta/T_c \rightarrow \infty$ , with  $T_c$  fixed. In this limit,  $\beta \rightarrow \infty$ ,  $T_c$  fixed, the chemical reaction is frozen for  $T < T_c$  and infinitely fast for  $T > T_c$ . In the region of low temperature,  $T < T_c$ , the chemical reaction is frozen and the reactants can mix. In the region where  $T > T_c$  the reactants can not coexist; one of the reactants has been completely consumed.

Therefore, we find three regions in the flow field where the chemical reaction term is zero. Namely,

- i) A frozen region,  $\Omega_f$ , where  $T < T_c$  and  $Y_1Y_2 \neq 0$ .
- ii) An equilibrium region,  $\Omega_1$ , without oxygen, where  $T > T_c$  and  $Y_2 = 0$ .
- iii) An equilibrium region,  $\Omega_2$ , without fuel, where  $T > T_c$  and  $Y_1 = 0$ .

The regions  $\Omega_f$  and  $\Omega_1$  are separated by an infinitely thin, rich, premixed flame, where all the oxygen reaching the flame by diffusion is burned with part of the fuel reaching the flame by diffusion. The regions  $\Omega_f$  and  $\Omega_2$  are similarly separated by a lean premixed flame. The regions  $\Omega_1$  and  $\Omega_2$  are separated by an infinitely thin diffusion flame.

The reaction terms appearing in Eqs. (10)-(12) take the form of Dirac delta functions along these flame sheets with strengths determined by the conditions of continuity of the temperature and concentrations there, together with the condition  $Y_1 = 0$  in  $\Omega_2$  and  $Y_2 = 0$  in  $\Omega_1$ . Notice that, in this limit of infinite activation energies, all the kinetic information is retained only in the parameter  $T_c$ , the premixed flame temperature that is very precisely determined by Eq. (17).

If the flame temperature  $T_c$ , defined by Eq. (21), is lower than the plate temperature, namely for values of  $D > D_d$  defined in order

of magnitude by the relation

$$D_d \beta^{-3} e^{-\beta} = 1 \quad (22)$$

then, the quench region disappears, and the diffusion flame reaches the plate. If the resulting value of  $T_c$  is larger than the non-dimensional adiabatic flame temperature,  $T_e = 1 + \gamma$ , the premixed flame does not lie in the Navier-Stokes region. Or, in other words, for values of the Damköhler number  $D$  lower than a lift-off Damköhler number  $D_l$ , defined in order of magnitude by the relation

$$D_l \{\beta/(1 + \gamma)\}^{-3} \exp \{-\beta/(1 + \gamma)\} = 1 \quad (23)$$

the diffusion flame is lifted off, away from, the Navier-Stokes flame attachment region.

The condition (23) for the flame lift off is roughly equal to the condition that the residence time through the stoichiometric, fuel/air, premixed flame is equal to the residence time,  $1/A_o$ , in the Navier-Stokes region.

For values of the Damköhler number in the interval  $(D_l, D_d)$  we therefore encounter a combustion regime, where the diffusion flame is attached to the Navier-Stokes region and ends in a V shaped premixed flame system. The premixed flame temperature is  $T_c$ , larger than the plate temperature and lower than the adiabatic flame temperature  $T_e$ . When the Lewis numbers of the reactants are equal to 1, the values of the concentrations and of the temperature are related to the local value of  $H$ . Due to the heat losses by heat conduction to the splitter plate,  $H$  is negative in the Navier-Stokes region and rises to zero when we move away from the region. Thus, the temperature  $T_f$  along the diffusion flame, that lies on the stoichiometric surface  $Z = Z_s = 1/(S + 1)$ , rises from the value  $T_c$  at the tip of the flame to the adiabatic flame value  $T_e$  at distances from the plate large compared with  $l_N$ .

For the precise evaluation of the values  $D_d$  and  $D_l$ , we should carry out a more refined asymptotic analysis, which is difficult, or a numerical integration of the equations with the Damköhler number  $D$  replaced by  $\delta D_d$  in one case and  $\delta D_l$  in the other to lessen the influence of  $\beta$  in the resulting value of  $\delta$ .

For values of  $D < D_l$ , there is another weakly burning solution of the problem, for which if the activation energy is large the chemical reaction can be neglected in the Navier-Stokes region. For values

of  $D$  in the interval  $(D_l, D_c)$ , with  $D_c$  to be defined later, we may encounter three solutions of the problem for the typically large values of the activation energy larger than a critical value  $\beta_c$ .

