An Econometric Analysis of Household Consumption Patterns: A Comparative Study Based on New Zealand and Italian Household Budget Data

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Abstract

Econometric analysis of household expenditure is a very important area of economic inquiry because the estimated demand parameters are particularly useful in many behavioural aspects of demand forecasting and in welfare issues. This paper analyses and compares expenditure patterns in New Zealand and Italy by estimating preference-consistent complete demand systems directly derived from utility or cost functions of increasing complexity.

Because demographic factors have been recognised as essential components of empirical models of household consumption since the early studies by Engel, we use information on the number of children in the household to test for demographic effects on demand, and check whether similar economic conclusions and model acceptance decisions are supported by the two different sets of data we use, which, for both countries, are derived from household consumption surveys pooled across several time periods.

We approach the problem of how to introduce demographic variables into the demand models' analytical framework in a variety of ways, from the simple addition of a few demographic variables to the long established Linear Expenditure System (LES), on to the theoretically more advanced technique of introducing the demographic variables directly into the demand system via the utility function, as we do in the case of the Demographic Cost Scaling model introduced in Chapter 4.

The estimated models have been compared and tested to identify the ones that are more likely to describe and interpret the data correctly. The ones that are selected are then used to compute the price, income and demographic variable elasticities, both for the whole sample of households considered in the surveys as well as for households of specific size.

The computed elasticities have been analysed and checked for consistency with the tenets of the theory of consumer behaviour, and whenever found to be in contradiction to them, efforts are made to find out whether this was due to social or economic reasons, specific to the economies of the countries under study, or, more simply, to model or data inadequacies.

Because most of the demand models considered in this study are highly non-linear, and their parameters have to be estimated by iterative methods, we took great care to check the iterative performance of the estimation algorithms we used by making sure that, when estimating a model's parameters from the data at hand, the iterative procedure always converged to the same set of parameter estimates from all, or most, of the sets of parameter values we had selected to start the estimation procedure with. Most of all we checked carefully that the models' iterative estimation procedures did not show *sensitive dependence on initial conditions* - generating a different set of parameter estimates for every set of starting values, even similar ones (these types of systems, impossible to control or predict, are also called *chaotic*).

Although many of the demand models we have analysed showed chaotic behaviour during estimation, which reflected their inadequacy to explain the empirical data, the parameter estimates resulting from the estimation procedure itself often appeared to have good statistical properties. Therefore, it became apparent that the behaviour of a model estimation procedure should be considered very carefully when choosing, among different non-linear models, the most appropriate ones to describe and explain a set of data, because such models are likely to reveal any existing model inadequacies better than the customary statistical tests performed *after* the model has been estimated. In fact, we found that a well behaved iterative estimation process almost always provides parameter estimates which satisfy statistical criteria, and fulfil the model economic expectations.

Another empirical problem we had to resolve was to try and find some guidelines on how far, in household consumption studies, commodities should and could be aggregated into broader categories. This of course is a very important issue as demographically augmented models, because of their complexity, are often estimated with respect to only a few highly aggregated commodity groups, under the implicit assumption of "separability". We checked whether or not such a high level of aggregation allows meaningful empirical analysis of consumer behaviour, and found that, at least in the case of Italy, increasing the number of consumption categories from four to six did not increase the explanatory power of the models.

There are two more interesting theoretical results we have obtained in this study: one is the rejection by all models, and for both countries, of the hypothesis of income-linearity of the Engel curves; and the second is the empirical rejection, again by all models and for both countries, of the negative semi-definitiveness of the Slutsky matrix. This, latter, is a theoretical requirement which is seldom fulfilled in practice.

The introduction of demographic variables into the demand models made it necessary to convert households of different size and composition to equivalent units, before their consumption patterns could be properly compared. To this end we computed, for both New Zealand and Italy, constant-utility household consumption equivalence scales, to convert the expenditures of households of different compositions and sizes to standardised consumption units, based on the consumption of a "reference household". To estimate such equivalence scales, we used both an expanded version of the Linear Expenditure System, discussed and estimated in Chapter 2, and, with less success, the DT-RNLPS model, explained and estimated in Chapter 4. The resulting commodity-specific equivalence scales are, to our knowledge, the first of their kind estimated with New Zealand household expenditure data.

The main problem encountered persistently in our work has been the inadequacy of the data which, for both countries, only reported cell averages, instead of individual household observations and, in the case of Italy give no information on household composition, only the number of members in a household. This drawback in regard to the adequacy and appropriateness of the available data makes our results in some areas open to question. But, one of the useful contributions our study makes lies in its drawing attention to the nature of the statistical information base provided by household budgets, both in Italy and New Zealand. Improvements in data collection and presentation can only take place if researchers communicate their difficulties to the statistical agencies responsible. Studies such as the present one are therefore an invaluable part of the interface between data gathering, presentation, and use.

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Introduction

I.1 The Analysis of Consumer Demand

The microeconomic theory of consumption[1] explains how a consumer, with a certain level of income, spends it, during a specified time period, on goods and services which satisfy his needs. More precisely, the theory tries to find a rational explanation of why, under certain behavioral and economic assumptions, a consumer purchases certain specific quantities of some goods but not of others.

The consumer's income and tastes, as well as market prices, are assumed as given. Each consumer endeavours to spend his income on the goods and services he chooses in such a way as to maximise his total utility while the marginal rate of substitution of each good diminishes as the purchased quantity of it increases, other things remaining the same.

Under the above assumptions, the theory shows that consumers will divide their incomes among the goods they purchase in such a way as to make the marginal utility of each good proportional to its price or, to put it in another way, to equalise the price-weighted marginal utilities of all goods they purchase. Any change in income, tastes or prices will change the composition of the combinations, or the *basket*, of goods chosen by the consumer.

Although economic theorists have long analysed the optimal behavior of consumers, and developed a variety of specific types of utility functions providing examples of demand systems which maximise consumers' utility, most of the models used were of little help in empirical work. As a result, it often happened that econometricians engaged in empirical research would estimate ad hoc models

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[1] For a general introduction to the topics briefly mentioned here see the review articles by Brown-Deaton (1972) and Blundell (1988), the classic introduction to the quantitative analysis of consumption by Phlips (1974), and the landmark work on consumer behavior by Deaton-Muellbauer (1980). In the first three sections of this Introduction we will refrain from giving specific, point by point references, but will instead discuss the relevant literature in a separate final section.

with little or no connection with utility maximisation. In more recent years, demand models to test empirical relationships satisfying the theoretical requirements of utility maximisation have come to be developed. This has not only provided a rational justification for using specific consumption functions but has also suggested the restrictions that can be imposed on the parameters of those functions to help both their estimation and testing. In fact, the shape of a consumption or demand function depends upon the properties of the consumer' s utility function.

To obtain empirical measurements of the relationship linking the amount a consumer spends on each good out of his budget with given market prices, it is necessary to specify the mathematical form of the utility function, and then to find a solution to the utility maximisation problem for a given set of prices. The maximising solution will be subject to the *budget constraint* that total expenditure must be less than or equal to the available income.

The solution to this constrained maximisation problem provides us with the optimal quantities to be purchased, and it consists of a set of equations, one for each good, which show the expenditures for all the goods in the consumer's budget as functions of all prices and income. These functions are the *demand equations* describing the behavior of the consumer in the market.

The mathematical conditions that have to be satisfied to obtain a global maximum of the utility function will in turn define the *restrictions* to be imposed on the demand equations themselves and show what limits are placed on the behavior of an individual consumer by the postulates of the theory.

We can distinguish two types of restrictions on the demand equations: *general restrictions,* resulting from utility maximisation, which apply to all forms of the utility function, and *particular restrictions* resulting from the specific forms and properties of the utility function.

Although general restrictions are of limited use from the point of view of the applied economist, as they often fail to be satisfied by the data, a well developed utility and consumer preference theory has been very successful in generating empirically useful *particular restrictions* , on consumer behavior, at least at the level of the individual consumer.

Let us examine very briefly the *general restrictions* which the utility theory imposes on demand equations.

Every demand equation must be *homogeneous of degree zero* in income and prices: if all prices and income increase by the same proportion, demand must remain unchanged. In other words, there is no money illusion.

The budget constraint has to be satisfied over the observed range of income and prices: over the time period under observation, the sum of the consumer expenditures on the different goods predicted by the demand equations must be equal to the total expenditure, i.e. the expenditure shares of individual goods must *add* to one.

Finally, the price derivatives of the demand equations, obtained from the first order conditions for a global maximum solution, and representing the changes in the quantities consumed consequent to changes in prices, can be broken down into two parts: the *income effect* and the *substitution effect* .

The *income effect* measures the variation in the quantities purchased due to the fact that a change in the price of one good also implies a change in real income for the consumer: a price increase (decrease) in any one good means that less (more) money will be available to purchase all other goods.

The *substitution effect* describes the effects of changes in relative prices on the quantities consumed, other than those operating through the changes in real income, consequent to the change in prices: if the price of one good rises (falls) then its relative price with respect to the prices of other goods will also rise (fall), and less (more) of it, but more (less) of its substitutes[2], will be consumed.

To impose all the general restrictions simultaneously, we need to derive the demand equations directly from a specific utility function so that the form of the demand equations will be such that it will satisfy all general restrictions automatically. The alternative is to specify the demand equations first and then constrain them to satisfy the general restrictions during estimation. Both approaches have been used extensively in applied studies.

The *particular restrictions* most commonly imposed on demand equations in empirical studies are *independence*, *separability* and *homotheticity*. As mentioned above, particular restrictions result from the specific forms and properties of the utility function assumed to describe consumer behavior. To discuss them, we need to examine the utility functions from which they derive.

The (strong) assumption that the utility provided by the consumption of one good is *independent* of the consumption of any other good, derives from *additive utility functions* of the form:

$$U_n = F_1(x_1) + F_2(x_2) + \dots + F_n(x_n) \qquad \text{(I.1)}$$

where F_i indicates a function peculiar to good i, and x_i the purchased amount of good i (see Phlips 1974, p.57).[3]

An additive utility function postulates that the utility provided by the consumption of one good is not influenced by the consumption of any other good; in such a case the marginal utility of any good is independent of the consumption of any other good. This means that a change in the demand for any good induced by a change in the price of any other good will be proportional to the change induced by a

[2] The substitution and income effects can be partitioned via the *Slutsky equation*; from the Slutsky equation it follows what Samuelson has called the *Fundamental Theorem of Consumption Theory*, see Green, 1971, p. 65-69.
[3] A more rigorous definition of independence can be found in Blackorby et al. 1978, p.159-165.

change in income, with the factor of proportionality being dependent on the good whose price has changed.

The assumption of independence, while indefensible for individual commodities, is much more acceptable for broad categories of commodities like Food or Housing, which is precisely the sort of information we find in statistical data, both at the macro (National Accounts) and micro (Household Budgets) levels. This makes additivity of the utility function (not to be confused with the adding-up general property) a very common restriction in empirical work.

Although there does not seem to be universal agreement on the terminology of *separability*, we will follow Cornes (1992, p.151-54) and say that n commodities purchased in quantities x_i (for i=1, 2,...n) are *weakly separable* [4] if we can partition the vector $\mathbf{x} = [x_1, x_2,..., x_n]$ into a set of m subvectors, so that $\mathbf{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2,..., \boldsymbol{x}_m]$, and represent the consumer's preferences by the utility function:

$$U_n = F[f_1(\boldsymbol{x}_1), f_2(\boldsymbol{x}_2),....,f_m(\boldsymbol{x}_m)] \qquad \text{(I.2)}$$

where the f are often called sub-utility functions. If preferences can be represented by an utility function of the form:

$$U_n = f_1(\boldsymbol{x}_1) + f_2(\boldsymbol{x}_2) + ... + f_m(\boldsymbol{x}_m) \qquad \text{(I.3)}$$

then we have *strong* (or *additive*) *separability*.[5]

Separability is a weaker assumption than additivity as it assumes independence only among groups of commodities instead of between individual commodities. From an economic point of view, a necessary (see Leontief, 1947) and sufficient (see Green, 1964, p.13) condition for separability is that the marginal rate of substitution between any two commodities within a group must be independent of

[4] A detailed discussion of separability can be found in Deaton and Muellbauer, 1980, ch. 5.

[5] If there is only one commodity in each group then strong separability, as defined in (I.3), also implies independence, as defined in (I.1).

the consumption of any other commodity in any other group. The above property is also called *weak separability. Strong separability* implies that the marginal rate of substitution between any two commodities, belonging to two different groups, is independent of the consumption of any other commodity in any other group. It also implies additivity between groups of commodities and is sometimes called *groupwise independence* or *additive separability.*

As additive separability is a very strong assumption, which is rarely supported by empirical evidence, weak separability is often postulated instead, together with restrictive assumptions on the number and types of price indices that need to be used to represent the prices of the various commodity groups.

Weak separability is a fundamental assumption underlying the two-stage budgeting procedure by which households first allocate their incomes among broad commodity groups like food, apparel and housing and, then, spend the budget share allocated to each group on the goods within the group with no further reference to expenditures and prices in the other groups. We will test for separability in Chapter 4.

A utility function is defined as *homothetic* if it can be written in the form

$$U_n = \mathrm{F}\left[\mathrm{f}(x_1, x_2, \ldots x_n)\right] \qquad (\mathrm{I}.4)$$

where F is a positive, finite, continuous and (strictly) monotonically increasing single-variable function with $F(0) = 0$, and f is a homogeneous function of degree r in n variables (see Phlips, 1974, p.86). Although derived from a homogeneous function, U is, in general, not homogeneous in $\mathbf{x}$ (Chiang, 1984, p. 423).

