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# The disjunctivities of ω-languages

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#### Abstract

An  $\omega$ -language over a finite alphabet X is a set of infinite sequences of letters of X. Consider congruences  $I_L$ and  $P_{\omega, L}$  on X\* and a congruence  $O_L$  on X<sup> $\omega$ </sup> introduced by an  $\omega$ -language L.  $I_L$ ,  $P_{\omega, L}$ , and  $O_L$  are called the infinitary syntactic-congruence, the principal congruence and the  $\omega$ -syntactic congruence of L, respectively. If  $I_L$  ( $P_{\omega, L}$ ,  $O_L$ ) is the equality then L is called an I-disjunctive (P-disjunctive, O-disjunctive, respectively)  $\omega$ language. Properties concerning such  $\omega$ -languages are explored and relations between these  $\omega$ -languages are also studied.

### The disjunctivity concerning the infinitary syntactic-congruence IL

Given an  $\omega$ -language L, by the infinitary syntactic-congruence  $I_L$  of L we mean the relation  $I_L$  on X\* given by  $u \equiv v(I_L) \Leftrightarrow \forall x, y \in X^*, x(uy)^{\omega} \in L$  iff  $x(vy)^{\omega} \in L$ . If  $I_L$  is the equality then L is called I-disjunctive. Every I-discrete I-dense  $\omega$ -language is I-disjunctive. An  $\omega$ -language is I-dense iff it contains an I-disjunctive language. A periodically generated  $\omega$ -language L is I-dense iff L can be expressed as a disjoint union of infinitely many I-disjunctive  $\omega$ -languages.

## The disjunctivity concerning the principal congruence $P_{\omega,L}$

Given an  $\omega$ -language L, by the principal congruence  $P_{\omega,L}$  of L we mean the relation  $P_{\omega,L}$  on X\* given by  $u \equiv v$   $(P_{\omega,L}) \Leftrightarrow \forall x \in X^*$  and  $\alpha \in X^{\omega}$ ,  $xu\alpha \in L$  iff  $xv\alpha \in L$ . If  $P_{\omega,L}$  is the equality then L is called Pdisjunctive. Every P-discrete P-dense  $\omega$ -language is P-disjunctive. A P-discrete  $\omega$ -language is P-disjunctive iff the set of all its finite subwords is X\*.

## The disjunctivity concerning the $\omega$ -syntactic congruence OL

Given an  $\omega$ -language L, by the  $\omega$ -syntactic congruence  $O_L$  of L we mean the relation  $O_L$  on  $X^{\omega}$  given by  $\alpha \equiv \beta$ ( $O_L$ )  $\Leftrightarrow \forall x \in X^*, x\alpha \in L$  iff  $x\beta \in L$ . If  $O_L$  is the equality then L is called O-disjunctive. If S is a left singular language then SL is O-disjunctive for any O-disjunctive  $\omega$ -language L. If P is a finite prefix code then L is an O-disjunctive  $\omega$ -language iff PL is O-disjunctive.

#### Families of Disjunctive $\omega$ -languages

Every I-closed I-disjunctive  $\omega$ -language is P-disjunctive while not every P-disjunctive  $\omega$ -language is I-disjunctive. Every O-disjunctive  $\omega$ -language is P-disjunctive while not every P-disjunctive  $\omega$ -language is O-disjunctive. Every O-disjunctive  $\omega$ -language is I-disjunctive while not every I-disjunctive  $\omega$ -language is O-disjunctive. Every P-disjunctive  $\omega$ -language is P-dense

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