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BOUNDS ON THE ARITHMETIC DEGREE

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS AT MASSEY UNIVERSITY

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Abstract

In this thesis we study the arithmetic degree theory of polynomial ideals. The main objectives are: (i) to show whether we can generalize a lower bound on the arithmetic degree of monomial ideals to the arithmetic degree of arbitrary homogeneous ideals; and (ii) to explain whether some known bounds for the geometric degree can be restated in terms of bounds on the arithmetic degree. We give a negative answer to all questions raised by constructing counterexamples. In some cases we provide a general method for constructing such counterexamples. Concerning properties of the arithmetic degree, we give a new Bezout-type theorem. Finally we take a brief look at open problems concerning the arithmetic degree under hypersurface sections.

Acknowledgements

- This thesis is dedicated to the memory of my supervisor Professor Wolfgang Vogel. His expertise, guidance and enthusiasm were an inspiration. He will be greatly missed.
- My special thanks to Dr. Le Tuan Hoa for his careful critical reviews regarding this work. The final manuscript was substantially improved by these reviews.
- I would like to thank Dr. Kee Teo for taking over the position as my supervisor, with all that entails, at such a late stage. I am very grateful.
- Thanks to Mark Johnston for the help he has given me in learning to use text editors and the appropriate compilers. His patience is appreciated.
- I am also grateful for all of the encouragement and support given to me by fellow residents of Rm 2.40, in particular to Nick, Mary, and Anton.
- Thanks to all of the staff in the Department of Mathematics at Massey University for creating a friendly and supportive environment in which to work.
- I could not begin to put into words the thanks owed to my husband Brian. Thank you for making my world a much better place over the past 7 years.
- Finally, I would like to thank my parents Tom and Kathy for all the support they have given me, both financially and emotionally, in my educational career and otherwise.

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