*Res. Lett. Inf. Math. Sci.*, 2004, Vol.**6**, pp109-122
Available online at http://iims.massey.ac.nz/research/letters/


# A stochastic spatial-temporal disaggregation model for rainfall

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A stochastic model for disaggregating spatial-temporal rainfall data is presented. In the model, the starting times of rain cells occur in a Poisson process, where each cell has a random duration and a random intensity. In space, rain cells have centres that are distributed according to a two dimensional Poisson process and have radii that follow an exponential distribution. The model is fitted to seven years of five-minute data taken from six sites across Auckland City. The historical five-minute series are then aggregated to hourly depths and stochastically disaggregated to five-minute depths using the fitted model. The disaggregated series and the original five-minute historical series are then used as input to a network flow simulation model of Auckland City's combined and wastewater system. Simulated overflow volumes predicted by the network model from the historical and disaggregated series are found to have equivalent statistical distributions, within sampling error. The results thus support the use of the stochastic disaggregation model in urban catchment studies.

## 1 Introduction

In a recent paper, a methodology based on a Neyman-Scott process was developed to enable the simulation of spatial-temporal rainfall time series data [7]. The methodology was tested using historical records of hourly data from the Arno Catchment in Italy; the results showing that the model was able to preserve extreme values at the 1 and 24h levels of aggregation, thus supporting the use of the model in hydrological catchment studies, e.g. flood studies [7]. A special case of the model is considered here for disaggregating hourly data to five-minute data.

A range of stochastic models for disaggregating rainfall data exist. For example, there are models for downscaling the output from deterministic global circulation models, e.g. [22, 23, 5, 19, 18, 13], and there are models aimed more specifically at producing fine resolution data for urban catchment studies e.g. [16, 6, 12, 8, 9, 17, 21]. The model that we consider here differs from previous disaggregation models in that it is a conceptual-stochastic model, using only a moderate number of parameters and incorporating random variables to represent 'rain cells' which are understood to occur in the physical process. Furthermore, the model used here can be applied in both space and time, which increases the range of possible applications. For example, the model could be used for generating spatially representative data at sites lacking data, i.e. spatial-temporal infilling.

The methodology used here is similar to that used in [14, 20, 4] in that 'within storm' rain cells will have arrival times that occur in a Poisson process. However, these papers apply a Bartlett-Lewis process to univariate rainfall time series, whilst our focus here is on disaggregating spatial-temporal data to fine resolutions.

## 2 Auckland City Catchment

The Auckland Region is situated on the North Island of New Zealand, with mainland Auckland City occupying 15,300 hectares on an isthmus between the Waitemata and Manukau Harbours (Figure 1). Auckland contains considerable industrial activity, extensive roadway, and is the largest city in New Zealand with a population of 368K, extending to 1.2 million people in the greater Auckland area. Auckland City is forecast to grow to 583K over the next 50 years [3].

There are three types of piped drainage networks in Auckland City: wastewater, stormwater and combined. Combined stormwater and wastewater networks remain in 18% of the city [1]. Due to interaction among the three types of networks, the assemblage results in two interrelated drainage systems that remove: (i) stormwater from within Auckland City to streams and harbours where it is discharged, and (ii) wastewater and combined flows to the Mangere wastewater treatment plant. In all approximately 2,500 outfalls discharge potentially impaired stormwater along 82km of coastline and numerous urban streams, which can include discharge from some 350 designed overflows structures on the combined and wastewater networks [2].

To fulfill environmental regulatory requirements and meet the demands of the rising population, Auckland City Council and Metro Water Limited initiated an integrated catchment study. The objective of the study is to develop a comprehensive understanding of the Auckland City drainage system to enable engineers to redesign and upgrade the system to meet regulatory requirements that minimize the pollution to receiving water courses and flooding due to excess loads.

A network model of the trunk and larger wastewater and combined pipes was assembled using the 'MOUSE' hydraulic model developed in [11, 10]. The network model consists of some 3,122 manholes, 236 overflows (structures, pump stations, and lumped overflows) and 568 sub-catchments (Figure 1; [15]).