Preferences are homothetic, if, for some normalization of the utility function, utility increases at the same rate as the quantities purchased by the consumer: utility is generated under constant returns to scale (see Deaton and Muellbauer, 1980, p. 143). The concept of homotheticity, however, is more general than that of homogeneity because, although every homogeneous function is also homothetic, the reverse is not necessarily true.

From the close relationship between consumer utility and demand functions follows the important assumption that, in empirical studies, where we try to estimate the parameters of a demand system, the consumer's preferences must not change over the observation period, for otherwise, the demand system will change too, and its estimation will become impossible. In fact, consumer preferences are likely to change substantially over time, and such changes need to be considered in the demand model which will have to become dynamic, or intertemporal[6] .

For empirical economic analyses, a microeconomic theory of consumption is not enough if we want to consider the collective consumption of all goods available by all consumers, given the total available income and the market prices of the goods. To achieve this, the theory must be extended so that it relates to the collective aggregate demand of all consumers for the aggregated commodities.

The aggregation over consumers and commodities has been one of the major areas in the study of demand theory because, firstly, most statistical data relate either to groups of consumers or to the totality of consumers and, secondly, it would be impractical to deal with the thousands of distinguishable commodities which would correspond to single homogeneous goods.

The problem of aggregation is complex, and a simple way to resolve it would be to follow Hicks (1956, p.55), and assume that a preference approach is only plausible when applied to statistical averages, and when the hypothetical consumer postulated by this approach simply reflects the average behavior of groups of people. This way we can formulate our aggregate demand relationships directly from the theory of individual consumer behavior, and the main use of the theory becomes one of suggesting restrictions facilitating the estimation and interpretation of the demand equations that result from the theory itself.

Aggregation over commodities does not present too many problems. Though the formal restrictions for grouping commodities are very stringent, there exist

[6] A good introduction to intertemporal consumer theory is Deaton (1992). For a more elementary exposition see Phlips (1974, Part 2).

approximation procedures which need much weaker assumptions, and are sufficiently accurate in most contexts (see Brown and Deaton, 1972, p. 1170). The error caused by aggregation, if the commodity groups are homogeneous enough, is usually small.

Although the full set of conditions required to ensure aggregation is rather complex (see Deaton and Muellbauer, 1980, ch. 6), the essential condition is that the utility function should be strongly, or additively, separable into "branches" each of which is homogeneous. This ensures that when prices and incomes change, the expenditures on each commodity within a group remain in the same proportions; 'the demand functions for these groups must then be subject to the restriction of additivity' (see Brown and Deaton, 1972, p. 1170).

The conditions for perfect aggregation - homogeneous commodities, and all consumers together behaving as a single consumer - are very stringent as they require separability, and also that all individual demand functions be linear and parallel in income. As these conditions are unlikely to be met in practice, because they require an unreasonable degree of uniformity between individual consumers and commodities, in all applied work, errors of aggregation should be expected and taken into account. However, all applied work is subject to errors, and errors of aggregation are not necessarily more significant than errors of measurement, omission and estimation. In fact, some authors consider aggregation errors to be of little relevance (see Houthakker-Taylor, 1970, p.200), and an unavoidable part of the difficult process involved in moving from theory to application.

A theoretical breakthrough was achieved by Muellbauer (1975 and 1976) who showed that non-linear aggregation was possible for demand systems included in the general family of Price Independent Generalised Linear (PIGL) models derived from an indirect utility function of the form:

$$U = F\{[y^{\alpha} - a(\boldsymbol{p})^{\alpha}] \;/\; [b(p)^{\alpha} - a(\boldsymbol{p})^{\alpha}]\} \tag{I.5}$$

where y represents income, and a($\mathbf{p}$) and $b(\boldsymbol{p})$ are linear homogeneous and concave functions of prices. PIGL systems can encompass many empirical demand systems (see Blundell 1988, p. 27-29), including the Almost Ideal Demand System (or AIDS) which we discuss and estimate in Chapter 3.

In general, when aggregating micro data, whatever the demand system in use, it is always advisable, in order to minimise aggregation error, to make the groups into which consumers are aggregated as homogeneous as possible by taking into account as many demographic characteristics (such as income, profession, and the size and age composition of the consumption unit) as possible. This will avoid losing too much detail and will preserve some individual characteristics, thus improving the chances of obtaining good estimates of the model parameters and consistent predictions from the model itself.

I.2 Demographic Effects on Household Consumption

I.2.1 The Analysis of Family Budgets

Applied consumption analysis is mostly based on two sorts of data: time series of aggregate consumption, incomes and prices, usually derived from the National Accounts; and cross-sectional data usually coming from surveys of household expenditure. Survey data are collected by asking a number of families or households to provide a list of all their expenditures over a certain period of time, the households being normally selected by random sampling. In some instances, the surveys are repeated regularly over many years so that, after a while, the available consumption information consists of time series of cross-sectional data. It is this latter sort of data that the present study is concerned with.

There are two main advantages that household budget data have over time series. The first is that, as the data refer to a specific time period, market prices can be assumed constant and therefore the relationship between consumption and income can be considered in isolation. The second is that they permit much wider variations in income between households than is possible for the type of macro data available from time series. This larger variation in observed consumers' incomes allows more precise measurements of consumers preferences than those which can be obtained from the macro data.

Since the pioneering studies by Ernst Engel, who was the first economist to formulate empirical laws describing the relationship between the consumption of certain commodities and the consumer' s income, the analysis of household budgets has enjoyed great prominence in applied econometric research. In fact, some of the most notable advances in econometric research methodology have been achieved in studies involving demand analysis based on household budget data.

Econometric studies of household expenditure enjoy considerable importance because of the usefulness of the estimated demand parameters in several policy issues. These range from the purely behavioral aspects of demand forecasting to welfare issues of poverty and inequality measurement, which depend crucially on the estimated equivalence scales and on demographic demand parameter estimates. Meaningful discussion of tax design and tax reform also requires reliable estimates of consumer behavioural parameters. Moreover, as the analytical literature on optimal taxation suggests, it is important to experiment with general demand functional forms which allow for realistic consumer behaviour, since assumptions like linearity and separability that are commonly employed in demand studies severely distort the estimates and often prejudge the policy issues that the welfare analyst is supposed to investigate.

From the tenets of the theory of consumer behaviour, we may assume that the demand of a single consumer for each commodity can be considered as a function of the consumer's income and all market prices, and that, for each specific time period, prices can be considered constant. This relationship is commonly called the consumer's Engel curve for any commodity i and can be written as

$$x_i = C\,(y \mid p_1, p_2, \ldots, p_n) \qquad \text{(I.6)}$$

where C is any suitable functional relationship[7] derived from the household's utility

[7] For an introductory discussion of the possible functional forms C can assume and their properties see Phlips (1974), p.105-115 and Brown-Deaton (1972), p. 1173-86.

function.

The estimation of the parameters of (I.6) from household budget data rests on the assumption that, on average, differences in consumption between families at different income levels depend only on their current incomes. Any other differences in the consumption patterns of individual households must be considered as random disturbances and be taken into account by adding a random component to (I.6). This assumption, like those for error-free aggregation alluded to above, suggests that, when aggregating raw survey data, the grouping of households should aim to minimise, within groups, the variation in factors such as the family size and composition, geographical location and occupation which might affect consumption preferences in any significant way.

Conversely, when estimating Engel curves, demographic factors which might affect consumption, such as the household size and composition, must be taken into account to avoid biased estimates of the effects of income.

I.2.2 Demographically Extended Models

Demographic variables have been recognised as essential components of empirical models of household consumption since the early studies by Engel, and the problem of how to introduce them in the analytical framework of demand models has been approached in a variety of ways.

The simple expedient of using per capita expenditure to capture family size overlooks the possibility of economies of scale (there is little or no difference in the cost of watching television or cooking a meal for two persons instead of one) and/or the different consumption needs of an adult and a child. A better solution, first adopted by Engel, is to deflate expenditure according to the household composition by means of *adult equivalent scales* where a male adult has a weight of one and all other components less than one, for example.

While in earlier studies (e.g. Stone, 1953) the scales were pre-fixed (normative) and, therefore, subject to the criticism that they might not have reflected market behaviour correctly, in subsequent studies (e.g. Brown, 1954 and Prais-Houthakker, 1955), the scale weights were considered as extra parameters of the demand model to be estimated from the data.

The equivalence-scale approach to introducing demographic effects into demand systems can be carried out within the framework of classical consumer theory in a manner consistent with utility theory. Alternatively, the demographic effects can be included directly into the demand system as explanatory variables or indirectly as modifications of the model parameters. Yet another approach consists in introducing functions of demographic variables, prices and expenditures directly into the cost function of a demand system. All three approaches have been extensively used in empirical studies. How to introduce demographic variables in empirical demand systems will be discussed at length in Chapter 4 of the present work.

I.3 Aims of the Present Work

One of the principal motivations of this study is to compare the results of the analysis of household expenditure patterns in New Zealand and Italy by estimating *preference-consistent complete demand systems* from household budget data pooled across different survey periods. We will test for linearity in the Engel curves, and provide evidence of non-linearity in income (or total expenditure). We will use information on the number of children in the household to test for demographic effects on demand, and to check whether similar economic conclusions and model-acceptance decisions are supported by different data.

We will analyze and estimate preference-consistent demand models directly derived from utility functions. As a consequence of this the properties and characteristics of the demand models themselves will derive from those of the utility functions from which they have been obtained.

The origins of the present work can be found in a paper (Chatterjee, Michelini and Ray, 1994) where the authors, for the first time, used the New Zealand Household

Expenditure and Income Survey (HEIS) data, to estimate utility based demand models of the type described in Chapter 4 of the present study. The research was further developed in Michelini and Chatterjee (1995) and in Michelini , Chatterjee and Ferrari (1996).

To our knowledge, the HEIS data have not yet been fully utilised by New Zealand researchers in empirical analyses, by means of econometric models, to quantify the consumption behaviour of New Zealand households. It is the aim of the present study to make a contribution by applying time series data from HEIS to a succession of demand models of increasing complexity, all of them rooted in specific utility functions.

The main thrust of our work is not theoretical but of an applied nature. All the models we use are well established in the literature as are most of the econometric estimation techniques we use. We will limit the theoretical discussions of the models, and the statistical properties of their parameter estimators, to the minimum required for clarity of presentation and for a basic understanding of the theoretical arguments involved. More in-depth discussions and proofs will be omitted in favour of references to the existing literature.

The models considered will be essentially static; no effort has been made to make them dynamic or to analyse multi-period consumption patterns which would have required some consideration of the household saving and labour supply decisions. This would have greatly complicated the econometrics involved in the formulation of the models; also, the required data would not have been readily available. This is not considered to be the main purpose of our work. Similarly, we have kept to rather simple assumptions the stochastic components of our models, and have avoided exploring the Data Generating Process approach suggested by Hendry (1993, p.77) which, if implemented, would have required endless testing of a very large number of possible error structures. Once again, this is not the main purpose of our work.

Our main effort has been the collection and manipulation of the data, the setting up of the computer data banks and estimation programs (especially those required to compute the elasticities for the non-linear models) and the careful scrutiny of the performance of the iterative estimation algorithms which, in most applied studies, do not seem to receive the attention they deserve (for an exception see Nelson, 1992, p. 1307). Most of all, in describing and interpreting data, we have tried to compare the performance and reliability of the different demand models we have employed, and also to check the effects on model performance, if any, of the "quality" of the available data.

Researchers familiar with the problems associated with the iterative solution of non-linear systems - and the dynamic properties of the iterative solution algorithms involved - are well aware that very often the current state of the system is crucially dependent on its initial conditions. Dynamic systems, which evolve differently for small differences in initial conditions (or show *sensitive dependence* on initial conditions) and therefore are practically impossible to control or predict for any length of time, are often called chaotic.

A system is defined as chaotic if at any stage of its development its current state crucially depends on its initial conditions. More specifically, we define as chaotic any system that, given any initial condition x and, given a region around it no matter how small, we can always find in this region an alternative initial condition y from which the system will develop differently from the way it would have developed from x (for a more rigorous definition of chaotic systems see Devaney, 1992, ch. 10).

We are convinced that, whenever we encounter chaotic behaviour in the estimation of the parameters of a non-linear model by an iterative solution algorithm, we should check very carefully that the model is really suitable to represent the phenomenon under study. If the final estimates crucially depend on initial conditions then any set of parameter estimates can be achieved by an appropriate choice of the initial parameter values required to start the iteration process itself. Parameter estimates obtained from such a chaotic system are likely to be meaningless mathematical accidents. Even if such parameter estimates seem to be economically acceptable, the model they quantify cannot be accepted as a reliable representation of the phenomenon under study.

Particularly revealing is the analysis done by Devaney (1992, ch. 13) of how the solution of even simple non-linear equations by the Newton-Raphson iterative method - often used in econometric packages to estimate the parameters of non-linear models - can be sensitively dependent on initial conditions and show chaotic behaviour of the type we will often encounter when estimating some of the models considered in the present study.

During estimation of the demand systems analysed in this study, we took great care to check the iterative performance of the estimation algorithms by making sure that the iterative procedure converged to the same set of parameter estimates for (almost) any set of parameter starting values and *convergence criterion*[8] we adopted. Whenever different starting values generated different sets of parameter estimates, we checked the corresponding values of the Likelihood Function (LF) and chose, as the Maximum Likelihood (ML), the estimates corresponding to the highest value of the LF. It is only after such careful checking, repeated over and over again, that we could be reasonably sure to have obtained ML estimates.

