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## ABSTRACT

This study examines the development of children's notational schemes including their use of informal nonstandard notations and formal standard notations. A Year 5/6 class of students, their teacher and the researcher were involved in a collaborative teaching experiment in the context of qualitative developmental research. 'Experiment' refers not to untried or unusual instruction, but rather to collaborative analysis and planning of the students' mathematical activity. In order to gain information about children's notation of number computations data was gathered through interviewing, observing, and analyzing work samples of six case study students.

This research study documents the emergence and development of notational schemes from children's problem-solving activities. The ways of symbolizing that emerged in the classroom evolved from the need to clarify and communicate thinking. Children represented their mathematical ideas using a variety of notational forms, both informal and formal. Within the classroom children used notational schemes as a 'thinking device' to help them make sense of their developing mathematical knowledge.

Classroom practice intellectually engaged children with key mathematical ideas. Children increasingly became engaged in genuine dialogical encounters making reference to their own and others' explanations as captured by the notational schemes. As a result, notational schemes served to support shifts in children's mathematical understanding and development.

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