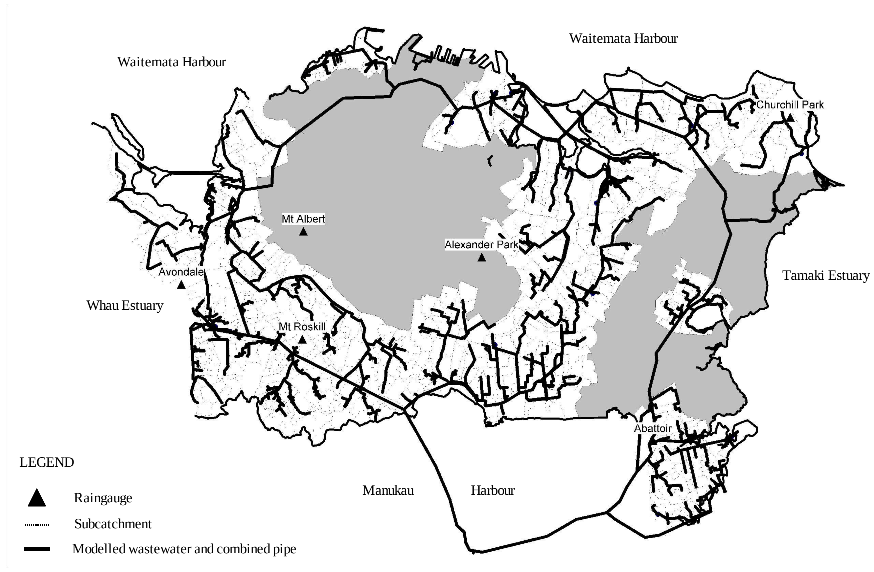


Figure 1: Auckland City catchment: Network model and location of rain gauges (records are for the period: 1993-99)

An understanding of rainfall is crucial to the success of the integrated catchment study. Available records from sites in Auckland City include five-minute data from six sites for the period 1993-99 (Figure 1). As the historical data is limited in extent, a spatial-temporal stochastic model [7] is proposed for simulation of long records of multisite 1h data which could be used as input to the network model to assess system performance under a range of conditions. However, rainfall data at finer resolutions than 1h are needed because of the rapid rainfall event response of an urban drainage system. Consequently, a stochastic disaggregation model is required to enable simulated 1h series to be disaggregated to five-minute series. A similar approach was adopted for single sites in the United Kingdom, where a disaggregator based on an algorithm developed in [21] was used to disaggregate simulated hourly time series [9]. The focus in this paper is to propose a suitable model for stochastically disaggregating multisite 1h data and validate the model with respect to the intended application using a wastewater and combined network model for Auckland City.

## 3 Model Definition

Consider a stochastic process of rain cells:

$$\{(U_i, V_i), S_i, L_i, X_i, R_i\},$$

where for the $i$th cell: $(U_i, V_i)$ forms a two-dimensional Poisson process with rate $\varphi$ (per km$^2$); $(U_i, V_i)$ and $R_i$ form discs in two-dimensional space, where $(U_i, V_i)$ is the disc centre and $R_i$ is an independent random variable representing the disc radius; $S_i$ is the arrival time of the cell which occurs in a Poisson process with rate $\lambda$; $L_i$ is an independent random variable representing the cell lifetime, so that the cell terminates at a time $S_i + L_i$; $X_i$ is an independent random variable representing cell intensity, which remains constant throughout the cell lifetime and over the area of the disc. Rain cells can thus be thought of as cylinders in three-dimensional space with heights given by $X_i$. Furthermore, the total intensity at time $t$ and location $\mathbf{x} \in \mathbb{R}^2$, denoted as $Y(\mathbf{x}, t)$, is the sum of the intensities of all cells active at time $t$ and overlapping point $\mathbf{x}$. The stochastic process of rain cells is similar to that used in [7], with the exception that the cells arrive in a Poisson process instead of a Neyman-Scott process.