Many of the demand models we have analysed in Chapter 3, 4 and 5 show sensitive dependence on initial conditions, and they generated a different set of parameter estimates for every set of starting values. Although some of those final estimates had a very low LF, and were obviously not ML, some had high and very similar LF values and it was difficult to identify among them the true ML estimates. The quandary was made worse by the fact that most of those "quasi" ML estimates were quite different from one another and all looked statistically "good" (small standard errors, good fit, etc.). We ended up by choosing, in an almost arbitrary manner, the ones looking more appealing from an economic point of view.

From results such as these, it became apparent that one of the most important criteria in deciding how appropriate a specific model was in describing and explaining a set of data must be the behaviour of its estimation procedure.

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[8] If we stop the iteration when all the differences between two successive sets of estimates of θ are such that:

$$|\,\theta(t+1) - \theta(t)\,| \,/\, |\,\theta(t)\,| < c$$

where c>0 is a small and pre-specified constant, then we call c the *convergence criterion*. We experimented with four alternative values for c: 0.01, 0.001, 0.0001 and 0.00001.

Sometimes this seemed even more important than the customary statistical tests performed on the model itself or its parameters, or even the parameter restrictions suggested by economic theory. We are convinced that any econometric model showing chaotic behaviour during estimation should not be accepted as a valid explanation of a set of data and economic hypotheses, even if it can been validated by a whole battery of statistical tests *after* it has been estimated.

We will give special attention in the course of our work to the behaviour of the iterative estimation procedures used in computing the parameters of complex non-linear models, an issue often overlooked by many researchers in the validation of econometric models. We wish to stress its importance, and make it clear that we are convinced it should become standard practice in applied econometric research - a practice very rarely followed in most applied econometric research where the behaviour of the iterative estimation algorithm is rarely mentioned.[9]

We proceed in this study to discuss and estimate a variety of demand models, starting from the long established Linear Expenditure System (LES), with a few demographic variables added to its basic form, going on to the theoretically more sophisticated demand models where the demographic variables enter the models either directly, as modifications of their variables or, indirectly, via the utility or cost functions.

We then compare the results from the estimated models to find those among them which are more likely to be able to describe and interpret the data correctly, and then from them, we compute, for specific variable values, the price, income and demographic-variable elasticities.

The computed elasticities are then analyzed and checked for consistency with the tenets of the theory of consumer behaviour and, if found to contradict them, are further explored to see if this is due to social or economic reactions specific to the economies under study or, more simply, to model inadequacies.

[9] An exception to this practice is Nelson , 1988.

In consideration of the fact that we are analysing similar consumption data for two different countries, we check for consistency of consumer behaviour in Italy and New Zealand. If large discrepancies occur in the behaviour of consumers in the two countries, we investigate whether the supposedly different behaviours are really due to economic and social factors or, once again, to data and/or model inadequacies.

We also consider as important our effort to analyse together two sets of data, obtained from very similar sampling techniques, but quite different in other aspects like the length of time covered, the number of consumption categories, the number of income classes and household sizes, and the criteria for grouping the households themselves, for example. In the end, it became apparent that household consumption data based on cell averages - where the cells are very large and group together households of very different demographic characteristics - were unsuitable for detailed analyses of household consumption behaviour, with the help of complex non-linear models with a high level of parameterisation, even if their sampling coverage is extensive. In fact, such data may produce paradoxical results, such as showing decreasing aggregate consumption expenditures when household sizes increased.

We aim to verify whether or not the assumption of Separability is sustainable in the case of Italy. To achieve this, we use an alternative and more detailed set of data where household consumption is classified according to a larger number of less aggregated commodity groups, making it possible to break up some of the commodity groups considered in Chapter 4 into their component parts and thereby to obtain a larger and less aggregated demand system.

Once both systems have been estimated, by testing the hypothesis that the disaggregated model is not significantly different from the aggregated model, we implicitly test for Separability.

Finally, in the last Chapter of this study we estimate, from an expanded version of the Linear Expenditure System, and from the demographically expanded models discussed in Chapter 4, constant-utility household consumption equivalence scales for both New Zealand and Italy. Consumption equivalence scales are needed to convert the expenditure of households of different composition and size to standardised consumption units based on the consumption of a "reference household", which in our case is the two-adult household for New Zealand, and the two-component household for Italy.

The resulting commodity-specific equivalence scales estimated here for New Zealand are, to our knowledge, the first of their kind, and they make a very useful contribution to this area of applied socio-economic research.

I.4 A Brief Review of the Relevant Literature

The literature on consumption and demand analysis is so large that a comprehensive review would need a separate publication by itself. In fact, even if we try to compile a review restricted to the specific arguments treated in the present study we will almost certainly omit some important work. As a consequence, we only mention those works which are directly relevant to our study or suggest alternative or contrary developments. Our listing of references tries to follow the order in which topics are treated in the preceding three sections.

Among the many introductory texts on consumer behaviour and utility theory we found most helpful are those by Green (1971 and 1976), Theil (1975), Phlips (1974), Henderson and Quandt (1980, Ch. 2), Deaton and Muellbauer (1980), and the classic work by Hicks (1956, reprinted 1986) as well as the more recent work by Lancaster (1991).

Also very helpful has been a survey article by Brown-Deaton (1972) and a more recent one by Blundell (1988). Another useful discussion of the more recent developments of consumer theory - both at a microeconomic and macroeconomic level - can be found in Deaton (1992), where the arguments are developed at an intuitive level with a minimum of formal econometrics. Deaton's approach however is dynamic unlike ours, which is essentially static, being based on an intertemporal theory of consumption and life-cycle models of income, savings and consumption, as observed earlier.

A number of interesting essays on demand analysis, index numbers theory and households labour supply written by some of the pioneers of consumption theory (like Gorman, Johansen, Theil, Nerlove and Atkinson) can be found in a collection edited by Deaton (1981), which also contains a full bibliography (40 pages) of the works of Sir Richard Stone, one of the founders of modern applied demand analysis.

A more historical perspective of the theory of consumer behaviour and of the "consumer society", with criticisms of accepted ideas and current interpretations of the underlying theories, can be found in Fine and Leopold (1993). Also, Miller (1995) edits a collection of essays which offer alternative insights into consumption from socio-political and psychological perspectives.

Further readings on utility theory and its applications to consumer behaviour include Houthakker (1950), Ellsberg (1954), Gorman (1953, 1959 and 1976), Deaton (1974), Modigliani and Brumberg (1979). For an attempt at an empirical integration of ordinal and cardinal utilities based on data other than those derived from observations of demand behaviour see Van Praag (1994) and the references therein.

Turning to the problems associated with additive preferences (for a statement of the problem and some definitions see Houthakker, 1960a) and separability, Blundell and Ray (1984) have analysed the effects of additive preferences and separability in demand systems, while Blundell and Walker (1984) consider the possibility of non-separability of preferences between household members and commodities. Deaton et al. (1989) consider the possibility that 'adult goods' are separable from 'children goods' (*demographic separability*). Attanasio and Weber (1989) have studied intertemporal separability between consumption and leisure, and Epstein and Zin (1989, 1991) intertemporal separability between consumption and risky assets. The conjunction of direct and indirect separability is discussed by Blackorby and Russell (1991). For a general discussion of separability and further references, see Blundell's review article (1988, p.18-22) and the books by Blackorby et al. (1978) and Cornes (1992).

The problem of separability has also been considered in models of international trade (see Armington, 1969; Hickman and Lau, 1973; Deppler and Ripley, 1981; Ranuzzi, 1981) where trade allocation models assume a two-stage budgeting procedure in which each importer's (exporter's) total imports (exports) of some commodities are explained by one set of variables (e.g. the price index of all import prices), and then these totals are allocated among sources according to some other explanatory variables (e.g. the specific prices of the various commodities). Separability can also be assumed between imported and domestic products (see Brenton, 1989). For a contrary view on the assumption of separability in international trade, see Winters (1984, 1985 and 1992).

There is a vast literature on the theoretical and practical problem of aggregating individual consumer behaviour over groups of consumers and whole communities. A good understanding of the problem and its solution can be found not only in the general introductory books on consumer behaviour mentioned above but also in Working (1943), Gorman (1953, 1959), Green, (1964), Muellbauer (1975, 1976 and references therein), Anderson (1979), Blundell (1988, p.27-30) and Jorgenson (1990), a more advanced treatment can be found in Blackorby et al. (1978). Special coverage of the effects of household composition and the resulting conditions of aggregation is given in Jorgenson et al. (1980), Gorman (1981), Lau (1982) and Stoker (1984). A critique of the practice of using aggregated data for the analysis of consumer behaviour can be found in Hall and Mishkin (1982), while Blundell and Meghir (1987) discuss the estimation of demand equations from non-aggregated data. A comprehensive collection of the works of Gorman on both separability and aggregation can be found in Blackorby and Shorrocks (1996).

Quantitative analysis of budget and cross-section data have a long history in empirical economics going back to Engel's (1895) studies on the consumption patterns of Belgian working-class families. Successive path breaking works have included some of the most sophisticated analytical techniques of their time and pointed the way for further theoretical developments. For a small selection, mention can be made of Allen and Bowley (1935), Working (1943), Houthakker (1952), the now classic analysis of pre-war British family budgets by Prais and Houthakker (1955), the Houthakker (1960) paper on the estimation of Engel curves satisfying the general restrictions of demand theory, the book by Balestra (1967) containing one of the early examples of the pooling of time series with cross-sectional data, and finally, the extension to budget data of the linear and quadratic expenditure systems of Pollak and Wales (1978).

Household budget data can be used to investigate the measurement of welfare (Muellbauer 1974, Pollak and Wales 1979, Pollak 1991, Blundell et al 1994), the definition of equivalence scales (Nelson 1988 and 1992, Blundell-Lewbel 1991, Ray 1993, Valenzuela 1995) and their international comparisons (Phipps and Garner 1994, Coulter et al. 1992), the effects of children on consumption (Ray 1983 and 1985, Muellbauer 1977, Dickens et al. 1993), as well as in studies on optimal taxation, family welfare, income inequality and redistribution, the incidence of poverty (Apps 1994, Blundell et al. 1994, Kakwani 1986, Muellbauer 1974a, Ray 1988, and for New Zealand, Brashares 1993, and Chatterjee and Ray 1996).

An alternative explanation of household behaviour has been introduced by some authors (see Bourguignon-Chappori, 1994) who criticise the standard microeconomic practice of assuming a single utility function for the whole household, which then becomes the basic decision unit. As an alternative and more realistic assumption they suggest that a household should be considered as a group of individuals, with different sets of preferences, among whom a collective decision process takes place. Results by Nelson (1988, p. 1311-12) support this view as they seem to show that the assumption of identical tastes within households should be rejected.

Special attention in household budget studies is often given to methods of modelling the number of children in the household, and their age, as variables. This is because of the effects that the presence and the ages of children might have not only on household consumption but also on household income, as the presence of children strongly affects women labour supply. A recent survey by Browning (1992) covers this area quite exhaustively, and is not repeated here. An interesting recent approach based on a general class of cost-of-children indices is suggested by Blackorby and Donaldson (1994) and by Bradbury (1994) for Australia.

How to introduce demographic variables which involve, essentially, the size, age and sex composition of the family, has been a major issue in this area of demand analysis for a long time. The simplest solution is the application of *adult equivalence scales.* First introduced by Engel, equivalence scales got into mainstream demand analysis after the seminal work on the analysis of family budgets by Prais and Houthakker (1955).

The literature on equivalence scales is very extensive and we can only try and mention some of the most recent contributions we actually consulted for the present study, like those by Banks et al 1994, Blackorby and Donaldson (1989), Blundell and Lewbel 1991, Blundell et al. 1994, Dickens et al. 1993, Kakwani 1977, Lewbel 1989, Nelson 1992, Ray 1986.

Recent empirical contributions specific to New Zealand are those of Easton 1980 and 1995, Chatterjee and Ray 1994, Jensen 1988, Rutherford et al. 1990, Smith 1989. Recent studies focusing on Australia are those of Binh and Whiteford 1990, Bradbury 1994, Griffiths and Valenzuela 1996, Kakwani 1980, Valenzuela 1995. Some international comparisons can be found in Coulter et al 1992, Phipps and Garner 1994, Deaton et al. 1989,

The equivalence scale models consistent with utility theory were proposed by Barten (1964), and further developed by Gorman (1976) and Muellbauer (1974, 1977). These were applied by Pollak and Wales (1980, 1981), Blundell (1980), and more recently by Alessie and Kapetyn (1991).

Introduction of the demographic variables necessitates the conversion of households of different size and composition to "equivalent units" before their spending behaviour can be analysed. The extensive literature on equivalence scales has therefore been studied to select an appropriate estimation procedure for our two countries. The methodology is explained and the results reported in Chapter 6.

The alternative approach to equivalence scales is to model demographic effects either directly into the demand system as explanatory variables (see for example Blundell and Walker, 1984) or indirectly as modifications of the model parameters (see Blundell and Walker, 1986 and Ray, 1986).

In another approach Lewbel (1985) suggests the introduction of functions of demographic variables, prices and expenditures directly into the cost function of the demand system. Lewbel's approach seems to be midway between the ad hoc modification of specific models and the general, non-model specific, equivalence scales. This method of introducing demographic variables directly into the demand system is quite general and can be shown to include most other demographic scaling techniques as special cases.

