

STATISTICAL METHODS FOR ESTIMATING TEPHRA SOURCE AND DISPERSAL

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF
DOCTOR OF PHILOSOPHY
IN
STATISTICS
AT MASSEY UNIVERSITY, PALMERSTON NORTH,
NEW ZEALAND.

Emily Kawabata

2016

Copyright is owned by the Author of the thesis. Permission is given for a copy to be downloaded by an individual for the purpose of research and private study only. The thesis may not be reproduced elsewhere without the permission of the Author.

Abstract

Tephra refers to any pyroclastic fragments ejected from a volcanic vent and its dispersal is one of the major hazards with explosive eruptions. The attenuation of tephra fall thickness is most commonly estimated after contouring field measurements into smooth isopachs. I explicitly describe the variability in thickness by using a semiempirical tephra attenuation relation as a link function. This opens the way to fitting models to actual tephra observations through maximum likelihood estimation (MLE). The method is illustrated using data published from the 1973 Heimaey eruption in Iceland.

Complex eruptions commonly produce several phases of tephra fall from multiple vents. When attempting to precisely reconstruct past eruptions from the geological record alone, separate phases are often indistinguishable. Augmented by a mixture framework, the MLE attenuation model was able to identify the sources and directions of tephra deposition for the 1977 Ukinrek Maars eruption in Alaska, US, from only the tephra thickness data. It was then applied to the unobserved 1256 AD Al-Madinah eruption in Saudi Arabia.

The estimation of the spatio-temporal hazard from a monogenetic volcanic field is critically dependent on a reconstruction of past events. The Auckland Volcanic Field (AVF) has produced about 50 volcanoes in the last 250,000 years. Although inconsistent, age data for many of these volcanoes exist from various dating methods with various reliabilities. The age order of some pairs is also known due to the overlaying of lavas

(stratigraphy). A discussion is provided on how informative priors are obtained via expert elicitation, on both the individual volcano ages, and the reliabilities of the dating methods. A possible Bayesian model for reconciling the available inconsistent volcano age data to estimate the true eruption ages is also discussed.

To improve these eruption age estimates, some of the volcanoes can be correlated with the better dated tephra layers recovered from five maars in the field. The likelihood of any combination of volcano and tephra, incorporating the spatial variability based on the attenuation model and temporal components, is evaluated and is maximised numerically using linear programming. This statistical matching provides an improvement in the volcano age-order model and age estimates of the volcanoes in the AVF.

Acknowledgements

I would like to thank Professor Mark Bebbington, Professor Shane Cronin, and Dr Ting Wang for their guidance and showing me the joy of doing research.

I would like to thank Professor Stephen Marsland, Associate Professor Geoff Jones, Dr Christopher Tuffley, Dr Jonathan Marshall, Professor Martin Hazelton, and Dr Jonathan Godfrey for their time and support.

I would like to thank Dr Rebecca Green and Dr Kate Richards for their friendships and company.

Publications arising from this thesis

- **Kawabata, E., Bebbington, M., Cronin, S., and Wang, T. (2013).** *Modelling thickness variability in tephra deposition. Bulletin of Volcanology*, 75(8): 1–14.
- **Kawabata, E., Cronin, S., Bebbington, M., Moufti, M., El-Masry, N., and Wang, T. (2015).** *Identifying multiple eruption phases from a compound tephra blanket: an example of the AD1256 Al-Madinah eruption, Saudi Arabia. Bulletin of Volcanology*, 77(1): 6.
- **Kawabata, E., Bebbington, M., Cronin, S., and Wang, T. (under review).** *Optimal likelihood-based matching of volcanic sources and deposits in the Auckland Volcanic Field. Journal of Volcanology and Geothermal Research.*

Contents

Abstract	iii
Acknowledgements	v
Publications arising from this thesis	vii
1 Introduction	1
1.1 Motivation	1
1.2 Overview	4
2 Literature Review	9
2.1 Volcanological background	9
2.1.1 Volcanic eruptions	9
2.1.2 Tephra	11
2.1.3 Grain size	12
2.2 Tephra spatial hazard models	13
2.2.1 Isopachs	13
2.2.2 Numerical models	16
	ix

2.2.3	Empirical models	17
2.2.4	Semiempirical models	19
2.2.5	Fresh versus old tephra	23
2.2.6	Volume estimation	24
2.2.7	Residuals, sensitivity analysis and robustness	27
2.2.8	Recreation of past events	28
2.3	Dating methods	29
2.3.1	Radiocarbon dating	29
2.3.2	Potassium-argon dating	30
2.3.3	Argon-argon dating	30
2.3.4	Paleomagnetic declination	33
2.3.5	Optically stimulated luminescence dating	33
2.3.6	Thermoluminescence dating	34
2.4	Summary	34
3	Tephra Dispersal - One Component	35
3.1	Introduction	35
3.2	The Heimaey eruption	36
3.3	Sampling error	36
3.4	Tephra attenuation	39
3.5	Results	45
3.6	Sensitivity analyses	49
3.7	Discussion	50