For the purpose of model fitting and simulation, some assumptions are made about the distributions of the random variables used for the rain cells: $X_i$ is taken to be an independent weibull random variable with parameters $\theta$ and $\alpha$ and survivor function $P(X_i > x) = e^{(x/\theta)^\alpha}$; the cell lifetime $L_i$ and radius $R_i$ are taken to be independent exponential random variables with parameters $\eta$ and $\phi$ respectively. Under these assumptions, $\phi^2 = 2\pi\varphi$, which reduces the number of parameters in the model to five: $\lambda$, $\phi$, $\eta$, $\theta$, and $\alpha$.

Rainfall data are usually available as discrete time-series, so that it is necessary to consider the aggregated stochastic process:

$$Y_k^{(h)}(\mathbf{x}) = \int_{(k-1)h}^{kh} Y(\mathbf{x}, t)dt \tag{1}$$

so that $Y_k^{(h)}(\mathbf{x})$ is the rainfall depth in the $k$th time interval of duration $h$ at location $\mathbf{x}$.

Statistical properties of $Y_k^{(h)}(\mathbf{x})$, up to third-order, follow directly by taking $C \equiv 1$ in [7], equations (5)-(7). These properties then follow as:

$$\mu_h = E\left\{Y_k^{(h)}(\mathbf{x})\right\} = \lambda E(X)h/\eta \tag{2}$$

$$\begin{aligned} \gamma_{\mathbf{x},\mathbf{y},h,l} &= \operatorname{cov}\left\{Y_k^{(h)}(\mathbf{x}), Y_{k+l}^{(h)}(\mathbf{y})\right\} \\ &= \gamma_{\mathbf{x},\mathbf{x},h,l} - \lambda\{1 - P(\phi, d)\} E(X^2)A(h,l)/\eta^3 \end{aligned} \tag{3}$$

$$E\left[\{Y_k^{(h)}(\mathbf{x}) - \mu_h\}^3\right] = 6\lambda E(X^3)(\eta h - 2 + \eta h e^{-\eta h} + 2e^{-\eta h})/\eta^4 \tag{4}$$

where:

$$\gamma_{\mathbf{x},\mathbf{x},h,l} = \gamma_{\mathbf{y},\mathbf{y},h,l} = 2\lambda E(X^2)A(h,l)/\eta^3 \tag{5}$$

$$A(h,l) = \begin{cases} (h\eta + e^{-\eta h} - 1) & \text{for } l = 0 \\ \frac{1}{2}(1 - e^{-\eta h})^2 e^{-\eta h(l-1)} & \text{for } l > 0 \end{cases} \tag{6}$$

$$P(\phi, d) = \frac{2}{\pi}\int_0^{\pi/2}\left(\frac{\phi d}{2cosy} + 1\right) exp\left(\frac{-\phi d}{2cosy}\right) dy \tag{7}$$

$P(\phi, d)$ is the probability that a cell overlaps some point $\mathbf{x}$ given that it overlapped a point $\mathbf{y}$, at spatial separation $d = ||\mathbf{x} - \mathbf{y}||$. The moments of the weibull distribution for the $X_i$ are given by: $E(X^r) = \theta^{r/\alpha}\Gamma(1 + r/\alpha)$.

In the above, spatial-temporal stationarity is assumed, e.g. $\gamma_{\mathbf{x},\mathbf{x},h,l} = \gamma_{\mathbf{y},\mathbf{y},h,l}$ in equation 5.

## 4 Fitted Model

When fitting the model to five-minute time series (i.e. with $h = 5mins$ in equations 2–4), it is convenient to work with the following dimensionless functions, which do not depend on the cell intensity scale parameter $\theta$:

$$\begin{array}{llcl} \text{Coefficient of variation:} & \nu(\lambda,\eta,\phi,\alpha) & = & \gamma_{\mathbf{x},\mathbf{x},5,0}/\mu_5 \\ \text{Autocorrelation (lag 1):} & \rho(\lambda,\eta,\phi,\alpha) & = & \gamma_{\mathbf{x},\mathbf{x},5,1}/\gamma_{\mathbf{x},\mathbf{x},5,0} \\ \text{Cross-correlation:} & \rho_{\mathbf{x},\mathbf{y}}(\lambda,\eta,\phi,\alpha) & = & \gamma_{\mathbf{x},\mathbf{y},5,0}/\gamma_{\mathbf{x},\mathbf{x},5,0} \\ \text{Coefficient of skewness:} & \kappa(\lambda,\eta,\phi,\alpha) & = & E[\{Y_k^{(5)}(\mathbf{x}) - \mu_5\}^3]/\gamma_{\mathbf{x},\mathbf{x},5,0}^{3/2} \end{array} \tag{8}$$

