

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

**CONSERVATIVE
PERMUTATION MODELS OF
TWO-DIMENSIONAL FLUID
FLOW**

A thesis presented in partial fulfilment of the requirements for
the degree of

Master of Science

in

Mathematics

at

Massey University

Paul Geoffrey Turner

1997

Abstract

At a meeting of Massey University Mathematics Department staff and students on 20 March 1997, a project to model fluid flow by constructing successive permutations of lattice cells was discussed. Several assumptions about the fluid being modelled were necessary to make the problem manageable, the chief ones being that the fluid would be two dimensional, ideal and incompressible.

Two square lattice models were developed, one Eulerian and the other more Lagrangian, in which the lattice cells were each initially assigned a value of a vorticity function. The time evolution of these models consisted of finding a permutation of the cells that was “close” to the fluid flow, then permuting the fixed initial vorticity values according to this lattice map. Justification of this method followed from consequences of the assumptions, including advection of vorticities, and the invertibility and area-preserving nature of the fluid flow map.

Reliance was placed on two previously published papers: one containing a result guaranteeing that such permutations are possible in certain circumstances, and the other providing the key to their practical construction. An essential algorithm in the latter paper relies on a theorem concerning the selection of a set of distinct elements from several sets.

Also as a result of the assumptions, the enstrophy, total vorticity and kinetic energy of such hypothetical fluid flow is conserved, although tests neither conclusively confirmed nor denied all conserving properties of the models.

Acknowledgements

I wish to express my gratitude to my supervisor, Dr. Robert McLachlan who parted the waters of many a confused sea for me, to my mother for her endurance, and to the students and other staff members who frequently gave of their knowledge and too-short time when I became stuck. Without the assistance of these people I would still be stuck.

Contents

1	Introduction	1
2	Models from the Past	12
3	A First Permutation Model	22
4	An Eulerian Permutation Model	34
5	Explorations	41
6	Conclusion	53

List of Figures

1.1	Vorticity contours	3
1.2	Flow 1	4
1.3	Tessellations	8
1.4	Periodic boundaries	10
2.1	Flow 2	15
3.1	Flow compared to permutation	26
3.2	Neighbourhood	27
3.3	Flow 3	32
4.1	Vorticity surface	35
4.2	Flow 4	38
5.1	Flow 5	43
5.2	Model A energy	44
5.3	Model B maximum and minimum vorticity	46
5.4	Model B consecutive distributions of vorticities	48
5.5	Model A consecutive distributions of vorticities	49
5.6	Model B enstrophy	50
5.7	Model B energy	51

LIST OF FIGURES

vii

6.1 Flow 6 55