The introduction of demographic variables in empirical demand systems can proceed in a variety of ways. Some of the best known and most widely used are Demographic Scaling and Demographic Translating (Pollak and Wales, 1981), as well as various Cost Scaling techniques based on generalisations of the Gorman (1976) approach of adding a fixed cost term to the Barten (1964) model, see for example Ray (1986), Ray (1993b), Chatterjee et al. (1994) and Bollino et al. (1995). The introduction of demographic effects into preference-consistent complete demand systems will be the focus of Chapter 4, in the present work.

The issue of how to appraise the estimation of non-linear models by iterative methods is often played down in econometric literature. We are aware of very few papers in which the actual performance of the estimation algorithm is openly discussed. Instances such as Nelson (1988), where the performance of the estimation algorithm is considered in the evaluation of the results, are the exception rather than the rule. Because of the characteristics of the iterative algorithms used to estimate non-linear models it often happens that the researcher might end up with many sets of estimates all of which appear to be equally acceptable either statistically or economically or both: quite a few examples of such apparently inconsistent results can be found in several chapters of the present work.

Contrary to the situation for the empirical applications of iterative methods to the estimation of non-linear models, the theoretical literature on iterative solution methods is quite vast, both in the field of econometrics and numerical optimisation. A good introduction to the econometrics of non-linear estimation are Eisenpress and Greenstadt (1966), Goldfeld and Quandt (1972), Quandt (1983) and Gallant (1987). From a numerical optimisation viewpoint, interesting discussions of iterative solution methods can be found in Crockett and Chernoff (1955), Hartley (1961), Ortega and Rheinboldt (1970), Bard (1974) and the references therein. A review of the application of numerical optimisation techniques in econometric estimation, with a comprehensive list of references, can be found in Judge et al. (1985, Appendix B, p.951-79). Davidson and MacKinnon (1993, Chapter 6) present an analysis of the properties of Gauss-Newton non-linear regression; Cossarini and Michelini (1971) discuss the characteristics of some of the maximisation algorithms most commonly used in non-linear regressions.

Finally, we found the concepts and analytic tools adopted in the area of *Chaotic Dynamical Systems* very useful towards understanding the inner workings of the iterative optimisation techniques needed in non-linear estimation. A first non-technical introduction to *Chaos Theory* is Kellert (1993), more rigorous but still approachable expositions are those of Gleick (1987) and Devaney (1992). Creedy and Martin (1994) present a selection of applications of chaos theory to non-linear economic models with special attention to the analysis of financial markets. Another good conceptual introduction to the application of chaos theory in the analysis of financial markets can be found in Peters (1991), with some evidence that, contrary to accepted wisdom, the standard random walk model does not describe markets well, and the hypothesis of efficient markets is unsupported by empirical evidence.

Chapter 1

The Data, Description and Comparisons

1.1 A Comparative Description of the Surveys

Until relatively recently, the information content of New Zealand household budget data was quite limited, particularly in relation to the composition of the households and the relationship between their levels of expenditure and income. This lack of data has greatly hampered the analysis of New Zealand household consumption patterns. As Giles and Hampton (1985, p.461) have noted in the context of their Engel curve analysis on New Zealand expenditure data, "our work to date has been limited by the absence of detailed data on expenditure behaviour by households of different types for each income group".

In recent years, however, Statistics New Zealand has released household expenditure data where households are cross classified by expenditure and family composition (see New Zealand Department of Statistics, 1992). Together with these new data on household consumption, Statistics New Zealand has started making available commodity specific price indices. The availability of this type of data now permits the estimation of demographically extended demand functions, if only in a limited way.

Similar household budget data have been available for Italy for many years (see ISTAT, 1960, 1968, 1986). The surveys there are also taken more often, and the budgets cover a wider variety of commodities at a lower level of disaggregation and for more classes of households sizes than is the case for New Zealand. As better information is also available on the prices of the commodities considered in the Italian households budgets, we end up with two sets of data with similar coverage, but one of which, Italy, appears to be of better quality.

To facilitate understanding of the similarities and differences of the Italian and New Zealand data, we describe, in the next section, the main technical characteristics of the two surveys in a comparative framework. To facilitate comparison, description and comments on the similarities and differences between the surveys conducted in the two countries, we describe them together, in "boxes" placed side by side. Whenever the survey characteristics are almost identical we show only one "box" in the middle of the page.

1.1.1 Main Sources

NEW ZEALAND

Household Expenditure and Income Survey, Department of Statistics, Wellington, 1992. The data span a period of nine years from 1983/84 to 1991/92.

ITALY

Rilevazione sui consumi delle famiglie italiane. Istruzioni per la raccolta dei dati. ISTAT, Roma, 1986. The data span a period of thirteen years from 1981 to 1993.

1.1.2 Commencement of the Surveys

The survey commenced on 1 July 1973. From 1983/84 it was renamed "Household Expenditure and Income Survey" (HEIS).

The "Family Budget Survey" (Rilevazione sui consumi delle famiglie italiane) commenced in 1968 and was substantially modified in 1973. From 1980, the survey was enlarged to households income and savings. There were earlier non-systematic surveys (See the ISTAT References).

1.1.3 Survey Target Population

Resident private households living in permanent dwellings. Excluded from the survey are all non-residents + cohabitations + residents temporarily overseas or staying in non-private dwellings + residents staying at other private dwellings. Households eligible to participate in the 1990/91 survey were approximately 1,080,600.

Resident private households living in permanent dwellings. Excluded from the survey are members of cohabitations like barracks, hospitals, etc.+ foreign residents. Households eligible to participate in the 1990/91 survey were approximately 20m .

1.1.4 Survey Population

Survey target population less (i) residents in islands other than the North and the South Islands (ii) residents in very remote locations in the North and the South Islands and (iii) households selected in the survey sample in the previous survey year.

Same as the survey target population.

1.1.5 Definition of "Household"

(Same for the two countries)

(a) A single individual living in a dwelling who makes his or her own housekeeping arrangements;

(b) A group of persons living in or sharing a dwelling for most of the reference period who participate in the consumption of goods and services purchased for joint use by members, or who, if not dependent upon a household member, contribute some portion of income towards the provision of essentials of living for the household as a whole.

1.1.6 The sample Selection Process

1.1.6.1 The sample design

(Same for the two countries)

The sample design is a two-stage sample-selection method. The first stage consists of a stratified collection of geographical areas to be surveyed in each month of the survey year(s) called Primary Sampling Units (PSUs). The PSUs are non-overlapping. The second stage sampling frame consists of lists of dwellings produced from field enumeration of the sampled PSUs.

1.1.6.2 The sample selection procedure

Steps in the stratification and sample selection:

1. The North and the South Island are stratified into 29 major (geographical) superstrata.

2. Each superstratum is subdivided by multivariate techniques into a total of 94 strata.

3. A sample of at least two PSUs is selected from each urban stratum per calendar quarter, selection being on a rotation basis. One PSU is selected from each rural stratum per quarter.

4. Within each sampled PSU, a systematic random sample of households is selected.

5. Each urban PSU selected is revisited in the next quarter, with a different sample of households being selected from the PSU. Rural PSUs are not revisited. The number of households selected in a particular PSU is chosen to attain the required accuracy with the smallest possible sample size.

Two stage, stratified sample selection:

The first stage consists of local councils subdivided into two groups: (1) city councils with a population of 50,000 inhabitants or more, and (2) other councils. The first group participates in the survey every month; the second group is stratified by region according to the size of population, geographic factors and prevailing economic activity. Three councils are drawn out from each of the above strata. Councils in the second group are divided into three sub-groups: councils belonging to the first sub-group are surveyed during the first month of each quarter, councils belonging to the second sub-group are surveyed during the second month of each quarter, and councils belonging to the third sub-group are surveyed during the third month of each quarter. Approximately 550 councils are included in the survey - 150 in the first group and 400 in the second, for a total of 135 strata. Each month the survey is carried out in 150 councils belonging to the first group and 135 to the second. The second stage of the sample consists of households, approximately 3,250 of which are surveyed each month. Households are randomly selected from municipal registers and participate in the survey only once. The sample is a proportional one with a household sampling rate of about 0.2 percent.

1.1.7 The Technical Features of the Sample Design

1.1.7.1 Objectives to be met

(Same for the two countries)

(a) To provide expenditure statistics for use in the revision of the CPI;
(b) To provide expenditure statistics to be used in the National Accounts;
(c) To provide selected socio-economic statistics on households and their members.

1.1.7.2 Estimate of sampling errors

Sampling errors (expressed as percentages of the estimates) vary from the lowest relative size of 5% for Food, Household Operation and Other Goods to the highest relative size of 19% for Housing, with a 7% for Total Net Expenditure.

Sampling errors (expressed as percentages of the estimates) vary at the group level from the lowest relative size of 1.4 - 1.6% for Housing and Food to the highest relative size of 6.2% for Health.

1.1.8 Data-Collection Procedures

(Only small differences between the two countries)

1.1.8.1 Type of contact

Personal visits by interviewers to all selected households living in the private dwellings defined above.

1.1.8.2 Eligibility

Eligible primary households which (i) refuse to participate, (ii) are not able to keep diaries, (iii) are unable to supply the information, (iv) have their participation discontinued by the interviewer, and (v) were invited to participate in the HEIS within the preceding twelve months are substituted for.

1.1.8.3 Length of contact with participating households

Contact extends over a period of just over two weeks. During this time, each household member aged 15 years or over keeps an expenditure diary, recalls major purchases (generally for goods and services costing $200 or more) made in the previous 12 months and provides income and employment data.

Contact with each participating household extends over a period of just over ten days. During this time, each household keeps an expenditure diary for ten consecutive days. At the end of the month, the interviewer surveys the household on purchases made during the entire month or in the preceding quarter and collect information on individual members of the household as well as on type of dwelling, income and savings.

1.1.9 Survey Response in 1990 / 1991

A total of 2,934 randomly selected households participated in the survey, 2,244 from primary addresses and 690 from substitute addresses. In all, interviewers visited 4,727 addresses of which 598 did not correspond to an eligible household and 1,195 did not participate in the survey. The total non-participants were 1,793, giving an overall response rate of 71,1%.

Approximately 39,000 randomly selected households participated in the survey.

1.1.10 Data Collection Documents

Four types of documents:

1. Household Questionnaire: to gather information on the composition of the household and on the demographic characteristics and education experience of each household member.
2. Expenditure Questionnaire: for details of expenditure and sales in areas such as housing, home maintenance, household operation, transport, holidays, health and education. Regular commitments (such as rates, rent and telephone rental) are collected using the 'latest payment' approach.
3. Income Questionnaire: issued to each household member aged 15 years or over, for details of current and past employment in the previous 12 months and of income by resource. Regular income data are collected by means of the 'current approach'.
4. Expenditure Diary: issued to each household member aged 15 years or over, for recording details of items bought and of money otherwise spent on each of the following 14 days.

Three types of documents:

1. Expenditure Diary: for recording the purchases made by household during a ten days period.
2. Summary of All Expenditures: compiled during the concluding interview at the end of the month.
3. Notebook of Goods and Services produced and consumed within the Households: only for those households (mostly in the farming sector) producing for self-consumption.

1.1.11 Expenditure Classification

Goods and services are identified by four-digit Item Reference Numbers based on the commodity groups, sub-groups and sub-sub-groups of the Consumers Price Index.

There are eights groups:

0 Food (592 Item Reference Numbers)
1 Housing (175 Item Reference Numbers)
2 Household Operation (345 Item Reference Numbers)
3 Apparel (216 Item Reference Numbers)
4 Transportation (152 Item Reference Numbers)
5 Other Goods (394 Item Reference Numbers)
6 Other Services (243 Item Reference Numbers)
7 Refunds, Sales and Trade-ins (115 Item Reference Numbers)

Example:

Group	Sub-group	Sub-Sub-group
0	00	000-003
Food	Fruit	Fresh fruit

Goods and services are identified by four-digit Reference Numbers according to NACE classification of products.

There are nine groups:

0 Foods and beverages
1 Tobacco
2 Apparel
3 Housing, Fuel and Electricity
4 Furniture and Home articles
5 Health Services and Expenditure
6 Transportation and Communications
7 Leisure, Education and Culture
8 Other Goods and Services

1.1.12 Weighting of Survey Data

The weight attributed to a responding household is the inverse of its probability of selection. Thus, a surveyed household which had a 1 in 300 probability of selection has a weight of 300, which effectively means that the household represents itself and 299 others in the survey population. The sum of each surveyed household's weight is the estimated number of households in the survey population.

The weight attributed to a responding household is in principle given by the product of the probability a council has to be included into the first stage by the probability a household has to be selected for the second stage, according to the sample design.

The procedure consists of two steps and may be summarised as follows: firstly for stratum i of region j, a coefficient c_{ij} is calculated

$$c_{ij} = P_{ij}/p_{ij}$$

where P_{ij} is the total population in that stratum and p_{ij} is the total number of peoples belonging to the households surveyed in that stratum.

Then a second coefficient C_{ijs} is calculated:

$$C_{ijs} = c_{ij}(F_{ijs}/f_{ijs}) \qquad (s = 1,2,\ldots,7)$$

where F_{ijs} represents the number of households of size s in region j and f_{ijs} is an estimate of the number of households of size s in region j obtained from of the sample composition.

