CONSERVATIVE PERMUTATION MODELS OF TWO-DIMENSIONAL FLUID FLOW

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Abstract

At a meeting of Massey University Mathematics Department staff and students on 20 March 1997, a project to model fluid flow by constructing successive permutations of lattice cells was discussed. Several assumptions about the fluid being modelled were necessary to make the problem manageable, the chief ones being that the fluid would be two dimensional, ideal and incompressible.

Two square lattice models were developed, one Eulerian and the other more Lagrangian, in which the lattice cells were each initially assigned a value of a vorticity function. The time evolution of these models consisted of finding a permutation of the cells that was "close" to the fluid flow, then permuting the fixed initial vorticity values according to this lattice map. Justification of this method followed from consequences of the assumptions, including advection of vorticities, and the invertibility and area-preserving nature of the fluid flow map.

Reliance was placed on two previously published papers: one containing a result guaranteeing that such permutations are possible in certain circumstances, and the other providing the key to their practical construction. An essential algorithm in the latter paper relies on a theorem concerning the selection of a set of distinct elements from several sets.

Also as a result of the assumptions, the enstrophy, total vorticity and kinetic energy of such hypothetical fluid flow is conserved, although tests neither conclusively confirmed nor denied all conserving properties of the models.

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