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The interaction of multiple bodies and water waves

with the application to the motion of ice floes

A thesis presented in partial fulfillment of the requirements for the degree of

Master of Science

in

Mathematics

at Massey University, Albany,

New Zealand

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2002

Abstract

To understand the propagation of water waves through arrays of floating or (fully or partially) submerged bodies it is necessary to know how these bodies interact with each other under the influence of ambient waves. However, the conventional full diffraction calculation of the scattered wavefields of many interacting bodies requires a considerable computational effort.

In this thesis, a method is developed which makes it possible to quickly calculate the wave scattering of many interacting floating or (fully or partially) submerged, vertically non-overlapping bodies of arbitrary geometry in water of infinite depth. It extends Kagemoto and Yue's analysis for axisymmetric bodies in finite depth.

The idea of this method is to expand the water velocity potential into its cylindrical eigenfunctions such that the scattered potentials of the bodies are defined by a set of coefficients only. Representing the scattered wavefield of each body as an incident wave upon all other bodies, a linear system of equations for the coefficients of the scattered wavefields is derived.

Diffraction transfer matrices which relate the coefficients of the incoming wavefield upon a single body to those of its scattered wavefield play an important role in the process. The calculation of the diffraction transfer matrices for bodies of arbitrary shape requires the representation of the infinite depth free surface Green's function in the eigenfunctions of an outgoing wave. This eigenfunction expansion will be derived from the equivalent finite depth Green's function.

An important application of this interaction method is the propagation of ocean waves through fields of ice floes which can be modelled as floating flexible thin plates. Meylan's method of solution is used to calculate the motion of a single ice floe from which the solutions for multiple interacting ice floes are computed.

While the interaction theory will be derived for general floating or submerged bodies, particular examples are always given for the case of ice floes. Results are presented for ice floes of different geometries and in different arrangements and convergence tests comparing the finite and the infinite depth method are conducted with two square interacting ice floes where full diffraction calculations serve as references.

Acknowledgements

I would like to thank Dr Michael Meylan for the great support he has shown me during the completion of this thesis. He has been an excellent supervisor and I have learnt very much from his comments and suggestions.

I would also like to thank the German National Scholarship Foundation for financially supporting my research at Massey University.

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