

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

APPLICATIONS OF  
MATHEMATICAL PROGRAMMING  
ON FOUR NEW ZEALAND  
HORTICULTURAL HOLDINGS

by

A.N. RAE

Thesis Presented in Partial Fulfilment  
of the Requirements for the Degree  
of  
Master of Horticultural Science  
in  
Massey University

November 1968

## ACKNOWLEDGEMENTS

The author wishes to thank his two supervisors, Professor W.V. Candler and his successor, Professor A.R. Frampton, for their advice and encouragement received during this study. Special thanks are due to Professor Candler who obtained a solution to the quadratic programming study of Chapter 5 at Purdue University, Indiana, U.S.A. But for a lack of time a solution to the quadratic programme of Chapter 6 would have been obtained from the same source.

The author also wishes to express his thanks to the four horticulturalists who willingly gave of their time to allow the various programming models to be constructed.

Thanks are due to Mesdames J. Bishop and M. Clifford for typing, and to the library staff for their assistance in obtaining references.

Finally, thanks are due to members of the Horticulture Department, Massey University, with whom the author had discussions from time to time.

## CONTENTS

	Page
CHAPTER 1 INTRODUCTION AND THESIS OUTLINE	1
1.1 Introduction	1
1.2 Objectives of the Study	2
1.3 The Approach to the Study	3
1.4 Some Common Criticisms of Mathematical Programming for Horticulture	4
1.4.1 Problems of price and yield variability	4
1.4.2 Methods of overcoming price and yield variability	4
1.4.3 The problem of lack of farm records	5
1.4.4 Problems peculiar to process vegetable production	6
1.5 An Outline of the Thesis	6
CHAPTER 2 A DESCRIPTION OF LINEAR AND QUADRATIC PROGRAMMING	8
2.1 Linear Programming	8
2.1.1 Introduction	8
2.1.2 The assumptions	9
2.1.2.1 The linearity assumption	9
2.1.2.2 The additivity assumption	9
2.1.2.3 The non-negativity assumption	10
2.1.2.4 Single-value expectations	10
2.1.2.5 The convexity assumption	10
2.1.3 The objective function	11
2.1.4 The activities	11
2.1.5 The restraints	12
2.1.6 The non-negativity requirement	12
2.1.7 A summary of the linear programme problem	12
2.1.8 Disposal activities	12
2.1.9 The valuation of resources	13
2.1.10 Linear programme solutions and their interpretation	14
2.1.11 Sensitivity analysis	15
2.1.11.1 Stability of the optimum plan to price changes	15
2.1.11.2 The limits to which non-basic activities may enter the basis	16

2.2	Quadratic Programming	18
2.2.1.	Introduction	18
2.2.2	Statement of the quadratic programming problem	19
2.2.3	The related lagrangian problem	19
2.2.4	The partial derivatives	20
2.2.5	The related linear programming problem	22
2.2.6	An algorithm for solving quadratic programming problems	22
2.2.7	Interpretation of the solution	25
2.2.7.1	Real activities in the basis	25
2.2.7.2	Real activities not in the basis	25
2.2.7.3	Imputed values of scarce resources	25
2.2.7.4	Resources in disposal	25
2.3	Risk Programming	25
2.3.1	Introduction	25
2.3.2	Some examples of stochastic errors in horticultural production data	27
2.3.3	An outline of risk programming	28
2.3.4	A measure of risk	30
2.3.5	Mathematical statement of the risk programming problem	32
CHAPTER 3	LINEAR PROGRAMMING AND PROFIT MAXIMISATION ON AN OTAKI HORTICULTURAL HOLDING	34
3.1	Introduction	34
3.2	A Description of the Holding	34
3.2.1	Location and size	34
3.2.2	Leased land	35
3.2.3	Glasshouse area	35
3.2.4	Cropping practice	35
3.2.5	Soil types	35
3.2.6	Rainfall and irrigation	36
3.2.7	Disease problems	36
3.3	The Restraints	37
3.3.1	Problems of timing of production	37
3.3.2	Freehold land (restraints R1-R7)	39
3.3.3	Leased land (restraints R8-R15)	39
3.3.4	Labour availability (restraints R16-R39)	40
3.3.5	Cropping limits (restraints R40-R52)	42