The final weighting coefficient is:

$$w_{ijs} = c_{ij}C_{ijs}$$

1.1.13 Data Processing

Completed data-collection documents are checked and classified (coded) on a monthly basis. The fortnightly Diary recorded expenditure amounts are scaled up by 26.07 to give equivalent annual expenditure and are added to expenditure data from the Expenditure Questionnaire, which are already in annual specification. For estimates of average weekly expenditure per household in a particular category each relevant household's expenditure is multiplied by the household's weight, producing its weighted expenditure. The weighted expenditure from all surveyed households in the category are then aggregated, and weighted again, to produce the average weekly expenditure per household in the category concerned. In the 1990/91 survey, 53.0% of all annualised net expenditure came from the Expenditure Questionnaire, the remaining 47.0% from the Expenditure Diary.

Interviewers experience shows that roughly 30% of the expenditure diaries is "sufficiently" filled in, 60% is completed and 10% is empty.

A first hand-check is carried out at the local level, and then the results are sent to ISTAT, which provides for a further revision, using informatics procedures and techniques on a monthly basis.

1.2 The Data Used in the Present Study

Of the commodity groups listed in 1.1.11 above, we have considered four: Food, Household Operations, Apparel and Transport. For New Zealand, we have not considered Group 1 (Housing) because, as it includes principal repayments, it may, and sometimes does, assume negative values. For Italy, we have aggregated Group 3 (Housing, Food and Electricity) and Group 4 (Furniture and Home Articles) to match almost exactly the New Zealand Housing Operations group.

The only major discrepancy in the data is between the Italian Transportation group, which includes Communications, and the New Zealand one, which does not. Since eliminating the expenditure on communications from the Italian Transportation group would have implied not only a re-computation of the original data but also a revision of the group price index, we accepted the discrepancy as the lesser of two evils.

Another difference in the two sets of data is that while New Zealand households are grouped into five income classes (the five income quintiles), Italian households were grouped into fifteen income classes, the last of which was an open ended one. To make the data more comparable (and also to reduce the number of observations to speed up computations), we re-grouped the Italian households into five income classes, like New Zealand.

The New Zealand data set consists of twenty cross-sectional cells - five income classes and four households sizes[1] - and nine time series units, representing annual observations for the fiscal years (June to June) 1983/84 to 1991/92, giving a total of 180 observations.

The Italian data set consists of twenty five cross-sectional cells - five income classes and five households sizes[2] - and thirteen time series units, representing yearly observations from 1981 to 1993, giving a total of 325 observations.

An unfortunate characteristics both sets of data have is that they only report cell averages, often computed over dozens of households, instead of the kind of individual observations often utilised in similar studies (e.g. Ray, 1985 or Binh and Whiteford, 1990). This makes it very difficult to enforce, and or verify, the empirical application of the theoretical assumptions underlying the kind of complex non-linear demand model we propose to use in this study. The Italian data have the further drawback that they only report household sizes, with no mention of their composition. We are convinced that these very serious data deficiencies are the reasons for some of the estimation problems we will to encounter in the course of this study and are the main cause of most of the economically "wrong" results we will obtain.

For both countries the commodity specific price indices used in this study are those employed to compute the Consumers Price Index.

Finally, it must be pointed out at this stage that the comparisons among time series of cross-sectional data should be done with care, both because of sampling errors, and as a consequence of the design of the surveys themselves which have their own dynamics, due to the modifications that were introduced year by year. However, this

[1] Couples, couples with one and two children, single persons.
[2] Households with one, two, three, four and five components.

is a problem common to any analysis such as ours, and must be accepted as unavoidable.

Chapter 2

The Linear Expenditure System

2.1 The Theoretical Framework

The theory of consumer behaviour postulates that consumers behave "rationally" and choose among the consumption alternatives available to them in such a way as to maximise their satisfaction. Consumers are aware of the alternatives facing them and are able to evaluate their worth or *utility*. Thus, consumers have stable preference systems and the satisfaction they derive from the consumption of the various quantities of commodities they purchase, with the available income, can be described by means of a *utility function.* Although earlier economists, like Jevons and Walras, considered utility to be measurable or *cardinal,*[1] this is not strictly necessary and a rational consumer needs only to be able to rank commodities in order of preference by an *ordinal*[2] utility function. It follows that each individual consumer, within the budget constraint, selects the particular combination, or basket, of commodities for which the utility function takes the largest value.

If we consider n commodities and indicate by q_i (for i=1,..., n) the quantities of them bought at time t[3] and assume that the consumer's satisfaction is measured by the utility function:

[1] A cardinal utility function is such that every combination of commodities consumed has a number associated with it representing its utility. For a discussion of how to measure the cardinal dimension of the utility concept from sources other than demand behaviour, and how to relate it to ordinal utility see Van Praag (1994). For an earlier discussion of "measurable utility" see Ellsberg, 1954.

[2] An ordinal utility function does not assign numbers to measure utility, but simply ranks combinations of commodities in order of preference. The ranking is expressed mathematically by associating certain numbers with the various quantities of commodities consumed, but these numbers do not measure amounts of satisfaction. They only provide a ranking or ordering of preferences.

[3] The utility function is defined with reference to consumption during a specific time period t. We do not take into account what happens after t, consumers make their decisions for only one such period at a time. The possibility to transfer consumption from one period to another is not taken into account. For

$$U(\boldsymbol{q}) = U(q_1, ..., q_n) \qquad (2.1.1)$$

where $\boldsymbol{q}$ is a $(n \times 1)$ column vector of quantities consumed, then the consumer will chose a combination $\boldsymbol{q}^*$ which will make $U(\boldsymbol{q})$ as large as possible subject to the constraint:

$$\boldsymbol{p}' \boldsymbol{q}^* = y \qquad (2.1.2)$$

where $\boldsymbol{p} = (p_1, ..., p_n)$ is a column vector of prices and y represents the consumer's total expenditure or income. Constraint (2.1.2) simply states that the sum of the expenditures on the n commodities must be equal to the *given* amount of total expenditure.

The maximisation of the utility function (2.1.1) subject to constraint (2.1.2) under appropriate conditions will produce a system of n equations

$$\boldsymbol{q}^* = \boldsymbol{q}(\boldsymbol{p}, y) \qquad (2.1.3)$$

where $\boldsymbol{q}(.)$ represents a set of functions, the so-called demand equations, describing the quantities of the various commodities consumed under specific price/income conditions which maximise the consumer's satisfaction.

The conditions for maximisation are that (2.1.1) has continuous derivatives up to the third order with positive first derivatives, all variables in (2.1.2) are continuous, and finally that the $\boldsymbol{q}^*$ solution is strictly positive and unique. The positive sign of the first derivatives insures that an increase in the quantities consumed will generate an increase in utility.

The condition for positive first derivatives of the utility function and the existence of the third derivatives implies that for $i, j = 1, ..., n$ the $(n \times n)$ matrix of second derivatives

a discussion of multi-period consumption see Henderson and Quandt (1980, p.326-333) and Deaton (1992).

$$H = \{\delta^2 u / \delta q_i \delta q_j\} \quad (i, j = 1,2,\ldots,n) \tag{2.1.4}$$

called the Hessian, is symmetric and negative definite, this ensures that ***q**** corresponds to a constrained maximum rather than a minimum or a saddle point (See Theil, 1975, p.3).

To maximise (2.1.1) subject to the budget constraint (2.1.2) we form the Lagrangian function

$$U(q) - \lambda\left(\sum_{i=1}^{n} p_i q_i\right). \tag{2.1.5}$$

where λ is an as yet unknown Lagrangian multiplier. Differentiating with respect to q_i, the quantities of the commodities entering the consumer budget, and equating to zero we obtain the set of *n* equations

$$(\delta U / \delta q_i) = \lambda p_i \quad (\text{for } i = 1, \ldots, n) \tag{2.1.6}$$

Solving (2.1.6) for the n quantities q_i and λ gives the optimal consumption vector ***q**** subject to the budget constraint. A direct result of utility maximisation is that every demand equation must be homogeneous of degree zero: if all prices and income are multiplied by a positive constant, the quantities demanded must remain constant (see Theil, 1975, p.34).

To define the functional form of $\boldsymbol{q}(\boldsymbol{p}, y)$ for empirical studies, we need to choose a specific utility function. Following Klein and Rubin (1947-1948) we assume:

$$U(q) = \sum \beta_i \log(q_i - \gamma_i) \tag{2.1.7}$$

where β_i and γ_i are the parameters of the function. The q_i are the quantities consumed of the *n* commodities in the consumer's basket which must be positive and $q_i > \gamma_i$. The γ_i, if positive, may be interpreted as the minimum quantities needed for subsistence.

From (2.1.7) we can find the marginal utility of the ith commodity as the partial derivative of the utility function in q_i:

$$\delta U / \delta q_i = \beta_i / (q_i - \gamma_i) \tag{2.1.8}$$

From the condition that all marginal utilities must be positive, and with the constraints on the quantities q_i imposed in (2.1.7), it follows $\beta_i > 0$. If we normalise the β parameters by dividing each of them by their sum, then

$$\sum \beta_i = 1 \tag{2.1.9}$$

To derive the empirical demand functions, we equate the RHS's of (2.1.6) and (2.1.8) and obtain

$$\lambda(p_i q_i - \gamma_i p_i) = \beta_i \qquad (i = 1, \ldots, n) \tag{2.1.10}$$

which, summed over n and, considering (2.1.9) and the constraint (2.1.2), yields

$$\lambda = \left(y - \sum_{k=1}^{n} p_k \gamma_k \right)^{-1} \tag{2.1.11}$$

If we now solve (2.1.10) in λ and fit the result in (2.1.11) we obtain the well known Linear Expenditure System (LES), first used by Stone (1954), and more recently by Solari (1971), Deaton (1975), Carlevaro (1975) and many others. For the *ith* commodity the demand equation becomes:

$$p_i q_i = \gamma_i p_i + \beta_i y - \beta_i \sum_{k=1}^{n} \gamma_k p_k \tag{2.1.12}$$

Equation (2.1.12) describes the expenditure on each of the n commodities in a consumer's budget as functions of prices and the consumer's income or total expenditure.

Demand systems based on functional relationships like (2.1.12) are very attractive for the simplicity of their mathematical formulations and have been extensively used in empirical studies. However, they depend on a utility function like (2.1.7) which is very restrictive, and rather unrealistic, as it assumes that the marginal utility of every commodity depends exclusively on its own quantity (Theil, 1975, p.6).

We will apply the LES to our data as a first approximation to the description of households' consumption behaviour in Italy and New Zealand and also as a bench-mark against which to compare the more advanced demand systems which we will go on to study in the next few chapters.

In Section 2 of this chapter, we discuss the estimation of model (2.1.12) under various statistical assumptions; in Section 3, we show the parameter estimates for New Zealand and Italy; and in Section 4 we derive and discuss the price and income elasticities derived from the parameter values estimated from the data.

2.2 The Estimation of the LES Model

2.2.1 The Statistical Assumptions

If we divide both sides of model (2.1.12) by y[4], we obtain a system of demand equations in a budget-share[5] form which, being in a relative form, is easier to analyse than one involving absolute monetary values.

For the *jth* household in time period *t*, the model can be written:

$$w_{ijt} = \beta_i + (1-\beta_i)\gamma_i z_{ijt} - \beta_i \sum_{k \neq i}^{n} \gamma_k z_{kjt} + \varepsilon_{ijt} \qquad (j = 1,\dots,M \text{ and } t = 1,\dots,T) \qquad (2.2.1)$$

where $w_i = (p_i q_i / y)$, $z_i = (p_i / y)$ and ε is a random error component.

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4 Dividing (2.1.12) by the income variable should have the effect of reducing heteroskedasticity as it can be reasonably assumed that the variability of the error disturbance is approximately proportional to the level of income enjoyed by the consumer, see Theil, 1975, p.237.

5 For a discussion of a demand system in budget shares form see Theil, 1975, Chapter 1.5

When estimating (2.2.1) from household consumption data collected over several periods of time, the behaviour of the disturbances ε_{ijt} over the cross-sectional units, the households, is likely to be different from the behaviour of the disturbances for a given cross-sectional unit over time. Thus the disturbances will be mutually independent but heteroskedastic for the cross-sectional observations and will be autocorrelated over time. This will require the imposition of appropriate restrictions on Ω_i, the variance-covariance matrix of the disturbances in the *ith* equation.

The assumptions of cross-sectional heteroskedasticity and time-wise autocorrelation for the disturbances of the ith equation can be summarised as follows:

Heteroskedasticity:

$$E\left(\varepsilon_{jt}^2\right) = \sigma_j^2 \tag{2.2.2a}$$

Cross-sectional independence:

$$E(\varepsilon_j \varepsilon_k) = 0, \quad for \ \ j \neq k \tag{2.2.2b}$$

First-order autoregression:

$$\varepsilon_{jt} = \rho_j \varepsilon_{j,t-1} + \eta_{jt} \ \text{ with } \ \eta_{jt} \sim N\left(0, \xi_j^2\right) \tag{2.2.2c}$$

From (2.2.2c), it follows:

$$\varepsilon_{jt} \sim N\left\{0, \xi_j^2 / (1-\rho_j^2)\right\} \text{ and } E\left(\varepsilon_{j,t-1}\eta_{kt}\right) = 0 \ \text{ for all } j, k \tag{2.2.2d}$$

Assumption (2.2.2c) implies that the autoregression parameter can vary from one cross-sectional unit to another, so that the stochastic characteristics of the disturbances with respect to time can be summarised as being:

$$E\left(\varepsilon_{j,t}\varepsilon_{j,s}\right) = \rho_j^{t-s}\sigma_j^2 \ \text{ for } \ t \geq s \ \text{ and } \ E\left(\varepsilon_{j,t}\varepsilon_{k,t}\right) = 0 \ \text{ for } \ j \neq k \tag{2.2.2e}$$

For each of the n demand equations in the system, under assumptions (2.2.2), the variance-covariance matrix results to be:

$$\Omega_i = \begin{bmatrix} \sigma_1^2 V_1 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 V_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & \sigma_M^2 V_m \end{bmatrix} \tag{2.2.3}$$

where the autoregressive matrices V_j are made up as

$$V_j = \begin{bmatrix} 1 & \rho_j & \rho_j^2 & \cdots & \rho_j^{T-1} \\ \rho_j & 1 & \rho_j & \cdots & \rho_j^{T-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \rho_j^{T-1} & \rho_j^{T-2} & \rho_j^{T-3} & \cdots & 1 \end{bmatrix} \tag{2.2.4}$$

Assumptions (2.2.3) and (2.2.4) imply different variance-covariance matrices for each commodity in the system, but no covariance across equations as they are estimated one by one.

