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Gaussian Process based Model Predictive Control

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Abstract

The performance of using Model Predictive Control (MPC) techniques is highly dependent on a model that is able to accurately represent the dynamical system. The datadriven modelling techniques are usually used as an alternative approach to obtain such a model when first principle techniques are not applicable. However, it is not easy to assess the quality of learnt models when using the traditional data-driven models, such as Artificial Neural Network (ANN) and Fuzzy Model (FM). This issue is addressed in this thesis by using probabilistic Gaussian Process (GP) models.

One key issue of using the GP models is accurately learning the hyperparameters. The Conjugate Gradient (CG) algorithms are conventionally used in the problem of maximizing the Log-Likelihood (LL) function to obtain these hyperparameters. In this thesis, we proposed a hybrid Particle Swarm Optimization (PSO) algorithm to cope with the problem of learning hyperparameters. In addition, we also explored using the Mean Squared Error (MSE) of outputs as the fitness function in the optimization problem. This will provide us a quality indication of intermediate solutions.

The GP based MPC approaches for unknown systems have been studied in the past decade. However, most of them are not generally formulated. In addition, the optimization solutions in existing GP based MPC algorithms are not clearly given or are computationally demanding. In this thesis, we first study the use of GP based MPC approaches in the unconstrained problems. Compared to the existing works, the proposed approach is generally formulated and the corresponding optimization problem is efficiently solved by using the analytical gradients of GP models w.r.t. outputs and control inputs. The GPMPC1 and GPMPC2 algorithms are subsequently proposed to handle the general constrained problems. In addition, through using the proposed basic and extended GP based local dynamical models, the constrained MPC problem is effectively solved in the GPMPC1 and GPMPC2 algorithms. The proposed algorithms are verified in the trajectory tracking problem of the quadrotor.

The issue of closed-loop stability in the proposed GPMPC algorithm is addressed by means of the terminal cost and constraint technique in this thesis. The stability guaranteed GPMPC algorithm is subsequently proposed for the constrained problem. By using the extended GP based local dynamical model, the corresponding MPC problem is effectively solved.

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List of Abbreviations

ANN Artificial Neural Network
${\bf BFGS}$ Broyden-Fletcher-Goldfarb-Shanno
\mathbf{CG} Conjugate Gradient
CGP Convolved Gaussian Process
DGP Dependent Gaussian Process
DMC Dynamic Matrix Control
DOF Degree-of-Freedom
\mathbf{FM} Fuzzy Model
FP-SQP Feasibility-Perturbed Sequential Quadratic Programming
\mathbf{GA} Genetic Algorithm
\mathbf{GMV} Generalized Minimum Variance
GP Gaussian Process
GPC Generalized Predictive Control
GPDM Gaussian Process Dynamical Model
IAE Integral Absolute Error
IDC Inverse Dynamics Control
\mathbf{IGP} Independent Gaussian Process
KKT Karush-Kahn-Tucker
LGP Local Gaussian Process
LL Log-Likelihood

LMC Linear Model of Coregionalization LMI Linear Matrix Inequality LQR Linear-Quadratic Regulator **LTV** Linear Time-Varying **GP-LVM** Gaussian Latent Variable Model **MAE** Mean Absolute Error **MAP** Maximizing A Posterior MCMC Markov Chain Monte Carlo MFAC Model-Free Adaptive Control MIMO Multiple-Input Multiple-Output MISO Multiple-Input Single-Output **ML** Machine learning MLE Maximum Likelihood Estimation MPC Model Predictive Control mp-QP Multi-Parametric Quadratic Programs **MSE** Mean Squared Error **NLL** Negative value of Log-Likelihood **NLTV** Nonlinear Time-Varying **NMPC** Nonlinear Model Predictive Control **PCA** Principal Component Analysis **PFC** Predictive Functional Control **PFDL** Partial Form Dynamic Linearization **PSO** Particle Swarm Optimization **QP** Quadratic Programming **RBFN** Radial Basis Function Network **SMPC** Stochastic Model Predictive Control **SQP** Sequential Quadratic Programming **UAV** Unmanned Aerial Vehicle