3.4	The Activities	43
3.4.1	Production on leased and freehold land	43
3.4.2	Cauliflower (P1)	44
3.4.3	Cucumber (P2)	44
3.4.4	Tomato (P3)	45
3.4.5	Lettuce (P4)	46
3.4.6	Pumpkin (P5 and P10)	46
3.4.7	Rhubarb (P6)	46
3.4.8	Soleil d'Ors (P7)	46
3.4.9	Lettuce (P8)	47
3.4.10	Carrot (P9)	47
3.4.11	Cucumber (P11)	47
3.4.12	Cabbage (P12)	48
3.4.13	Cauliflower (P13)	48
3.4.14	Glasshouse cucumber (P14)	48
3.5	The Objection Function	49
3.5.1	Introduction	49
3.5.2	Calculation of gross margins	50
3.5.2.1	Expected prices and yields	50
3.5.2.2	Variable costs	52
3.5.2.3	Gross margins	52
3.5.3	Perennial crops and discounting	54
3.5.3.1	Present value of rhubarb (P6)	54
3.5.3.2	Present value of Soleil d'Ors (P7)	56
3.5.4	Calculation of annuities	59
3.5.4.1	Rhubarb (P6) annuity	59
3.5.4.2	Soleil d'Ors (P7) annuity	60
3.5.5	Overhead costs	60
3.6	Input-Output Coefficients	61
3.6.1	Freehold land restraints	61
3.6.2	Leased land restraints	61
3.6.3	Labour restraints	63
3.6.4	Cropping limit restraints	65
3.6.4.1	The glasshouse restraint (R52)	65
3.6.5	Dominated restraints	66
3.7	The Basic Matrix	66

3.8	The Solution	68
3.8.1	The cropping programme	68
3.8.2	Comparison of the linear programme solution with the grower's plan	68
3.8.2.1	Comparison of activity levels	68
3.8.2.2	Comparison of profits	71
3.8.3	Stability of the optimum plan	71
3.8.4	Unused resources	73
3.8.4.1	Land	73
3.8.4.2	Labour	74
3.8.5	Value of resources	75
3.8.5.1	Land	75
3.8.5.2	Labour	76
3.8.5.3	Cropping limits	76
3.8.5.3.1	The tomato cropping limit	77
3.9	A Parametric Solution	77
3.9.1	The reason for computing alternative plans	77
3.9.2	Parametric linear programming	77
3.9.3	Activity levels in the parametric solution	79
3.9.4	Resource values in the parametric solution	79
3.9.5	Supplementary, complementary and competitive relationships within the parametric solution	82
3.10	Adoption by the Grower of a Linear Programme Solution	85
3.10.1	Introduction	85
3.10.2	Comparison of the adopted plan with the grower's plan	86
3.10.2.1	Comparison of activity levels	86
3.10.2.2	Comparison of profits	88
3.10.2.3	Comparison of resource requirements	88
3.10.3	Stability of the adopted plan	91
3.10.3.1	Stable components of the plan	91
3.10.3.2	Unstable components of the plan	93
3.10.3.3	Crop which may become profitable	94
3.10.4	Unused resources	95
3.10.4.1	Land	95
3.10.4.2	Labour	96

3.10.4.2.1	Overhead labour	97
3.10.4.2.2	Replanting perennial crops	97
3.10.5	Value of resources	98
3.11	Increased Profits and Taxation	98
3.12	Grower's Comments on the Programming Solutions	101
3.12.1	Physical plans	101
3.12.2	Resource disposal levels	102
3.12.3	Shadow prices	103
3.12.4	Further comments on the adopted plan	105
CHAPTER 4	INVESTMENT IN PERENNIAL CROPS - A CAPITAL BUDGETING, INTERTEMPORAL LINEAR PROGRAMME	107
4.1	Introduction	107
4.1.1	The reason for an intertemporal approach	107
4.1.2	Brief review of intertemporal programming in farm management	108
4.1.3	An outline of the intertemporal programming matrix	110
4.2	Description of the Holding	110
4.2.1	Location and size	110
4.2.2	Present cropping practice	112
4.2.3	Future cropping practice	113
4.2.3.1	Treatment of the young apple and young asparagus crops in the model	114
4.2.4	Intercropping	114
4.2.5	Soil types	114
4.2.6	Irrigation	115
4.2.7	Disease problems	115
4.3	The Restraints	115
4.3.1	Land	115
4.3.2	Labour	117
4.3.3	Crops grown under contract for processing	119
4.3.4	Restrictions on fresh market crops	122
4.3.5	Rotation restraints	122
4.3.6	Cash and taxation restraints	124
4.3.6.1	Treatment of cash flows in the model	124
4.3.6.2	Tax-free cash available at the beginning of the development period	125
4.3.6.3	Net revenue from the young apple and asparagus crops	126
4.3.6.4	Withdrawals of tax-free cash	127