To obtain the sample estimates of these functions, data from the six sites (Figure 1) were pooled and the equivalent sample estimates calculated. Hours containing zero rainfall at all sites were removed before the calculation of the sample statistics so that the estimates were for 'wet' hourly sequences only. (The sample autocorrelation was calculated using adjacent values from 'unbroken' wet hourly series, using the sample mean and variance from all the pooled data.)

The sample estimates were calculated using a similar approach to that in [7] with the exception that one sample estimate was used for all seasons, so that the same fitted disaggregation model would be applied over all calendar months. On first sight this may seem an unreasonable over-simplication. However, it can perhaps be seen to be more reasonable based on the assumption that most of the seasonal variation in rainfall data is captured at higher levels of aggregation (e.g. hourly or daily levels), which will be preserved in the disaggregated series.

The parameters $\lambda, \alpha, \eta, \phi$ were estimated by minimising the following sum of squares:

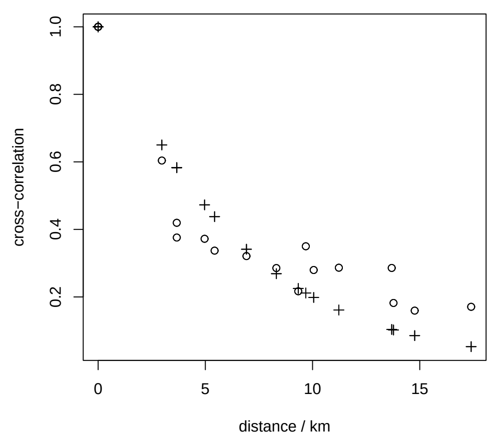


Figure 2: Fitted (+) and historical (○) cross-correlations against distance

$$SS = \left(1-\frac{\nu}{\hat{\nu}}\right)^2 + \left(1-\frac{\hat{\nu}}{\nu}\right)^2 + \left(1-\frac{\rho}{\hat{\rho}}\right)^2 + \left(1-\frac{\hat{\rho}}{\rho}\right)^2 + \left(1-\frac{\kappa}{\hat{\kappa}}\right)^2 + \left(1-\frac{\hat{\kappa}}{\kappa}\right)^2 + \sum_{(\mathbf{x},\mathbf{y})\in\mathbf{A}} \left[\left(1-\frac{\rho_{\mathbf{x},\mathbf{y}}}{\hat{\rho}_{\mathbf{x},\mathbf{y}}}\right)^2 + \left(1-\frac{\hat{\rho}_{\mathbf{x},\mathbf{y}}}{\rho_{\mathbf{x},\mathbf{y}}}\right)^2\right] \quad (9)$$

where $\mathbf{A}$ is the set of (15) pairs of points corresponding to the locations of the 6 sites.

Using the pooled sample mean, $\theta$ can then be estimated directly from:

$$\hat{\theta} = \left\{\frac{\hat{\mu}_5\hat{\eta}}{\hat{\lambda}\Gamma(1+1/\hat{\alpha})}\right\}^{\hat{\alpha}} \quad (10)$$

Using the above procedure, the following parameter estimates were obtained for the Auckland data: $\hat{\lambda} = 0.0721/min$, $\hat{\eta} = 0.838/min$, $\hat{\phi} = 0.407/km$, $\hat{\alpha} = 0.673$,

Table 1: Fitted Statistics

| Statistic | Sample Value | Fitted Value |
|---|---|---|
| Mean | 0.0708 | 0.0708 |
| Standard Deviation | 3.69 | 3.77 |
| Skewness | 8.95 | 10.1 |
| Autocorrelation | 0.138 | 0.151 |

and $\hat{\theta} = 0.123mm/min$. The sample and fitted statistics are shown in Table 1 and Figure 2, where it can be seen a reasonable fit is obtained, but with a slight over-estimation of skewness and autocorrelation (Table 1). There was also some slight under-estimation of the sample cross-correlations for those sites having the greater spatial separation (Figure 2). However, given that these discrepancies were only slight and that a very parsimonious model parametrization had been used, no further improvement in fit was sought.