Model (2.2.1), together with assumptions (2.2.2) to (2.2.4) about its disturbances, describes the budget consumption shares for the n commodities as linear functions of the consumer's income and the n prices, but because it is non-linear in the β and γ parameters, it cannot be estimated by ordinary least squares (OLS). The price parameters γ_i however can be linearised by fixing the β_i parameters a-priori and vice versa. If we want to avoid non-linear estimation techniques, the simplest way to estimate (2.2.1) is to do it in two steps: first estimate the income parameters $\beta_{i,}$ and then fit the estimated values of the β_i into (2.2.1) to linearise it and estimate the price parameters γ_i in isolation. The estimation procedure can be iterated until two successive iterations produce the same estimates for both sets of parameters.

2.2.2 The Kmenta Procedure

In situations where data are in the form of time series of cross-sectional observations, like in the present instance, Kmenta (1986, p. 616-635) describes a procedure for pooling such information in a way to obtain efficient and Maximum Likelihood (ML) estimators.

The pooling of cross section and time series data has a long tradition in the estimation of economic models going back to seminal papers like those by Theil and Goldberger (1960), dealing with the introduction into the estimation of regression parameters of prior information coming from alternative data sources, or by Balestra and Nerlove (1966) or Nerlove (1967) and (1971), dealing more specifically with the empirical application of the pooling techniques to the estimation of economic models.

The technique is extremely flexible and can take into account an almost endless variety of model specifications (see Judge et al., 1985, ch.13) and of error term structures (see Mundlak, 1978), and it has been applied, with very good results, in a variety of fields, from the estimation of the effects of changes in relative prices on a country share of international trade (Fair, 1981) to the demand for automotive fuel in the OECD (Baltagi and Griffin, 1983).

We intend to apply the pooling technique here because it is one of the most common, and most successful, estimation methods employed in the literature on the estimation of linear models based on cross sectional data obtained over several time periods.

The obvious alternative to the pooling techniques described in Kmenta 1971, and in the literature mentioned above, would be to estimate our demand model, with all the restrictions imposed by consumption theory and all the assumptions on its error term structure, made explicit, by a non-linear estimation method. We do not follow this route partly to keep to the traditional linear estimation methods used in the literature, and partly because we will re-estimate the LES model by non-linear methods in Chapter 4, where it will be one of the sub-models nested within the demographically extended NLPS model.

A consistent estimator of Ω_i in equation (2.2.3) can be found by first estimating by OLS model (2.2.1), to obtain unbiased and consistent estimates of its regression parameters, which can then be used to compute the regression residuals e_{jt}. From the regression residuals, consistent estimates of the autocorrelated model disturbances, we can obtain a consistent estimator of the autocorrelation parameters ρ_j, computed as the sample coefficients of correlation between successive regression residuals e_{jt} and $e_{j,t-1}$, which are bound to assume values in the [-1, +1] interval:

$$r_j = \left(\sum e_{jt} e_{j,t-1}\right) / \left[\left(\sum e^2_{jt}\right)^{1/2} \left(\sum e^2_{j,t-1}\right)^{1/2}\right] \quad (t = 2,3,\ldots,T) \tag{2.2.5}$$

The next step is to use r_j to transform the observations in (2.2.1) into a non-autoregressive process:

$$w^*_{ijt} = \beta_i + (1-\beta_i)\gamma_i z^*_{ijt} - \beta_i \sum_{k \neq i}^{n} \gamma_k z^*_{kjt} + \varepsilon^*_{ijt} \tag{2.2.6a}$$

where

$$w^*_{ijt} = w_{ijt} \sqrt{1-r_j^2} \qquad \text{for } t = 1 \tag{2.2.6b}$$

$$w^*_{ijt} = w_{ijt} - r_j w_{ijt-1} \qquad \text{for } t = 2, 3, \ldots, T \tag{2.2.6c}$$

and

$$z^*_{ijt} = z_{ijt} \sqrt{1-r_j^2} \qquad \text{for } t = 1 \tag{2.2.6d}$$

$$z^*_{ijt} = z_{ijt} - r_j z_{ijt} \qquad \text{for } t = 2, 3, \ldots, T \tag{2.2.6e}$$

(see Kmenta, 1986, p.619) so that, from the transformed model, we can obtain a consistent estimator of σ^2_j :

$$s_j^2 = v_j^2 / (1 - r_j^2) \tag{2.2.7}$$

where v_j^2 is a consistent estimator of ξ_j^2 obtained from

$$v_j^2 = \frac{1}{T-K} \sum_{t=1}^{T} u^2_{jt} \tag{2.2.8}$$

where the u_{jt} are the estimators of the η_{jt} residuals in (2.2.2c), (see Kmenta, 1986, p.620). Once we have consistent estimators for σ_j and ρ_j we can build up the n Ω_i matrices and estimate the regression model parameters by Generalised Least Squares (GLS)[6].

To actually estimate the LES parameters in (2.2.1) by the procedure described by Kmenta, 1971 (the *Kmenta procedure* from now on) we can proceed as follows:

i) Estimate the β_i first, assuming all $\gamma_i = 0$.

ii) Fit the estimated values of the β_i back into (2.2.1) and estimate the γ_i parameters from the new regression equation:

$$bw_i = \gamma_i bz_i - \sum_{k \neq i} \gamma_k (b_i z_k) \qquad (2.2.9)$$

where b_i represents the estimated value of β_i, $bw_i = (w_i - b_i)$ and $bz_i = (1 - b_i)\, z_i$

iii) Fit the estimated values of the γ_i back into (2.2.1) and estimate the β_i again from the resulting regression model

$$gw_i = \beta_i S \qquad (2.2.10)$$

where $gw_i = (w_i - g_i z_i)$ and $S = (1 - \sum g_i z_i)$.

In estimating model (2.2.1) in the manner described above, we can proceed in two ways: go through steps (i) to (iii) only once; or repeat them over and over again until successive sets of β and γ estimates become more or less the same and the estimation procedure is stopped (see Theil, 1975, p. 240). It must be kept in mind, however, that when iterating the estimation steps (i) to (iii) there is no certainty that successive sets of parameter estimates will stabilise to some specific values. As we shall see in

6 For an alternative estimation procedure, which first removes both autoregression and heteroskedasticity from (2.2.1), and then estimates it by OLS, see Kmenta, 1986, p. 620.

Sections 2.3.1 and 2.3.2, this seems to be the case for both the New Zealand and Italian data.

As it is applied here, the Kmenta procedure is iterative at four different levels. The first level is the application of GLS to the regression model as explained in equations (2.2.5) to (2.2.8). The second is the possibility to iterate the Kmenta procedure until two successive GLS sets of regression parameter estimates coincide; the third is represented by the three-step estimation of the parameters of the LES model (2.2.1) explained in equations (2.2.9) and (2.2.10), while the fourth is repeating the whole process over and over again trying to obtain identical (or similar enough) successive sets of LES parameters[7]. For the rest of this chapter we will call each iteration at the fourth level a "round".

2.2.3 The Extended Kmenta Procedure

The demand equations of system (2.2.1) cannot be estimated separately, one by one, by the Kmenta procedure, which is a single equation estimation method. Direct application of the Kmenta procedure would generate n different sets of γ estimates, one for each demand equation in the system. This would have contradicted one of the basic assumptions of the LES demand system, namely that the γ parameters must be the same in all equations.

A solution to this problem can be found in a property of the special class of multi-equation systems in which the regression coefficients in each equation are the same as the regression coefficients in all other equation. Such systems can be collapsed into a single equation with the number of observations equal to the number of observations in the system multiplied by the number of equations (see Kmenta, 1986, p. 637). In

[7] We must point out, however, that these types of iterative procedures are different from those normally used to estimate the parameters of non-linear models, we will apply in the next chapters, as they are not based on the maximisation of any objective function (usually the LF) but on successive efficient estimates of the regression parameters and their residuals variance-covariance matrices.

estimating this new system-wide equation, the original equations can be treated as additional cross-sectional units [8].

The equations in system (2.2.1) differ only because each one of them has a different β parameter, but we can let all equations in the system have the same regression coefficients by introducing all the β parameters in each equation, and then associate them with (180×1) vectors of dummy variables D_k (for k=1,2,...,n) such that in the ith equation it is: $D_k = 0$ for $k \neq i$ and $D_k = 1$ for k = i, as shown in (2.2.12).

Once all equations in (2.2.1) are of identical form, as they all contain the same parameters, the system can be collapsed into one single equation in which the w_i variables will be "stacked" one after the other, to generate a new dependent variable W with n extra cross-sectional units and $(M \times T \times n)$ observations.

In matrix form, the new dependent variable W, obtained by stacking the n share consumption variables w_i considered in system (2.2.1), can be described as:

$$W = \begin{bmatrix} w_{1jt} \\ w_{2jt} \\ \cdots \\ w_{ijt} \\ \cdots \\ w_{njt} \end{bmatrix} \quad \text{(for } i = 1,\ldots \text{n; } j = 1 ,\ldots, M \text{ and } t = 1,\ldots, \text{T)} \qquad (2.2.11)$$

where W is a $[(MTn) \times 1)$ matrix and w_{ijt} represents the original observation of the ith commodity consumption share by the jth household at time t.

[8] Once the demand system has been collapsed into a single equation the assumption of no cross-correlation of errors across commodities can be relaxed by introducing an heteroskedastic estimation procedure (e.g. White 1980a) in the first level - GLS - of the Kmenta procedure.

2.2.4 The Empirical Model

In the present study, we consider the consumption of four commodities only: V_1 = Food, V_3 = Household Operations, V_4 = Apparel and V_5 = Transport; obtained from the Italian and New Zealand budget survey data discussed in Chapter 1.

If, as in the case of New Zealand, we have four commodities (i=1, 3, 4, 5)[8] over M=20 cross-sectional observations and T=9 time periods, and collapse the demand system (2.2.1) into a single equation like (2.2.11), we can write it in matrix form as:

$$W = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix} + \begin{bmatrix} z_1 & 0 & 0 & 0 \\ 0 & z_3 & 0 & 0 \\ 0 & 0 & z_4 & 0 \\ 0 & 0 & 0 & z_5 \end{bmatrix} \begin{bmatrix} (1-\beta_1) & \gamma_1 \\ (1-\beta_3) & \gamma_3 \\ (1-\beta_4) & \gamma_4 \\ (1-\beta_5) & \gamma_5 \end{bmatrix} -$$

$$\begin{bmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 \\ 0 & 0 & \beta_4 & 0 \\ 0 & 0 & 0 & \beta_5 \end{bmatrix} \begin{bmatrix} 0 & z_3 & z_4 & z_5 \\ z_1 & 0 & z_4 & z_5 \\ z_1 & z_3 & 0 & z_5 \\ z_1 & z_3 & z_4 & 0 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix} \qquad (2.2.12)$$

where the I and 0 elements, representing the dummy variables, D_k, are (180 × 1) matrices of ones and zeros; the β_i and γ_i elements are the unknown parameters and the z_i are the original price observations, income-scaled. The matrix of dependent variables, the expenditure shares of commodities, will then be of dimensions (720×1)[9].

Before proceeding to the actual estimation of the demand system (2.2.12), we must take into consideration the fact that household consumption is a complex phenomenon, and besides income and prices, many other factors are likely to play a

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8 We do not use subscript 2 as it would refer to Housing, a commodity we are not considering in this study as it sometimes assumes negative values, see Section 1.2 above.

9 For an error-component approach to estimating models from pooled time series and cross-sectional observations see Mundlack (1978).

role in determining it: total assets, stocks of durable goods, level of indebtedness, past and expected future income and a number of demographic variables.

Because of data limitations, we can consider here only two of the above variables: previous year's income[10] and family size[11]. If we add them to (2.2.1) we obtain the linear model:

$$w_{ijt} = \beta_i + (1-\beta_i)\gamma_i z_{ijt} - \beta_i \sum_{k \neq i}^{n} \gamma_k z_{kjt} + \lambda_i q_{jt-1} + \theta_i s_{jt} + \varepsilon_{ijt} \qquad (2.2.13)$$

where $q_{jt\text{-}1}$ is the previous year's income of the *jth* household at time t, s_{jt} is the household size, and ε_{ijt} is a random error component. During estimation, (2.2.13) will have to be put into the same format of equation (2.2.12) and one equation dropped to fulfil the income parameters constraint $\Sigma\, \beta_i = 1$ and $\Sigma\, \lambda_i = \Sigma\, \theta_i = 0$.