	4.3.6.5	Tax-deductible overhead costs	128
	4.3.6.6	Total tax-deductible expenditures	129
	4.3.6.7	Taxation limits	129
	4.3.6.8	Summary of cash and taxation restraints	131
	4.3.7	Objective function accounting restraints	131
4.4		The Activities	131
	4.4.1	Introduction	131
	4.4.2	Intercropping activities	132
	4.4.3	Annual cropping activities	132
	4.4.3.1	Tomato	132
	4.4.3.2	Green bean	133
	4.4.3.3.	Beetroot	133
	4.4.3.4	Potato	133
	4.4.3.5	Pea	134
	4.4.3.6	Carrot	134
	4.4.3.7	Broad bean-kumara	134
	4.4.3.8	Kumara	134
	4.4.3.9	Mangold	135
	4.4.3.10	Ryegrass	135
	4.4.4	Existing perennial plantings	135
	4.4.4.1	Old asparagus	135
	4.4.4.2	Old peach	135
	4.4.5	New plantings of perennials	136
	4.4.5.1	Asparagus	136
	4.4.5.2	Apple	136
	4.4.6	Land transfer activities	137
	4.4.6.1	Annual cropland transfer	137
	4.4.6.2	Asparagus land transfer	137
	4.4.6.3	Peach land transfer	137
	4.4.6.4	Fallow	137
	4.4.7	Activities to hire labour	137
	4.4.8	Taxation and tax-deductible expenditures	138
	4.4.8.1	Taxation transfer activities	138
	4.4.8.2	Tax-deductions transfer	138
	4.4.9	Bank activities	139
	4.4.10	Final tax-free cash and final assets	139
4.5		Input-Output Coefficients	139

4.5.1	Introduction	139
4.5.2	Land restraints	140
4.5.3	Labour restraints	142
4.5.4	Process crop restraints	143
4.5.5	Fresh market crop restraints	145
4.5.6	Rotation restraints	145
4.5.7	Cash and taxation restraints	147
4.5.7.1	General description of coefficients	147
4.5.7.2	Derivation of marginal tax rates	149
4.5.8	Objective function accounting restraints	150
4.5.8.1	General description of coefficients	150
4.5.8.2	Derivation of asset values	152
4.5.8.2.1	Introduction	152
4.5.8.2.2	The procedure adopted	153
4.5.8.2.3	Optimum replacement times	153
4.5.8.2.4	Calculation of asset values	155
4.6	The Objective Function	156
4.7	The Basic Matrix	158
4.8	The Solution	158
4.8.1	A parametric solution	158
4.8.2	Activity levels: Plan 4	162
4.8.2.1	Intercropped and annual cropping activities	162
4.8.2.2	Perennial crops and land transfer activities	163
4.8.2.3	Activities to hire labour	164
4.8.2.4	Cash flows and taxation	165
4.8.3	Value of the objective function: Plan 4	168
4.8.4	Resources in disposal: Plan 4	168
4.8.5	Value of resources: Plan 4	170
4.8.5.1	Introduction	170
4.8.5.2	Annual cropland restraints	171
4.8.5.3	Labour restraints	171
4.8.5.4	Process and fresh market crop restraints	172
4.8.5.5	Tax-free cash restraints	173
4.8.6	Opportunity costs of growing excluded crops: Plan 4	174
4.9	Effect of Varying the Length of the Planning Horizon	176
4.10	Discussion on the Programming Results	177