3.8	Conclusion	54
4	Tephra Dispersal - Multiple Components	55
4.1	Introduction	55
4.2	Data	56
4.2.1	The Ukinrek Maars eruption	56
4.2.2	The Al-Madinah eruption	57
4.3	Methodology	61
4.3.1	Tephra modelling	61
4.3.2	Data imputation	62
4.3.3	Error distribution for old deposits	64
4.4	Results	66
4.4.1	The Ukinrek Maars eruption	66
4.4.2	The Al-Madinah eruption	68
4.5	Discussion	74
4.5.1	The Ukinrek Maars eruption	74
4.5.2	The Al-Madinah eruption	80
4.6	Conclusion	82
5	Auckland Volcanic Field	83
5.1	Introduction	83
5.2	Direct volcano age determinations	84
5.3	Volcano age constraints	88

5.4	Maar data	90
5.5	Spatio-temporal hazard estimates	92
5.6	Tephra hazard estimates	98
5.7	Correlating tephras to volcanoes	99
6	Bayesian Age Reconciliation	101
6.1	Introduction	101
6.2	Expert elicitation	102
6.2.1	Volcano ages	109
6.2.2	Dating method reliability	110
6.3	Discussion	125
7	Tephra ID matching	129
7.1	Introduction	129
7.2	Other volcano data	132
7.3	Age-order model	134
7.3.1	Likelihood equation	135
7.3.2	Tephra attenuation model	136
7.3.3	Age model	139
7.3.4	Linear programming	145
7.3.5	Age model averaging	147
7.4	Results	148
7.5	Sensitivity analyses	154

7.6	Discussion	156
7.7	Conclusion	159
8	Conclusions and Future Work	161
8.1	Statistical attenuation modelling	161
8.2	Statistical age ordering in volcanic fields	163
8.3	Future work	164
8.3.1	Bayesian age reconciliation model	164
8.3.2	Grain size	165
8.3.3	Stochastic models for monogenetic volcanism	165
8.3.4	Wind direction	166
	Bibliography	167
A	The marginal posterior probabilities of matching AVF volcanoes and their tephras	191

List of Figures

1.1	The Auckland region (inset), obtained from http://transportblog.co.nz/tag/population-growth/ , is highly populated and its geography presents significant challenges for evacuation planning.	2
2.1	An example of a maar: Ukinrek Maars in Alaska, US, obtained from http://volcano.si.edu/	11
2.2	An isopach map of the tephra deposits from the Fogo A Plinian deposit, Sao Miguel (Azores) (Bursik et al., 1992). The red star indicates the vent.	14
2.3	Contours of fitted thicknesses on the Heimaey data (Self et al., 1974). The observed thicknesses (cm) are shown. The vent is indicated by the origin.	21
2.4	Observed vs. fitted thicknesses of the two semiempirical models using the Heimaey data.	22
2.5	The relationship between thickness (cm) and distance (km) from the vent for the Heimaey eruption data.	25
2.6	Calibration curve based on Southern Hemisphere tree-ring measurements SHCal04 with one standard deviation interval (McCormac et al., 2004).	31
2.7	Radiocarbon age to calendar age conversion using http://radiocarbon.ldeo.columbia.edu/research/radcarbcal.htm	32

2.8	The two alternating states of the geomagnetic field (Bogue and Merrill, 1992).	33
3.1	The observed thickness (cm) and locations for the Heimaey eruption data. The vent is indicated by the triangle. The dotted line indicates the coast line prior to this eruption.	37
3.2	Nonparametric relative sampling error R and three fitted distributions. .	40
3.3	The relative errors from the two models. Top row is the model without wind (Equation 3.6) and bottom row the model with wind (Equation 3.5). Columns are for the lognormal, Weibull, and gamma error distributions, reading left to right. The displacements are relative to the vent location, indicated by the triangle.	47
3.4	The Monte Carlo residual bounds for tephra thickness. Top row is the model without wind (Equation 3.6) and bottom row the model with wind (Equation 3.5). Columns are for the lognormal, Weibull and gamma error distributions, reading left to right. Medians are shown by the dashed lines, 90% bounds by the filled areas.	48
3.5	A sensitivity analysis on the estimated parameters. The maximum likelihood estimates are shown by the dashed lines.	50
3.6	Relationships between eruptive column height H and the attenuation parameters.	52
4.1	The observed tephra thicknesses (cm) for the Ukinrek Maars eruption. The vents are indicated by the triangles.	58
4.2	Oblique aerial view of the largest cones along the north-northwest aligned fissure system produced during the 1256 AD Al-Madinah eruption. The three cones show Cones 6, 5, and 4 are labelled.	59