## 5 Disaggregation Algorithm

There are many possible implementations of the stochastic model in §3 which would enable the disaggregation of rainfall data. The approach adopted here attempts to reduce computional demands. Disaggregation is carried out for each wet hour by applying an algorithm which is summarised below; a schematic summary of the algorithm is given in Figure 3.

Constants:

| | |
|---|---|
| $N$ | Number of sites (6). |
| $NH$ | Number of hours to disaggregate. |
| $M$ | Number of intervals in each hour (12 for five-minute data). |
| $XTOL$ | Tolerance parameter ($0.5mm$). |
| $WTH$ | Width of catchment ($50km$; the catchment is treated as a square of area $WTH^2$). |

Variables:

| | |
|---|---|
| $H$ | A counter for the hour being disaggregated. |
| $Z(i,j,k)$ | Simulated data in the $k$th (five-minute) interval for the $j$th hour in an adjacent pair ($j = 1,2$) at the $i$th site ($N \times 2 \times M$ array). |
| $Y(i,j,k)$ | Simulated data, due to a single rain cell, in the $k$th (five-minute) interval for the $j$th hour in an adjacent pair ($j = 1,2$) at $i$th site ($N \times 2 \times M$ array). |
| $T(i,j)$ | Total historical rainfall in the $j$th hour at the $i$th site (array of size $N \times (NH + 1)$). |
| $G(i,j)$ | Simulated total rainfall in the $j$th hour of an adjacent pair ($j = 1,2$) at the $i$th site (array of size $N \times 2$). |
| $A(i)$ | Scaling factor for each site. |
| $S$ | Starting time of a rain cell. |
| $L$ | Lifetime of a rain cell. |
| $W$ | Intensity of a rain cell. |
| $X$ | x-coordinate for the centre of a rain cell. |
| $Y$ | y-coordinate for the centre of a rain cell. |

Initialise:

$$
\begin{array}{ll}
Z(i,j,k) = 0 & i = 1,\ldots,N;\ j = 1,2;\ k = 1,\ldots,M. \\
Y(i,j,k) = 0 & i = 1,\ldots,N;\ j = 1,2;\ k = 1,\ldots,M. \\
G(i,j) = 0 & i = 1,\ldots,N;\ j = 1,2. \\
T(i, NH+1) = 0 & i = 1,\ldots,N. \\
H = 0. &
\end{array}
$$

Algorithm:

1. Read in the hourly totals $T(i,j)$ for each site ($i = 1,\ldots,N$; $j = 1,\ldots,NH$).
2. Set $H = H + 1$. If $H = NH + 1$ terminate the algorithm.
3. Set $Z(i,1,k) = Z(i,2,k)$ for $i = 1,\ldots,N$; $k = 1,\ldots,M$. (The five-minute series in the first hour is set to zero plus the rain due to previous overlapping cells.)
4. Set $Z(i,2,k) = 0$ for $i = 1,\ldots,N$; $k = 1,\ldots,M$. (The five-minute series in the second hour is set to zero.)
5. Set $Y(i,j,k) = 0$ for $i = 1,\ldots,N$; $j = 1,2$; $k = 1,\ldots,M$. (The five-minute series due to a single cell starts as zero.)
6. Generate variables for a rain cell:
   (a) A starting time $S$ taken from a uniform distribution over the interval $(0, M)$, i.e. $S \sim U(0, M)$.
   (b) A cell lifetime, $L \sim exp(\hat{\eta})$.
   (c) A cell intensity, $W \sim weibull(\hat{\alpha}, \hat{\theta})$.

(d) A cell radius, $R \sim exp(\hat{\phi})$.

(e) A cell centre, $(X, Y)$;
$X \sim U(-WTH/2, WTH/2)$, $Y \sim U(-WTH/2, WTH/2)$.