4.10.1	Physical plans and problems with changed contracts	177
4.10.2	Resource use and disposal levels	179
4.10.3	Problems associated with the variability of input-output coefficients	180
4.10.4	Value of resources	180
4.10.5	Summary	181
CHAPTER 5            QUADRATIC PROGRAMMING AND PROFIT MAXIMISATION		183
5.1	Introduction	183
5.2	The Approach Adopted	185
5.3	The Restraints	186
5.3.1	Glasshouse restraints	186
5.3.1.1	Propagating-house (restraints R1-R13)	186
5.3.1.2	Growing-house (restraints R14-R26)	187
5.3.1.3	Shade-house (restraints R27-R36)	188
5.3.2	Labour (restraints R37-R41)	189
5.3.3	Land (restraint R42)	190
5.3.4	Summary of resource supplies on the hypothetical nursery	190
5.4	The Activities	191
5.4.1	<i>Telopea speciosissima</i> (P1)	191
5.4.2	<i>Acacia baileyana</i> (P2)	191
5.4.3	<i>Passiflora</i> 'Crackerjack' (P3)	192
5.4.4	<i>Banksia grandis</i> (P4)	192
5.4.5	<i>Photinia robusta</i> (P5)	192
5.4.6	<i>Eucalyptus ficifolia</i> (P6)	193
5.4.7	<i>Stachyurus praecox</i> (P7)	193
5.4.8	<i>Cistus purpureus</i> 'Brilliance' (P8)	193
5.4.9	<i>Protea cynaroides</i> (P9)	193
5.4.10	<i>Tibouchina grandiflora</i> (P10)	194
5.4.11	<i>Azalea indica</i> 'Salmonea' (P11)	194
5.4.12	<i>Viburnum japonicum</i> (P12)	194
5.4.13	<i>Rhododendron</i> 'Christmas Cheer' (P13)	195
5.4.14	<i>Weigela florida variegata</i> (P14) and <i>Forsythia</i> 'Karlsax' (P5)	195
5.4.15	<i>Azalea occidentalis</i> (P16)	195
5.4.16	<i>Magnolia stellata</i> (P17)	196
5.4.17	<i>Callicarpa dichotoma</i> (P18) and <i>Hypericum</i> 'Hidcote Gold' (P19)	196

5.5	Input-Output Coefficients	196
5.5.1	Problems of multi-period production	196
5.5.2	Glasshouse restraints	198
5.5.3	Labour restraints	198
5.5.4	Land restraint	200
5.5.5	Dominated restraints	201
5.6	Matrix of Activity Resource Requirements	201
5.7	The Objective Function	203
5.7.1	Introduction	203
5.7.2	Estimation of the demand functions	203
5.7.3	Variable costs	205
5.7.4	Total net revenue objective function	208
5.7.5	Overhead costs	209
5.8	The Solution	209
5.8.1	Introduction	209
5.8.2	Comparison of the optimum plan and the nurseryman's plan	210
5.8.2.1	Production levels and prices	210
5.8.2.2	Comparison of net revenue	213
5.8.2.3	Comparison of resource requirements	213
5.8.2.3.1	Glasshouse restraints	213
5.8.2.3.2	Labour and land restraints	216
5.8.3	Resources in disposal	216
5.8.4	Value of resources	216
5.8.5	The cost of including non-basic activities in the optimum plan	217
5.9	Comments on the Quadratic Programming Model and Results	218
5.9.1	Extension of the model to represent actual situations	218
5.9.2	Incorporation of demand cross-effects into the quadratic programme	219
5.9.3	Problems associated with the demand curves	220
5.9.4	Shadow prices and resource disposal levels	221
CHAPTER 6	RISK PROGRAMMING AND FRESH VEGETABLE PRODUCTION	222
6.1	Introduction	222
6.2	The Restraints	223
6.2.1	Land	223
6.2.2	Labour availability	224

