ADVANCES IN CLASSICAL AND QUANTUM WAVE DYNAMICS ON QUASIPERIODIC LATTICES

A dissertation submitted for the degree of
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in
Physics

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Declaration

This dissertation is a presentation of the research conducted between February 2013 and June 2016 at the Center for Theoretical Chemistry Physics within the New Zealand Institute for Advanced Study at Massey University (Albany campus, Auckland, New Zealand) while enrolled in the Doctor of Philosophy degree.
The degree was carried out under the supervision of Prof. Sergej Flach, Dr. Joshua D. Bodyfelt and Dist. Prof. Gaven J. Martin.
The material reported is, at the best of my knowledge, original (except where acknowledged), and it has not been submitted in whole or in part for a degree in any university.

Carlo Danieli
June 2016

The most merciful thing in the world, I think, is the inability of the human mind to correlate all its contents. We live on a placid island of ignorance in the midst of black seas of infinity, and it was not meant that we should voyage far. The sciences, each straining in its own direction, have hitherto harmed us little; but some day the piecing together of dissociated knowledge will open up such terrifying vistas of reality, and of our frightful position therein, that we shall either go mad from the revelation or flee from the deadly light into the peace and safety of a new dark age.

H.P. Lovecraft
The Horror in Clay

Abstract

Lattices and discrete networks are cornerstones of a number of scientific subjects. In condensed matter, optical lattices allowed the experimental realization of several theoretically predicted phenomena. Indeed, these structures constitute ideal benchmarks for light and wave propagation experiments involving interacting particles, such as clouds of ultra-cold atoms that Bose-Einstein condensate. Moreover, they allow experimental design of particular lattice topologies, as well as the implementation of several classes of spatial perturbations. For example, Anderson localization being observed for the first time in atomic Bose-Einstein condensate experiments and Aubry-André localization discovered with light propagating through networks of optical waveguide.

This thesis considers different types of lattices in the presence of quasiperiodic modulations, mainly the celebrated Aubry-André potential. Particular attention will be given to spectral properties of models, localization features of eigenmodes and the transition from delocalized (metallic) eigenstates to localized (insulating) ones within the energy spectrum. We additionally discuss the relation between the model's properties and the dynamics of particles hopping along the lattice.

After introducing the linear discrete Schrödinger equation, we first discuss the spectral properties of the Aubry-André model. We then study the transition between metallic and insulating regimes of a class of quasiperiodic potentials constructed as an iterative superposition of periodic potentials with increasing spatial period. Next, we discuss the Aubry-André perturbation of flat-band topologies, their energy-dependent transition (mobility edge), which can be expressed in analytical forms in case of specific onsite energy correlations, highlighting existence of zeroes, singularities and divergences. We then discuss two cases of driven one-dimensional lattices, namely an Aubry-André chain with a weak time-space periodic driving and an Anderson chain with a quasiperiodic multi-frequency driving. We show anaytically and numerically how drivings can lift the respective localization and generate delocalization by design. Finally we discuss the problem of the possible generation of correlated metallic states of two interacting particles problem in one dimensional Aubry-André chains, under a coherent drive of the interaction.

Publications

The thesis is based on the following publications:

Published:

- H. Hatami, **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Quasiperiodic driving of Anderson localized waves in one dimension*", Phys. Rev. E **93**, 062205 (2016);
- **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Flatband engineering of mobility edges* ", Phys. Rev. B **91**, 235134 (2015);
- **C. Danieli**, K. Rayanov, B. Pavlov, G. Martin and S.Flach, "*Approximating metal-insulator transitions* ", Int. Journal of Mod. Phys. B **29**, 1550036 (2015);
- J. D. Bodyfelt, D. Leykam, **C. Danieli**, X. Yu and S.Flach, "*Flat bands under correlated perturbations* ", Phys. Rev. Lett. **113**, 236403 (2014);
- L.Morales-Molina, E.Doerner, **C.Danieli** and S.Flach, "*Resonant metallic states in driven quasiperiodic lattices: Aubry-Andre localization by design*", Phys. Rev. A **90**, 043630 (2014).

Selected Conference Presentations:

- **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Flatband engineering of mobility edges* ",Australian and New Zealand School in Ultracold Physics, Otago University, Dunedin, New Zealand, December 2016;
- **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Flatband engineering of mobility edges* ", IONS, KOALA, University of Auckland, Auckland, New Zealand, November 2015;
- **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Flatband engineering of mobility edges* ", Conference on Frontiers of Nanoscience, ICTP, Triest, Italy, August 2015;
- **C. Danieli**, K. Rayanov, B. Pavlov, G. Martin and S.Flach, "*Approximating metal-insulator transitions* ", Seminar at the Center for Theoretical Physics of Complex Systems, Daejeon, South Korea, July 2015;

- L.Morales-Molina, E.Doerner, **C.Danieli** and S.Flach, "*Resonant metallic states in driven quasiperiodic lattices: Aubry-Andre localization by design*", NZIAS-MPIPKS Tandem Workshop, Nonlinear Physics at the Nanoscales: a Cross-Fertilization on Stochastic Methods, Rotorua, New Zealand, February 2015;
- **C. Danieli**, J. D. Bodyfelt and S. Flach, "*Flatband models with correlated onsite perturbation* ", INMS Postgraduate Conference, Massey University, Albany, New Zealand, September 2014;
- L.Morales-Molina, E.Doerner, **C.Danieli** and S.Flach, "*Delocalization in driven quasiperiodic lattices*", lll Dynamic Days South America, Vina de Mar, Chile, November 2014;
- S. Flach, **C. Danieli**, J. D. Bodyfelt , D. Leykam, P. Matthies and A. S. Desyatnikov, "*Perturbing flat-band*", Let's Face Chaos Through Nonlinear Dynamics, University of Maribor, Maribor, Slovenia, July 2014;
- **C. Danieli**, K. Rayanov and S.Flach, "*Approximating metal-insulator transitions* ", Dodd-Wall Symposium, Otago University, Dunedin, New Zealand, October 2013;
- **C. Danieli**, K. Rayanov and S.Flach, "*Approximating metal-insulator transitions* ", INMS Postgraduate Conference, Massey University, Albany, New Zealand, October 2013;
- **C. Danieli**, K. Rayanov and S.Flach, "*Approximating metal-insulator transitions* ", Advanced Workshop on Non-equilibrium Bosons: From Driven Condensates to Non-Linear Optics, ICTP, Triest, Italy, August 2013.

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