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THE COOLING OF SPENT CARBON ANODES IN THE
ALUMINIUM SMELTING INDUSTRY

A THESIS PRESENTED IN PARTIAL FULFILMENT
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Abstract

As part of the New Zealand Aluminium Smelters (NZAS) upgrade, a hot butt cleaning system has been proposed, this would remove the bath from anodes as they are removed from the cells. It is expected that the time to cool for hot cleaned anodes would be significantly less than for current method of allowing the butts to cool before the bath is removed.

In this project a mathematical model of the cooling process of both the clean and dirty anodes is developed. This model will aid in the investigation of the hot butt cleaning system by showing the difference in cooling times between the clean and dirty anodes.

The temperature profiles within both clean and dirty anodes is calculated for one-, two- and three-dimensional models. Temperature changes in the anodes with time are also compared to experimental data.

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Nomenclature

All constants and variables used in this thesis are defined when first used. Commonly used notation is summarized here.

a, b, c, d, e, f, g	diffusivity coefficients [m^2/s]
A	surface area [m^2]
A, B, C, D	defined variables
A_h	horizontal downward facing surface area [m^2]
Bi	Biot number [—]
c	heat capacity [kJ/kgK]
c_1, c_2, c_3	defined constant
D	diameter of sphere [m]
E_g	energy generated in a system [J]
E_{in}	energy transferred into a system [J]
E_{out}	energy transferred out of a system [J]
E_s	energy stored in a system [J]
$F(x), G(t)$	defined function
Fo	Fourier number [—]
g	gravitational acceleration [m/s^2]
h	heat transfer coefficient [W/m^2K]
h_c	convection heat transfer coefficient [W/m^2K]

h_r	radiation heat transfer coefficient [W/m^2K]
H	height [m]
H	characteristic length [m]
H	dimensionless heat transfer coefficient [-]
k	thermal conductivity [W/mK]
k_x	thermal conductivity in x-direction [W/mK]
k_y	thermal conductivity in y-direction [W/mK]
k_z	thermal conductivity in z-direction [W/mK]
L	length [m]
L_m	longest linear dimension [m]
L_x	x dimension [m]
L_y	y dimension [m]
L_z	z dimension [m]
Nu	Nusselt Number [-]
\bar{P}	mean horizontal perimeter [m]
Pr	Prandtl Number [-]
q	rate of heat transfer [kW]
q_c	rate of convection heat transfer [kW]
q_r	rate of radiation heat transfer [kW]
q''	rate of heat transfer [kW]
$q_1, q_2, q_3, q_4, q_5, q_6$	rate of heat transfer from specific direction [kW]
Ra_H	Rayleigh Number [-]
Re	Reynolds Number [-]
t	time [s]

t_o	time normalisation constant [s]
\bar{t}	normalised time [-]
T	temperature [K]
T_i	initial temperature [K]
T_o	temperature normalisation constant [K]
T_s	surface temperature of anode [K]
T_∞	ambient temperature [K]
\bar{T}	normalised temperature [-]
U_∞	air speed [m/s^2]
v_o	length normalisation constant [m]
V	volume of body [m^3]
x, y, z	spatial coordinate
x_o	length normalisation constant [m]
$\bar{x}, \bar{y}, \bar{z}$	normalised spatial coordinate [-]
$X(x), Y(y), Z(z)$	defined function
z_f	thickness of body [m]

Greek

α	thermal diffusivity [m^2/s]
β	coefficient of thermal volumetric expansion [K^{-1}]
δ	ratio of timesteps to grid size squared [s/m^2]
Δt	size of timestep [s]
Δx	distance between mesh points in x-direction [m]
Δy	distance between mesh points in y-direction [m]
Δz	distance between mesh points in z-direction [m]
ϵ	emissivity [-]

λ, μ, ν	defined variables
μ_s	viscosity of air at surface temperature [kg/sm]
μ_∞	viscosity of air at ambient temperature [kg/sm]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
σ	Stefan-Boltzmann Constant [W/m^2K^4]

Subscripts

b	bath
c	carbon
i	grid points
j	grid points
k	grid points
s	steel

Superscripts

m	timesteps
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