6.2.3	Rotations and cropping limits	225
6.3	The Activities	227
6.3.1	Production on freehold and leased land	227
6.3.2	Spring carrot and parsnip	227
6.3.3	Crown pumpkin	227
6.3.4	Buttercup pumpkin and butternut pumpkin	227
6.3.5	Brassica activities	227
6.3.5.1	Winter cauliflower and Winter cabbage	227
6.3.5.2	Spring cauliflower and Spring cabbage	228
6.3.6	Winter lettuce	228
6.3.7	Spring lettuce	228
6.4	Input-Output Coefficients	228
6.5	The Objective Function	230
6.5.1	Introduction	230
6.5.2	Activity prices and yields	230
6.5.3	Variable costs	233
6.5.4	Activity net revenues	233
6.5.5	Net revenue variances and covariances	234
6.5.5.1	The variance of activity net revenues	234
6.5.5.2	The covariance of activity net revenues	236
6.5.5.3	The variance - covariance matrix	237
6.5.6	Overhead costs	237
6.6	Construction of the Initial Risk Programming Matrix	238
6.7	The Maximum Risk Solution	239
6.7.1	Introduction	239
6.7.2	Comparison of the maximum risk solution with the grower's plan	239
6.7.2.1	Comparison of activity levels	239
6.7.2.2	Expected net revenue and net revenue variance	240
6.7.2.3	Comparison of resource requirements	242
6.7.3	Value of resources	242
6.7.4	Marginal opportunity costs	244
6.8	Summary of the Risk Programming Model	245
CHAPTER 7	SUMMARY OF RESULTS AND COMMENTS ON MATHEMATICAL PROGRAMMING IN HORTICULTURE	247
7.1	Introduction	247
7.2	The Linear Programming Applications	247

7.3	The Quadratic Programming Applications	249
7.4	The Potential of Mathematical Programming in the New Zealand Horticultural Advisory Service	250
7.4.1	The general use of programming techniques	250
7.4.2	The use of farm records	251
7.4.3	The role of market intelligence	252
7.5	Conclusions	253
APPENDIX A	DATA RELATING TO THE INTERTEMPORAL LINEAR PROGRAMME OF CHAPTER 4	254
A.1	Price, Yield and Variable Cost Estimates for Annual Crops	254
A.2	Price, Yield and Variable Cost Estimates for Perennial Crops and their Optimum Replacement	257
A.2.1	Apple	257
A.2.2	Asparagus	258
A.2.3	Peach	261
A.3	Part of the Basic Matrix	263
APPENDIX B	DATA OF THE RISK PROGRAMMING MODEL OF CHAPTER 6	267
B.1	Description of the Data	267

## LIST OF TABLES

Table		Page
2.1	Initial Simplex Tableau for Quadratic Programming	23
3.1	Labour Restraints and Supply	41
3.2	Cropping Limit Restraints	43
3.3	Price and Yield Assumptions	51
3.4	Variable Costs	53
3.5	Gross Margins	54
3.6	Variable Costs - Rhubarb (P6)	55
3.7	Present Value of Rhubarb (P6)	56
3.8	Variable Costs - Soleil d'Ors (P7)	57
3.9	Present Value of Soleil d'Ors (P7)	58
3.10	Overhead Costs	60
3.11	Labour Input - Lettuce Activity (P8)	64
3.12	The Basic Matrix	67
3.13	The Cropping Programme	69
3.14	Comparison of Activity Levels and Farm Profits Between the Optimum and the Grower's Plan	70
3.15	Gross Margin Stability Limits	72
3.16	Land in Disposal	73
3.17	Labour in Disposal	74
3.18	Land Shadow Prices	75
3.19	Cropping Limit Shadow Prices	76
3.20	The Parametric Solution - Activity Levels and Farm Profits	80
3.21	The Parametric Solution - Value of Scarce Resources	83
3.22	The Adopted Cropping Programme	86
3.23	Comparison of Activity Levels and Farm Profits Between the Adopted and the Grower's Plan	87
3.24	Land Requirements of the Adopted and the Grower's Plan	89
3.25	Labour Requirements of the Adopted and the Grower's Plan	90
3.26	Gross Margin Stability Limits - Adopted Plan	92
3.27	Price Limits - Adopted Plan	93
3.28	Yield Limits - Adopted Plan	94
3.29	Land in Disposal - Adopted Plan	95
3.30	Labour in Disposal - Adopted Plan	96