7. Calculate a simulated series $Y(i, j, k)$ at each site due to the generated rain cell ($i = 1, \ldots, N; j = 1, 2; k = 1, \ldots, M$). Note that the cell contributes to the simulated series for the $i$th site if, and only if, the distance from the cell centre to the site is less than (or equal to) the cell radius $R$. In addition, note that the starting time $S \leq M$ of the cell occurs in the first hour ($j = 1$) but the cell may overlap into the next hour ($j = 2$).

8. Aggregate the simulated (five-minute) series to a pair of hourly totals $G(i, j)$: Set $G(i, j) = \sum_{k=1}^{M} Y(i, j, k)$ for $i = 1, \ldots, N, j = 1, 2$.

9. If the either of the resultant pair of hourly totals ($j = 1, 2$) obtained by adding the series $G(i, j)$ to the hourly total $\sum_{k=1}^{M} Z(i, j, k)$, exceed either of the historical hourly totals $T(i, H)$ or $T(i, H+1)$ (to within $XTOL$; excluding $H+1$ when $H = HT$) at any site, then discard the generated cell and return to step 5; i.e. return to step 5 if $G(i, j)+\sum_{k=1}^{M} Z(i, j, k)-T(i, H-1+j) > XTOL$ for any site $i$ or any $j = 1, 2$.

10. Update the simulated (five-minute) series $Z(i, j, k)$ at each site by adding $Y(i, j, k)$, i.e. set $Z(i, j, k) = Z(i, j, k) + Y(i, j, k)$ for all $i$ and $j = 1, 2$.

11. Repeat steps 5–10 until until the hourly series $T(i, H)$ and the aggregated simulated (five-minute) series ($\sum_{k=1}^{M} Z(i, 1, k)$) are closely matched at each site, i.e. repeat steps 5–10 until $|\sum_{k=1}^{M} Z(i, 1, k) - T(i, H)| < XTOL$ for all $i = 1, \ldots, N$.

12. Scale the simulated (five-minute) series to achieve an exact match to the hourly data: Set $Z(i, 1, k) = A(i)Z(i, 1, k)$ where $A(i) = T(i, H)/\sum_{k=1}^{M} Z(i, 1, k)$.

13. Save the disaggregated data $Z(i, 1, k)$ to a file.

In the above algorithm, cells are generated over the catchment area $WTH^2$. This area is chosen to be larger than the actual drainage area being studied to allow for cells that may have centres outside the drainage area but which overlap the drainage area.

The algorithm does not use $\hat{\lambda}$ and therefore introduces a small bias due to discarding larger rain cells. A more precise algorithm involves repeatedly simulating a random number of cells over the catchment using $\hat{\lambda}$ until a close match to the hourly totals at each site is obtained. However, as this was computationally too demanding, the above algorithm was adopted for its practicality.

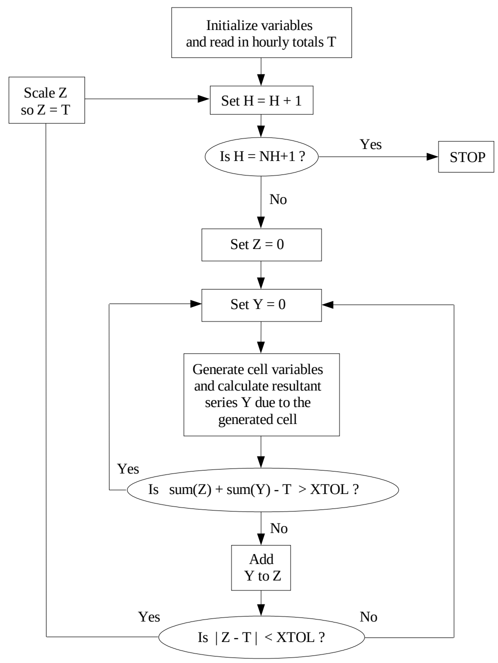


Figure 3: Flow diagram of stochastic disaggregation algorithm

# 6 Model Validation

The historical five-minute series were aggregated to hourly series. These hourly series were then disaggregated to five-minute series using the fitted stochastic model and the disaggregation algorithm.

To verify the disaggregation algorithm, and to assess the bias, sample standard deviations of the simulated five-minute series were compared with the standard deviations of the historical five-minute series at each site (Table 2). A slight underestimation is apparent, which is likely to be due to the tendency for the retention of smaller cells to achieve a match to the hourly totals (Table 2). However, as the bias is only small, the fitted model and implementation were retained for further validation against properties of interest in the intended application.