#### 4. Triple Flame Downstream of the Navier-Stokes Region

For large values of the activation energy, there is a second, unstable, solution of the problem of Eqs. (8)-(16) for values of the Damköhler number  $D$  larger than  $D_l$ . In this solution a triple-flame front (also formed by a premixed flame, with rich and lean branches, and a trailing diffusion flame) is established at a distance  $l_F$  from the splitter plate that, when measured with  $l_N$ , is large compared with unity if  $D$  is moderately large compared with  $D_l$ .

When  $D$  begins to rise above  $D_l$ , the leading edge  $l_F$  of the unstable flame front begins to move downstream of the Navier-Stokes to the lower deck region in the wake of the plate and, for larger values of  $D$ , to the Goldstein region.

When the, unstable, triple-flame front lies, for values of  $D$  close to  $D_l$ , in the Navier-Stokes region, the flame feels the effects of heat losses to the splitter plate. These effects determine, also for the main stable solution, the flame front position and shape.

The heat lost to the splitter plate from the unstable flame becomes negligible when, for larger values of  $D$ , the flame front moves downstream to distances large compared with  $l_N$ . In this case, in the triple-flame front structure, we must retain upstream heat conduction and diffusion effects, but only in a preheat transport zone upstream of the reaction sheet of thickness  $l_p$  small compared with the local thickness  $l_M$  of the mixing layer. The local velocity of the premixed flame relative to the flow, determined by the upstream concentration and the temperature  $T_o$ , takes its maximum value where the upstream mixture is close to stoichiometric and decreases rapidly away from this surface. For this reason the flame front is curved with a radius of curvature  $l_M/\beta$ .

The boundary layer approximation can be used to describe the concentration field upstream of the triple-flame front region.

The flow in the nose region of the flame front, at distances from it of the order  $l_M/\beta$ , is rotational downstream of the thin premixed flame with overpressures that deflect the incoming streamlines and slow the flow in the stoichiometric streamline. For this reason the flame front propagation velocity  $U_p$ , relative to the unperturbed flow,

is larger than, although of the order of, the planar stoichiometric flame velocity.  $U_p$  is given, in order of magnitude, in dimensional variables by the relation

$$U_p^2 = \alpha_o (RT_e/E)^3 BY_{FO} e^{-E/RT_e}. \quad (24)$$

This velocity is equal to the characteristic flow velocity in the viscous sublayer of the Goldstein region, given by Eq. (4) or, equivalently, by

$$u_G/U_N = (x/l_N)^{1/3}. \quad (25)$$

Therefore, the unstable flame front distance  $l_F$ , when it lies in the Goldstein region, is given, using Eqs. (24) and (25), by the relation

$$(l_F/l_N)^{2/3} = (\alpha_o/U_N^2)(RT_e/E)^3 BY_{FO} e^{-E/RT_e}. \quad (26)$$

or, equivalently, by the relation

$$l_F/l_N = (D/D_l)^{3/2}. \quad (27)$$

The solution with the triple flame in the Goldstein region is unstable but determines where we should place (at  $x < l_F$ ) an ignition source to insure that upstream propagation of the generated flame front to the splitter plate is possible.

The Goldstein region ends when the thickness of the mixing layer becomes equal to the initial boundary layer thickness  $l_B$ , that is, at a distance  $l_G = R_B l_B$ . The characteristic fluid velocity along the stoichiometric surface, which should be used to calculate the flame front lift-off distance, becomes constant for values of the distance larger than  $l_G$ . Thus, for a value  $D_b/D_l$  of  $D/D_l$ , of order  $R_B$ , the unstable triple-flame leaves the Goldstein region, and the lift-off distance position is not well determined until, due to the merging of mixing layers in jet flows, the velocity along the stoichiometric surface begins to decrease with the downstream distance. Thus, at  $D = D_b$ , we encounter a bending bifurcation to a new, triple-flame, stable, solution branch with higher values of  $l_F$  for  $D < D_b$ . See Chung and Lee (1991).

## 5. Spontaneous Ignition Regime

For values of the Damköhler number below a critical value,  $D_c$ , that will be determined below, there is a third type of solution of the problem of Eqs. (8)-(16).