2.3 The Estimation Results

2.3.1 The Estimation Results for New Zealand

To obtain the first set of income parameters β_i, we estimated model (2.2.1) in the "stacked" form (2.2.12), first under the assumption of a different autoregressive coefficient ρ_j for each cross-section, which implies that all the $\boldsymbol{V_j}$ autoregressive matrices in (2.2.4) are different; and, secondly, under the assumption of an equal autoregressive coefficient ρ for all cross-sections, which implies that $\boldsymbol{V_j}=\boldsymbol{V}$, i.e. all the autoregressive matrices in (2.24) are the same.

10 In estimating model (2.1.12) the income variable y is represented by total current household expenditure *on the four commodities considered* and previous year's income q is represented by total household expenditure on *all* commodities considered in the budget for the previous period. For the same household the two income variables y and q might be strongly correlated over time, therefore multicollinearity might prove to be a problem in estimating these two parameters.

11 Some Authors have suggested that children scale effects on consumption are non-linear and therefore family size should enter the demand function in log form, see Browning 1992, p.1436. We have tried this transformation but the overall results did not change, apart from some reductions in the number of iterations required to obtain stable estimates within the Kmenta procedure. We kept the household size variable in its original form for ease of interpretation of the results.

The first assumption was clearly the preferred one[12] and became the basic assumption on the distributional structure of the model's stochastic component.
We then plugged this first set of β_i estimates into equation (2.2.9) to estimate the γ_i parameters. We added to the model the two extra explanatory variables appearing in equation (2.2.13) as well as a time trend variable - to take into account the effects of non-economic variables like changes in tastes or social attitudes - with values from 1 to 9 for New Zealand and 1 to 13 for Italy, to represent the time series units spanned by the data for the two countries. We then reformatted all new variables according to (2.2.12) and applied the Kmenta procedure to the single equation resulting from the stacking procedure.

In estimating model (2.2.13) for New Zealand by the iterative Kmenta procedure, it proved rather difficult to achieve convergence[13] within the GLS procedure itself, especially for parameters β_5 and γ_5. At the end of Section 2.2.2 we have called this stage of the iterative estimation procedure "the first iteration level", and it was described by equations (2.2.5) to (2.2.8).

For both these parameters, the iterative procedure, after generating alternatively increasing and decreasing estimates, appeared to get locked into a loop and kept generating steadily increasing values, but only by very small increments of the third or fourth decimal figures. After experimenting with different values of the *convergence criterion* c[14], we found that convergence was more easily reached when $c = .001$.

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[12] We used the Likelihood ratio test and the Akaike Information Criterion to choose between the two assumptions, and $\rho_j = \rho$ was clearly rejected. We also found that under the assumption of different ρ_j the goodness of fit too was better, the Durbin-Watson test excluded the possibility of autocorrelation in the regression residuals and the Jarque-Bera Lagrange Multiplier test confirmed their normality. Under the assumption $\rho_j = \rho$ both the Durbin-Watson and the Jarque-Bera tests rejected the null hypothesis.

[13] One of the reasons might have been the very large dimensions of the matrix of explanatory variables, including the dummies, which was a (720 x 13) matrix, with 80 cross-sections and nine time periods

[14] If we stop the iteration when all the differences between two successive sets of estimates θ are such that:

$$|\theta(t+1) - \theta(t)| / |\theta(t)| < c$$

The parameter estimates obtained in the first round of the Kmenta procedure are shown in Table 2.3.1. In the first column of the table we list the four commodities considered, in the second column the estimates for the income parameters β, in the third column the estimates for the price parameters γ, in the fourth the estimates for

TABLE 2. 3. 1

Parameter Estimates from the Kmenta Procedure. New Zealand [(a)]

Com. ties (1)	β_i (2)	γ_i (2)	θ_i (2)	λ_i (2)
Food	.1847 (.00951)	18.593 (.6115)	.0346 (.00144)	-.0033 (.00151)
Housing	.2430 (3)	23.142 (.8461)	-.0122 (3)	.0030 (3)
Apparel	.0978 (.00471)	-1.456 (.4594)	.0054 (.00114)	.0016 (.00081)
Transport	.4745 (.00972)	-61.029 (2.8081)	-.0278 (.00221)	.0047 (.00221)

LL (4)(5)	**Rm** (5)(6)	**Rp** (5)(7)	**DW** (5)(8)	**LM** (5)(9)
1683.68	.978	.964	1.979	7.43 [2]
1779.85	.917	.916	1.822	6.52 [2]

(a) A time trend variable is included with values from 1 to 9
(1) A different autoregression parameter ρ_i is estimated for each cross-sectional cell
(2) Standard errors in brackets
(3) This parameter has been obtained as a residual to satisfy the add-up condition
(4) Value of Log-Likelihood function
(5) Value in upper row refers to β_i estimates, in lower row to γ_i estimates
(6) Raw-moment R-square. Single system-wide equation.
(7) R-square between observed and predicted. Single system-wide equation.
(8) Durbin-Watson statistic
(9) Jarque-Bera Lagrange Multiplier normality test, χ square distributed, d.f. in square brackets.

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where c>0 is a small and pre-specified constant, then we call c the *convergence.criterion.*

the household size parameters θ, and in the fifth the estimates for the lagged income parameters λ. In brackets, underneath the parameter values, we report their standard errors. In the lower part of the table we report the values of the Log-Likelihood Function (LL), those of two indices of fit: the raw moment R-square and the R-square between the observed and predicted values of the dependent variable[15], the (approximate) Durbin-Watson statistic and the Jarque-Bera (1987) test of normality of the regression residuals.

All the estimates have very small standard errors - a fact which seems to exclude multicollinearity - and they are significantly different from zero. The fit is extremely good and there are no indications of autocorrelation or non-normality of the regression residuals.

Considering how good the first round estimates were, we could have expected similarly good estimates in the following rounds. However it turned out that the second, third and fourth round estimates of the β and γ parameters, instead of getting closer, from one round to the next, kept diverging.

In the case of the income parameters, the problem seemed to be β_5 which got larger and larger, while the other β.s kept getting smaller and smaller. At the third round, β_5 became greater than one and β_1 and β_3 became negative. In the case of the price parameters, it was the negative γ_5 that kept decreasing until, by the fourth round, its absolute value was less than .0001. Most of the goodness of fit indices, standard errors and other tests were difficult to interpret as they tended to contradict each other. We do not show here the parameter estimates for rounds two, three and four to save space.

2.3.1.1 An Alternative Estimation Procedure

To improve on the Kmenta estimates, we tried an alternative estimation procedure

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[15] As Buse (1973) shows, the OLS R-square statistic, cannot be used in GLS estimation.

which could constrain the γ parameters in system (2.2.13) to be the same in all equations. The alternative estimation procedure we tried was a constrained version of the so-called Seemingly Unrelated Regressions (SUR), a multi-equation estimation method suggested by Zellner (1962).

Under the standard assumptions of the classical normal linear regression model, the OLS estimators of the regression coefficients are unbiased and efficient. This result depends on the understanding that the model represents the totality of the information available about the regression model. If some information about the characteristics of the model has not been taken into account then the OLS properties are not necessarily valid any more. Such a situation will arise if in the demand system (2.2.13) the disturbances in the demand equation for the *ith* commodity are correlated with the disturbances of the demand equation for the *kth* commodity. The resulting covariances of the disturbances of the demand equations in the system will be:

$$E(\varepsilon_i \varepsilon_k) = \sigma_{ik} \qquad (\text{for } i, k = 1, 2, \ldots, n) \qquad (2.2.14)$$

Under this assumption, the variance-covariance matrices, Ω_i will not be diagonal any more and, instead of having zeros, they will have the above covariances as their off-diagonal elements. In the classical normal linear regression model, under assumptions (2.2.14) the OLS estimators will generate estimates which are unbiased but not efficient. To obtain efficient estimators, the n equations must be estimated together as a system. The estimation procedure suggested by Zellner (1962) was to estimate the covariances $E\{\varepsilon_i \varepsilon_k\}$ from the OLS residuals of the n original equations in the system estimated one by one and then use these estimates, which are unbiased and consistent, to build up an estimate of the variance-covariance matrix for the whole system of n equations, say $\boldsymbol{W}$. Once $\boldsymbol{W}$ is known, it can be used to re-estimate the whole system by GLS obtaining efficient estimates for the model parameters.

This estimator is also called a *two-stage Aitken estimator* and it is asymptotically efficient and normally distributed. In small samples, two-stage Aitken estimators appear to be unbiased and efficient relative to OLS estimators (see Judge et al., p.175).

We tried a SUR estimation for the linearised version (2.2.9) of (2.2.14) where the β parameters had been obtained by the extended Kmenta procedure and the γ parameters were constrained to be the same across all equations. These alternative estimates are shown in Table 2.3.2.

All estimates obtained from the SUR procedure are again highly significant, the goodness of fit, as measured by the raw moment R-square and the R-square between the observed values of the dependent variable and those predicted by the model, is good and the hypothesis of diagonal covariance matrix (ie that the equations in the system are unrelated) is clearly rejected by a Likelihood Ratio test (shown at the bottom right corner of Table 2.3.2).

To choose between these two sets of estimates is not easy as both appear to be statistically sound. Only by looking at the respective values of the LL and the Akaike Information Criterion (Judge et al. 1985, p.870) could we establish a preference for the Kmenta estimates over the SUR estimates.

We will retain both the Kmenta and the SUR estimates and use them to compute the New Zealand price, income and household-size elasticities. A direct comparison of the resulting elasticities might help in choosing the "preferred" model.

2.3.2 The Estimation Results for Italy

As we did for New Zealand, we tried to estimate the parameters of model (2.2.13) for Italy by going through the three steps described in Section 2.2.2, equations (2.2.9) and (2.2.10), repeatedly, hoping to get sets of estimates converging to the same values after a few rounds.

In fact, we got sets of acceptable estimates only in the first and second rounds. After the second round, as in the case of New Zealand, successive parameter estimates tend to either increase or decrease steadily from round to round. For example all the price parameters β become very small in absolute value, with the exception of β_1 which tends to unity. Therefore, the only two sets of estimates which are statistically and

TABLE 2.3.2

Estimates for the γ parameters from the SUR Procedure. New Zealand[(a)(b)]

Com. ties	β_i (1)(2)	γ_i (2)	θ_i (2)(3)	λ_i (2)(3)
Food	.1847 (.00951)	30.624 (4.535)	.0336 (.00161)	-.0078 (.00168)
Housing	.2430 (4)	-37.590 (6.196)	-.0056 (4)	-.0032 (4)
Apparel	.0978 (.00471)	-11.178 (2.555)	.0064 (.00105)	.0049 (.00249)
Transport	.4745 (.00972)	-47.677 (12.061)	-.0344 (.00201)	.0061 (.00197)

LL(5)	AIC(6)	Rb (7)	Rb (8)	LR(9)
1278.78	-4.68	.964	.955	273.44 [3]
	-3.48	.952	.655	
		.983	.939	

(a) A time trend variable is included with values from 1 to 9
(b) Values for β_i from Kmenta procedure
(1) A different autoregression parameter ρ_i is estimated for each cross-sectional cell.
(2) Standard errors in brackets
(3) Estimated by SUR together with the γ parameters
(4) This parameter has been obtained as a residual to satisfy the add-up condition
(5) Value of Log-Likelihood function
(6) Akaike Information Criterion, upper row Kmenta estimates, lower row SUR estimates
(7) Raw-moment R square, for β_i regressions. Three separate equations.
(8) Raw-moment R square, for γ_i regressions. Three separate equations.
(9) Likelihood Ratio test of diagonal covariance matrix, d.f. in square brackets

economically acceptable are those from either the first or the second round of the Kmenta procedure. For the sake of comparison we show both of them in Table 2.3.3 and Table 2.3.4 .

TABLE 2.3.3

Parameter estimates, first round, Kmenta procedure. Italy [(a)(b)]

Com. ties (1)	β_i (2)	γ_i (2)	θ_i (2)	λ_i (2)
Food	.3916 (.00635)	22.9860 (.35320)	.0173 (.00182)	-.0028 (.00106)
Housing	.1451 (3)	-2.0760 (.06571) -	-.0034 (3)	- .0012 (3)
Apparel	.1877 (.00238)	-4.9852 (.11241)	-.0058 (.00083)	.0006 (.00030)
Transport	.2757 (.00497)	-9.5849 (.21560)	-.0081 (.00138)	.0034 (.00072)

LL(4)(5)	**Rm** (5)(6)	**R2**(5)(7)	**DW** (5)(8)	**LM**(5)(9)
2733.68	.973	.961	2.053	75.02 [2]
2833.26	.944	.941	1.905	85.28 [2]

(a) A time trend variable is included with values from 1 to 13
(b) Expenditure in LIt. 1000
(1) A different autoregression parameter ρ_i is estimated for each cross-sectional cell
(2) Standard errors in brackets
(3) This parameter has been obtained as a residual to satisfy the add-up condition
(4) Value of Log-Likelihood function
(5) Value in upper row refer to β_i estimates, in lower row to γ_i estimates
(6) Raw-moment R-square. Single system-wide equation.
(7) R-square between observed and predicted. Single system-wide equation.
(8) Durbin-Watson statistic
(9) Jarque-Bera Lagrange Multiplier normality test, χ square distributed, d.f. in square brackets.