To validate the model for the intended application, the historical five-minute rainfall time series were input into the pipe network model (Figure 1) and the spill volumes from each of 236 overflows calculated for each year. In addition, the stochastically disaggregated (simulated) five-minute series was also input into the same network model and the overflow volumes resulting from these series also found for each year. This resulted in 236 spill volumes for each year for both the historical and simulated five-minute rainfall series. Summary statistics for these series are given in Table 3, from which it is evident that the disaggregated series produces results comparable to the historical series.

Table 2: Standard Deviation of Five-Minute Rainfall Series

| Site | Historical Series | Disaggregated Series |
|---|---|---|
| 1 | 0.0989 | 0.0965 |
| 2 | 0.109 | 0.102 |
| 3 | 0.108 | 0.0991 |
| 4 | 0.0974 | 0.0930 |
| 5 | 0.108 | 0.0986 |
| 6 | 0.104 | 0.0974 |

Table 3: Overflow Statistics ($m^3$ per year)

| Statistic | Historical Series | Disaggregated Series |
|---|---|---|
| Largest | $4.007 \times 10^5$ | $4.088 \times 10^5$ |
| Second Largest | $1.693 \times 10^5$ | $1.686 \times 10^5$ |
| Mean | $1.358 \times 10^3$ | $1.379 \times 10^3$ |
| Standard Deviation | $1.132 \times 10^4$ | $1.160 \times 10^4$ |

The null hypothesis that there was no difference between spill volumes generated from the historical series and those generated from the simulated (disaggregated) series was tested using a paired sample t-test on the data for the 236 overflows for the 7-year period (a total of 1652 pairs of differences). The mean difference in spill volumes was $21.1m^3$, with a corresponding p-value of 0.19 indicating that there was no statistical evidence for a difference in spill volumes due to the historical and simulated rainfall data.

A Kolmogorov-Smirnov (KS) test was used to compare the historical and simulated distributions of overflow volumes for each year (i.e. 7 KS tests, one for each year). Each KS test gave a non-significant p-value, with the worst fit (for 1997) having smallest p-value of 0.73, which is not statistically significant. In addition, when combining the overflow volumes for all the years together, the KS p-value was 0.26, which is also not significant. Consequently, there was no statistical evidence to reject the null hypothesis that the historical and simulated overflow distributions were the same, providing good support for the use of the stochastic model in the Auckland catchment study. A quantile plot, excluding the largest two values which are in Table 3, is given in Figure 4, from which it is clear that the simulated overflow distribution closely fits the overflow distribution due to historical rainfall data.

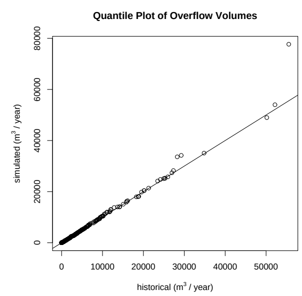


Figure 4: Quantile plot for historical and simulated overflow distributions; p-value of KS test statistic is 0.26.

## 7 Conclusions

The results in §6 indicate that the stochastic model and disaggregation algorithm can be used with confidence for the intended application, as there were no significant differences between the historical and simulated overflow distributions.

There is scope for improving the disaggregation algorithm through using the Poisson arrival rate. However, this is computationally demanding and was not necessary for this application.

The methodology readily extends to the problem of infilling missing values. To achieve this, the disaggregation algorithm could be modified so that cells are generated until any available data are approximately matched.

A further development of the methodology would include disaggregating daily records to hourly records. This issue is currently under investigation.

## Acknowledgements

Auckland City Council (ACC) and Metro Water Limited (Metrowater) funded the work as part of the Integrated Catchment Study (ICS). Team members from both organisations have contributed to the ICS, environmental programmes, and work contained in this paper. Several members are thanked for comments to improve the paper. The authors thank ACC and Metrowater for permission to present the paper. Discussion and interpretations are those of the authors of work in progress and should not be construed as those of ACC or Metrowater.

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