In this regime, analyzed by Jackson and Hussaini (1988) for laminar supersonic mixing layers without accounting for the wake effect associated with the original boundary layer thickness, the effects of the chemical reaction can be neglected in the Navier-Stokes region where upstream diffusion and conduction are important. The boundary layer approximations can be used downstream to describe how the heat release, due to the reaction, slowly increases the temperature to finally produce a thermal runaway. If this occurs within the Goldstein region, the distance  $l_I$  from the splitter plate at which the thermal runaway occurs is defined, in order of magnitude, by equating the residence time in the ignition region to the adiabatic explosion time. Namely, using the relation

$$1/t_I = u_G/l_I = \gamma(S+1)\beta Y_{oo} B e^{-E/RT_o} \quad (28)$$

together with Eq. (25). That is,  $l_I$  is given by

$$(l_I/l_N) = (A_o t_I)^{3/2} = \{\gamma(S+1)\beta D e^{-\beta}\}^{-3/2} \quad (29)$$

or

$$l_I/l_N = \left\{ A_o \frac{(RT_o)^2 \exp(E/RT_o)}{qE Y_{FO} Y_{oo} B} \right\}^{3/2} \quad (30)$$

This ignition length is to be compared with the unstable flame front lift-off distance  $l_F$ . If  $l_I < l_F$  for values of the Damköhler number larger than a critical value  $D_c$ ,

$$\frac{D_c}{D_l} = \frac{(1+\gamma)^{3/2}}{\beta^2 \sqrt{\gamma(S+1)}} \exp\left\{ \frac{\gamma\beta}{2(1+\gamma)} \right\}, \quad (31)$$

determined, in order of magnitude, by the condition  $l_I = l_F$ . The calculated spontaneous ignition length (30) does not have physical meaning because upstream flame propagation from the point of thermal runaway is possible.

## 6. Concluding Remark

We summarize the results in Fig. 2 where the flame stand-off distance for the V flame, the unstable triple-flame, and the spontaneous ignition lengths are represented schematically in terms of the Damköhler number  $D/D_l$ .

We find the results that we anticipated in the introduction.

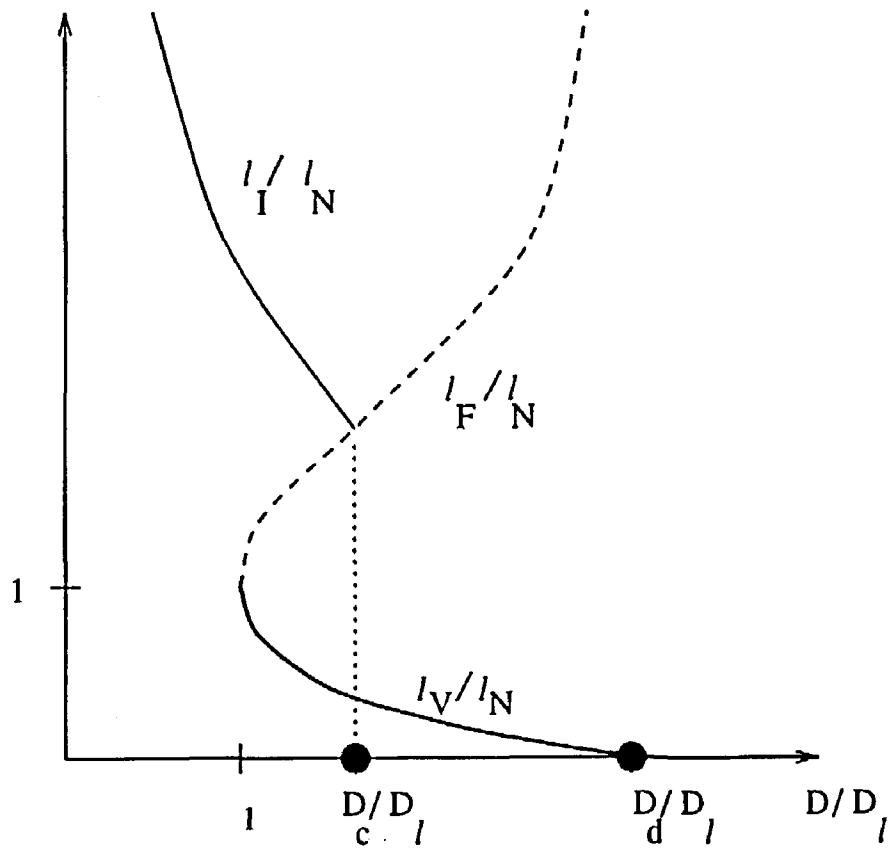


Figure 2. The flame lift-off distance in terms of the Damköhler number.

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