4.3	Tephra ash fall distribution map for the 1256 AD tephra blanket, with spot thickness measurements (cm) marked and automated isopachs drawn using a simple kriging process (Cleveland et al., 1992). The lava and spatter blankets are shown by the light and darker grey shading. The cones are indicated by the triangles. Inset map shows the position of the eruption in the Harrat Rahat lava field, western Saudi Arabia.	60
4.4	The observed thicknesses (cm) are in large type while the interpolated thicknesses (see text) are in smaller type for the Ukinrek Maars data.	63
4.5	Aleatory uncertainty in tephra thickness subject to weathering and compaction. The original (f) densities have the same mean and variance. The new (w) densities are then found by multiplying κ by 1.5 (to thin out the small tephra), and retaining the same mean (i.e., tephra is re-dispersed).	66
4.6	The 3cm isocontours for each component from the best 1-3 model are shown for the Ukinrek Maars eruption. W1 is the West Maar component, while the East Maar components are denoted E1-E3. Observed thicknesses (cm) and locations are also shown.	69
4.7	The relative errors at measurement locations for the best 1-3 model for the Ukinrek Maars data. The text is scaled by size of error.	70
4.8	Model component tephra lobes estimated for the 1256 AD Al-Madinah eruption. The 3cm isopachs for each component of the 3-1-1 model (top) and the 4-1-1 model (bottom). The 5, 10, 20, and 50cm isopachs are generated from the complete model. The relative (aleatory) errors at measurement locations are shown by the shape and size of symbols.	73
4.9	Sensitivity analyses of imputation done for the Ukinrek Maars data.	77

4.10	The relationship between log thickness $\log T$ and square root of the area \sqrt{A} (after Pyle 1989) for the entire modelled composite tephra fall. Integration yields a total volume estimate of 0.0077km^3 ($0.0024 \pm 0.0004\text{km}^3$ DRE).	81
5.1	The Auckland Volcanic Field (Bebbington and Cronin, 2011). The cored maars are indicated by stars.	85
5.2	Examples of tephra layers in Auckland maar sediment cores (Molloy et al., 2009).	91
5.3	Temporal and spatial hazards in the AVF.	96
5.4	Long-term annual probability of base surge impact as estimated from BET_VH application (Sandri et al., 2012).	97
6.1	The questionnaire template.	103
6.2	Histograms of generated volcano ages from the expert elicitation and fitted densities. The number in the parenthesis indicates the number of experts who provided an age estimate.	111
6.3	Stacked bar charts of the reliabilities of the dating methods. The left is the reliability of returned sample and the right is the reliability of returning the correct age.	126
6.4	Scatterplots showing the correlation in the responses between the two reliability questions on the dating methods. Each dot represents a response from an expert.	128
7.1	The Auckland Volcanic Field. The cored maars are indicated by circles and bold text. Volcanoes in italics were not considered as candidates in the matching procedure. Symbols scale with the cube root of the tephra volume.	131

7.2	Simulated ages (histograms) of selected volcanoes resulting from the constraints outlined in the text. Note the different vertical scale for Mt Victoria.	145
-----	---	-----

List of Tables

2.1	ϕ scale conversion for the grain size distribution.	13
3.1	Estimated parameters of the models with errors (1σ).	46
3.2	Residual error statistics from the models.	46
3.3	Parameter estimates for the sensitivity analysis on the Heimaey data. Errors are 1σ	49
4.1	Proportion of correctly identifying the error distribution before and after thinning using hypothetical thickness data.	67
4.2	The estimated AICs (lowest number indicates best fit) and model pa- rameters for the imputed Ukinrek Maars thickness data.	68
4.3	The estimated model parameters for the imputed Ukinrek Maars thick- ness data.	71
4.4	The AICs and total bulk fall volumes (sum of numerically integrated volumes of all identified lobes) of the fitted models for the 1256 AD Al- Madinah data. Model notation is such that 4-1-1 indicates four phases on Cone 6, one at each of Cones 5 and 4.	74
4.5	The mean estimated parameters (with standard deviation) of the Ukin- rek models fitted to the imputed data including bootstrap errors R^* . . .	79