3.31	Impact of Taxation on Increased Profits	100
4.1	Schematic Representation of the Basic Matrix	111
4.2	Process Crop Contracts	112
4.3	Land Restraints	116
4.4	Labour Requirements of Maintenance Work and the Young Apple and Asparagus Crops	120
4.5	Labour Restraints	121
4.6	Process Crop Restraints	121
4.7	Fresh Market Crop Restraints	123
4.8	Rotation Restraints	124
4.9	Tax-Deductible Overhead Costs	128
4.10	Cash and Taxation Restraints	130
4.11	Objective Function Accounting Restraints	131
4.12	Land : Input-Output Coefficients	141
4.13	Labour: Input-Output Coefficients	144
4.14	Rotation Restraints: Input-Output Coefficients	146
4.15	Cash and Taxation Restraints: Input-Output Coefficients	148
4.16	Marginal Tax Rates	150
4.17	Objective Function Accounting Restraints: Input-Output Coefficients	151
4.18	Asset Values of Perennial Crops	156
4.19	The Objective Function	157
4.20	Price Limits of the Final Assets Activity	159
4.21	Crop Activity Levels Which Vary in the Parametric Solution	160
4.22	Values of the Objective Function in the Parametric Solution	161
4.23	Intercropped and Annual Cropping Activities - Plan 4	162
4.24	Perennial Crops and Land Transfer Activities - Plan 4	163
4.25	Labour to be Hired - Plan 4	165
4.26	Cash Flows and Taxation - Plan 4	166
4.27	Resources in Disposal - Plan 4	169
4.28	Labour Shadow Prices - Plan 4	172
4.29	Shadow Prices of Process and Market Crop Restraints - Plan 4	173
4.30	Marginal Opportunity Costs of Growing Non-Basic Crops - Plan 4	175
5.1	Labour Restraints	190
5.2	Labour Input-Photinia (P5)	199
5.3	Planting Methods and Land Requirements	200



5.4	Activity Resource Requirements and Resource Supplies	202
5.5	The Demand (Average Revenue) Functions	206
5.6	Variable Costs	207
5.7	Output and Prices for the Optimum and the Nurseryman's Plan	211
5.8	Propagating-House Requirements of the Optimum and the Nurseryman's Plan	213
5.9	Growing-House Requirements of the Optimum and the Nurseryman's Plan	214
5.10	Shade-House Requirements of the Optimum and the Nurseryman's Plan	215
5.11	Land and Labour Requirements of the Optimum and the Nurseryman's Plan	215
5.12	Value of Resources	217
5.13	Marginal Opportunity Costs	218
6.1	Labour Restraints and Supplies	225
6.2	Cropping Restraints	226
6.3	Activity Resource Requirements and Resource Supplies	229
6.4	Wholesale Price and Quantity Sold - Crown Pumpkin	231
6.5	Prices, Yields and Gross Revenue - Crown Pumpkin	232
6.6	Net Revenue - Crown Pumpkin	233
6.7	Activity Net Revenues	234
6.8	Variance of Activity Net Revenues	235
6.9	Overhead Costs	237
6.10	Comparison of Activity Levels Between the Maximum Risk and the Grower's Plan	240
6.11	Expected Net Revenue	241
6.12	Resource Requirements of the Maximum Risk and the Grower's Plan	243
6.13	Value of Resources	244
6.14	Marginal Opportunity Costs	244
A.1	Gross Revenue from Annual Crops	255
A.2	Variable Costs of Annual Crops	256
A.3	Revenue, Costs and Optimum Replacement: Apples	259
A.4	Revenue, Costs and Optimum Replacement: Asparagus	260
A.5	Revenue, Costs and Optimum Replacement: Peaches	262
A.6	Sub-Matrix a	264
A.7	Sub-Matrix g	265
A.8	Sub-Matrix h	266
B.1	Quantities Sold, Acreages and Yields	268
B.2	Average Net Wholesale Prices, Yields and Gross Revenues	269

B.3	Variable Costs	270
B.4	The Net Revenue Variance - Covariance Matrix	271

LIST OF FIGURES

Figure		Page
2.1	An E - V Indifference System	29
2.2	Expected Income as a Function of Income Variance	29
3.1	Activity Land Requirements	38
3.2	The Parametric Solution	81