5.1	The available determined ages for the AVF volcanoes. The minimum ages for Hopua and Orakei Basin follow from the oldest observed tephra found in their cores. “Sea level” means that the volcano occurred above or below a certain sea level, which can be used to constrain the age. Nine ^{14}C ages from Maungataketake and two ^{14}C ages from Panmure Basin were combined via the method of Ward and Wilson (1978) to arrive at a mean age and error shown in this table.	87
5.2	Stratigraphy and contemporaneous events. $A > B$ to indicate that eruption A occurred before eruption B . An equality indicates (near) contemporaneous events, which may also be ordered. Where no reference is given, see Bebbington and Cronin (2011).	89
5.3	The posterior ages and thicknesses for the marker and AVF tephras found in the five cores in the AVF. The Maketu and Tahuna ages are from Molloy et al. (2009) and the Rotoehu from Danišík et al. (2012); other tephra ages are summarised by Lowe et al. (2013). Blank entries delineate the age limits of the recovered cores (censoring).	93
7.1	Wind directions (degrees anticlockwise from East) and volcano tephra volumes estimated from volcano tuff and scoria volumes (^a Allen and Smith (1994) ^b Kereszturi et al. (2013)) using Equation 7.1.	133
7.2	Notations for the age-order model.	135
7.3	Prior age distributions for each vent in the AVF, obtained from the available age determinations (Section 5.2). Where the age is given as an addition to that of another volcano these are the contemporaneous events.	142
7.4	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $\alpha = 2.0$, $\beta U = 0.5$. The global best arrangement is indicated by bold type.	150

7.5	Best global arrangement of assigning Volcano i to AVF tephra j . Scenarios are outlined in the sensitivity analysis section (Section 7.5). M stands for Maungataketake, NH stands for North Head; and TK stands for Three Kings. The six parameter sets are $A = \{\alpha = 1.5, \beta U = 0.5\}$; $B = \{\alpha = 2, \beta U = 0.5\}$; $C = \{\alpha = 2.5, \beta U = 0.5\}$; $D = \{\alpha = 1.5, \beta U = 1\}$; $E = \{\alpha = 2, \beta U = 1\}$; and $F = \{\alpha = 2.5, \beta U = 1\}$	155
7.6	Comparison of event order models.	160
A.1	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $A = \{\alpha = 1.5, \beta U = 0.5\}$	192
A.2	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $C = \{\alpha = 2.5, \beta U = 0.5\}$	193
A.3	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $D = \{\alpha = 1.5, \beta U = 1.0\}$	194
A.4	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $E = \{\alpha = 2.0, \beta U = 1.0\}$	195
A.5	Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $F = \{\alpha = 2.5, \beta U = 1.0\}$	196
A.6	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $A = \{\alpha = 1.5, \beta U = 0.5\}$	197
A.7	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $B = \{\alpha = 2.0, \beta U = 0.5\}$	198
A.8	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $C = \{\alpha = 2.5, \beta U = 0.5\}$	199

A.9	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $D = \{\alpha = 1.5, \beta U = 1.0\}$	200
A.10	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $E = \{\alpha = 2.0, \beta U = 1.0\}$	201
A.11	Marginal posterior probabilities for a volcano to have produced a given tephra for the Maungataketake and North Head scenario with $F = \{\alpha = 2.5, \beta U = 1.0\}$	202
A.12	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $A = \{\alpha = 1.5, \beta U = 0.5\}$.	203
A.13	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $B = \{\alpha = 2.0, \beta U = 0.5\}$.	204
A.14	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $C = \{\alpha = 2.5, \beta U = 0.5\}$.	205
A.15	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $D = \{\alpha = 1.5, \beta U = 1.0\}$.	206
A.16	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $E = \{\alpha = 2.0, \beta U = 1.0\}$.	207
A.17	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 9 scenario with $F = \{\alpha = 2.5, \beta U = 1.0\}$.	208
A.18	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $A = \{\alpha = 1.5, \beta U = 0.5\}$.	209
A.19	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $B = \{\alpha = 2.0, \beta U = 0.5\}$.	210

A.20	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $C = \{\alpha = 2.5, \beta U = 0.5\}$.	211
A.21	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $D = \{\alpha = 1.5, \beta U = 1.0\}$.	212
A.22	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $E = \{\alpha = 2.0, \beta U = 1.0\}$.	213
A.23	Marginal posterior probabilities for a volcano to have produced a given tephra for the Three Kings=AVF 10 scenario with $F = \{\alpha = 2.5, \beta U = 1.0\}$.	214
A.24	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $A = \{\alpha = 1.5, \beta U = 0.5\}$ 215
A.25	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $B = \{\alpha = 2.0, \beta U = 0.5\}$ 216
A.26	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $C = \{\alpha = 2.5, \beta U = 0.5\}$ 217
A.27	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $D = \{\alpha = 1.5, \beta U = 1.0\}$ 218
A.28	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $E = \{\alpha = 2.0, \beta U = 1.0\}$ 219
A.29	Marginal posterior probabilities for a volcano to have produced a given tephra for the AVF 9A/9B scenario with $F = \{\alpha = 2.5, \beta U = 1.0\}$ 220