We also tried the alternative SUR estimation procedure for the Italian data but the estimation results were totally unsatisfactory and are not reported here.

All the first round estimates reported in Table 2.3.3 have very small standard errors (which implies that they are significantly different from zero), the model fit is very good, as shown by the values of the raw moment R-square and the R-square between the observed and the predicted values of the dependent variable - the "stacked" W of equation (2.2.11) - and the Durbin-Watson statistic indicates absence of auto-correlation. The Jarque-Bera test (Jarque-Bera, 1987) however, suggests non-normality of residuals.

TABLE 2.3.4

Parameter estimates, second round, Kmenta procedure. Italy (a)(b)

Com. ties (1)	β_i (2)	γ_i (2)	θ_i (2)	λ_i (2)
Food	-.1945 (.00402)	4.1915 - (.19430)	.1628 (.00198)	.0017 (.00097)
Housing	.2043 (3)	-.0610 (.03196)	.1138 (3)	-.0015 (3)
Apparel	.2691 (.00519)	-2.5604 (1.79500)	-.0207 (.00074)	.0004 (.00030)
Transport	.7211 (.01931)	-6.5896 (1.79501)	-.2559 (.00363)	-.0006 (.00070)

LL (4)(5)	**Rm** (5)(6)	**Rp** (5)(7)	**DW** (5)(8)	**LM** (5)(9)
2663.09	.960	.910	2.064	23.69 [2]
2802.80	.978	.977	2.057	64.74 [2]

(a) A time trend variable is included with values from 1 to 13
(b) Expenditure in LIt. 1000
(1) A different autoregression parameter ρi is estimated for each cross-sectional cell
(2) Standard errors in brackets
(3) This parameter has been obtained as a residual to satisfy the add-up condition
(4) Value of Log-Likelihood function
(5) Value in upper row refer to β_i estimates, in lower row to γ_i estimates
(6) Raw-moment R-square. Single system-wide equation.
(7) R-square between observed and predicted. Single system-wide equation.
(8) Durbin-Watson statistic
(9) Jarque-Bera Lagrange Multiplier normality test, χ square distributed, d.f. in square brackets.

Similar comments apply to the second round estimates which appear in Table 2.3.4, and once again it is difficult to decide objectively which set of estimates is "better".

The only indication comes from the values of the LL, which are larger for the first round estimates than they are for the second round estimates. Therefore, as was done for New Zealand, we compute the price and expenditure elasticities for Italy from both sets of estimates.

2.4 The Elasticities

2.4.1 The Computation of the Elasticities

From the parameter estimates shown in Table 2.3.1 to 2.3.4, we have computed the price elasticities, the household size elasticities, the total expenditure[16] elasticities and the lagged income[17] elasticities for both countries.

All elasticities where computed at the means values of the dependent and explanatory variables entering model (2.2.13). The results for New Zealand are shown in Table 2.4.1 and 2.4.2, and those for Italy in Table 2.4.3 and 2.4.4.

2.4.2 The New Zealand Elasticities

Taking the New Zealand elasticities first, those obtained from the Kmenta estimates are shown in Table 2.4.1, and those obtained from the SUR estimates are shown in Table 2.4.2. The own-price elasticities are reported in the first column of the tables, the household size elasticities appear in the second column and the total and lagged expenditure elasticities in the third and fourth columns respectively.

All price elasticities obtained from the SUR estimates, with the exception of Food, are negative, this indicates that quantities consumed change in the opposite direction to prices, and all of them are less than unity in absolute value; a marginal increase (fall) in prices results in less than proportionate fall (increase) in quantities. Food, an essential commodity which *must* be consumed, has an elasticity which is positive and less than unity in absolute value (i.e. inelastic): a change in price will cause a less than

[16] Obtained by summing the expenditures for the four commodities considered

proportionate change in quantity and in the same direction as the change in price. The general impression gained from a first inspection of these price elasticities is that New Zealand households react rather strongly to changes in the prices of the basic commodity groups considered in this study.

TABLE 2.4.1

Price, Household Size, Expenditure and Lagged Income Elasticities for New Zealand Households. All elasticities computed at the means.
Average household size FS = 2.5
(Parameter estimates from the first round of the Kmenta procedure)

	Prices	H. Size	Tot. Exp.	Lag. Exp.
Food	.236	.285	.608	-.024
Housing	.335	-.113	.901	-.026
Apparel	-.062	.154	1.114	.041
Transp.t	-.673	-.205	1.400	.032

TABLE 2.4.2

Price, Household Size, Expenditure and Lagged Income Elasticities for New Zealand Households. All elasticities computed at the means.
Average household size FS = 2.5
(Price parameters γ estimated by SUR procedure)

	Prices	H. Size	Tot. Exp.	Lag. Exp.
Food	.388	.277	.608	-.058
Housing	-.545	-.052	.901	-.027
Apparel	-.475	.182	1.114	.127
Transp.t	-.526	-.254	1.400	.041

The price elasticities obtained from the Kmenta parameter estimates are similar to those obtained from the SUR estimates for Food and Transport, but different for

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[17] Obtained as the previous year expenditure on ALL commodities.

Household Operations (Housing) and Apparel. The elasticities for Apparel differ in size but have the same sign. On the contrary the price elasticity for Housing obtained from the SUR estimates is negative while that obtained from the Kmenta estimates is positive, both are less than unity.

A positive elasticity for housing operations expenditure does not seem economically meaningful, an increase in the price of household appliances is unlikely to encourage the consumers to spend more on them. A negative, but less than unity, price elasticity is more in accord with expectations: such an expenditure category is likely to belong to one of those classes of consumption which are so essential that consumers are often unable to reduce them at will (see Blundell, 1988, p.24); therefore, while it is perfectly reasonable to expect some decrease in housing expenditure when its price increases, it is unreasonable to expect a more than proportional decrease (represented by an elasticity larger than unity in absolute value).

Both the Kmenta and the SUR estimates for the Food price elasticities look acceptable and within expectations: the price elasticity for food, a necessary commodity, can be positive and inelastic because the quantity of food consumed cannot be easily reduced; price increases therefore have to be absorbed by the households by reducing the quantities purchased, but less than proportionately to the increase in price, with a consequent increase in the amount spent on food.

The Transport and Apparel own-price elasticities are negative, and inelastic, for both the Kmenta and the SUR estimates; an increase in their prices would cause a less than proportionate reduction in the expenditure for these two commodities. However, while the absolute values of the price elasticities for Transport found by the SUR and Kmenta procedures are similar, those for Apparel are very different. The Apparel elasticity found by the SUR procedure is much larger in absolute value than that found by the Kmenta procedure. Such a large difference is difficult to explain, but it might point to some degree of unreliability in the parameter estimates.

The household size elasticities are very similar for both estimation methods, and they are quite acceptable. For housing operations and transport the computed household size elasticities seem to indicate that as the household size increases there is a much less than proportionate decrease in the amount spent on these two commodities. Conversely, as the household size increases, there is an increase in the expenditure for food and clothing, but the increase is less than proportional to the increase in household size.

Coming now to the total expenditure, or income, elasticities (which are the same for both sets of estimates as they depend on the same parameter values) we find that two commodities - Food and Housing - are *necessities* (their income elasticity is <1, and when income changes they change in the same direction but less than proportionately), and two - Apparel and Transport - are *luxuries* (their income elasticity is >1, and when income changes they change in the same direction but more than proportionately). The less than proportionate increase in the expenditure for food, as total expenditure (income) increases, is a clear case of the Engel's law in operation. The total expenditure elasticities obtained in this study compare well with those found by Giles and Hampton (1985) for similar consumption categories and reported by them in Table 2, page 547.

Finally the Lagged Expenditure elasticities are all very small, which seems to show that the previous year expenditure patterns do not have a substantial effect on current year expenditure. There are some differences, both in size and sign, between the Kmenta and the SUR estimates, once again this might indicate some degree of unreliability in the parameter estimates.

Given the results for New Zealand of the elasticities computations, showing some degree of discrepancy between the two sets of estimates, and a few elasticity values difficult to explain on economic grounds (e.g. the positive price elasticity for Housing Operations), we feel the results from the LES model should be considered tentative, and only a first step in our analysis of New Zealand household consumption behaviour.

2.4.3 The Italian Elasticities

The Italian elasticities obtained from the first round of the Kmenta estimation procedure appear in Table 2.4.3; those obtained from the second round appear in Table 2.4.4. The price elasticities are reported in the first column of both tables, the household size elasticities appear in the second column, and the total and lagged expenditure elasticities in the third and fourth columns respectively.

Looking at the tables, it seems that the best estimates for the Italian elasticities are generated by model (2.2.13) parameter estimates obtained in the first round of the Kmenta procedure, as some of those generated by the second round estimates seem too large to be realistic. For example the household size elasticities for Housing and Transport obtained in the second round of the Kmenta procedure, and shown in Table 2.4.4, seems too large in absolute value to be acceptable: it does not seem possible that an increase (decrease) in household size will generate a more than threefold increase (decrease) in the expenditure in household operations and/or transportation, as implied by their household size elasticities.

Also the total expenditure elasticities for Apparel and Transport, found from the second round estimates, seems rather large in absolute value. However they are comparable with the elasticities for similar commodity groups obtained in other studies of Italian household consumption (see Leoni, 1967 and Ferrari, 1977, Tabella III.5 page 74-75) and therefore cannot be rejected as totally unreliable.

The price elasticities for all commodities, with the exception of Food, also compare rather well with those obtained by Ferrari (1977), they are all negative and much less than unity in absolute value, implying a less than proportionate and inverse relation to price movements. A negative, and small (in absolute value), price elasticity for Transport seems to confirm the empirical evidence in Italy where an increases in the cost of transport, consisting mostly of increases in the cost of fuel, which in Italy is one of the more expensive within the OECD, tend to have a very limited and temporary effect on the households' expenditure on transport.

Table 2.4.3

Price, Household Size, Expenditure and Lagged Income Elasticities for Italian Households. All elasticities computed at the means.
Average household size FS = 3.0
(Parameter estimates from the first round of the Kmenta procedure)

	Price	H. Size	Tot. Exp.	Lag. Exp.
Food	.504	.093	.705	-.070
Housing	-.044	-.095	1.356	-.155
Apparel	-.277	-.133	1.435	.063
Transp.t	-.374	-.118	1.334	.228

Table 2.4.4

Price, Household Size, Expenditure and Lagged Income Elasticities for Italian Households. All elasticities computed at the means.
Average household size FS = 3.0
(Parameter estimates from the second round of the Kmenta procedure)

	Prices	H. Size	Tot. Exp.	Lag. Exp.
Food	.092	.879	-.350	.042
Housing	-.013	3.191	1.909	-.194
Apparel	-.178	-.475	2.057	.042
Transp.t	-.254	-3.618	3.489	-.040

Food has a positive price elasticity for both sets of estimates, but the elasticity obtained in the second round is much smaller in absolute value. A positive price elasticity is contrary to economic expectations but it can come about in a dynamic context when the consumer is reacting not just to a change that has taken place, but to expectations that changes in a similar direction are to be expected in the near future too. Considering the rapid inflation experienced in the Italian economy during many of the years covered in this study, the expectation of higher prices might have been a prime reason for the observed positive price elasticities.

As for New Zealand the Lagged Expenditure elasticities are rather small, with the possible exception of Housing and Transport in Table 2.4.3 and Housing in Table 2.4.3, and suggest that also for Italy the previous year expenditure does not have a substantial effect on current expenditure. The negative elasticity for Housing, found for both sets of estimates, could have an interesting interpretation: families who have spent a lot on their homes in the previous year will spend less in the current year because if a large proportion of the previous year household operations expenditure was on durables, like white-wares or furnishings, then no more of such items needs to be purchased in the current year.

Finally for Italy, as well as for New Zealand, it is difficult to choose the "best" set of elasticity estimates because not only there appear to be substantial discrepancies between those obtained from the two alternative sets of estimates shown in Tables 2.3.3 and 2.3.4, but we have also obtained a few elasticities the values of which are difficult to explain on economic grounds (e.g. the very large household size elasticity for Housing and Transport obtained from the SUR estimates). Therefore we feel that the results, and the conclusions, derived from the application of the LES model to the Italian data should be considered tentative.

Chapter 3

The Almost Ideal Demand System

3.1 The Theoretical Framework

A new demand model, general enough to be comparable with the linear demand systems discussed in Chapter 2, but with some theoretical and practical advantages over the latter was suggested by Deaton and Muellbauer (1980b) who called it the "Almost Ideal Demand System", or AIDS, for short.

The AIDS system is based on a specific class of preference ordering, originally suggested by Muellbauer (1975 and 1976) in two theoretical papers on consumer behaviour and social preferences, which permits aggregation over consumers, and represent market demand as the result of rational decisions by a representative consumer.

Muellbauer (1975, 1976) gives the conditions for the existence of a representative consumer which allow a more general behaviour than the parallel Engel curves which are required if average demand are to be functions of average total budget expenditure (see Brown and Deaton, 1972, p.1168), and shows that for the h family type and the ith commodity the individual budget share equations have the *generalised linear* form (Muellbauer, 1975, p.526):

$$w_{ih}(y_h,p) = v_h(y_h,p)\, A_i\,(p) + B_i\,(p) \qquad (3.1.1)$$

where it must be $\Sigma_i\, A_i = 0$ and $\Sigma_i\, B_i = 1$.

In the special case where y is independent of prices and the budget shares depend on log(y), the share equations become:

$$w_{ih}(y_h,p) = \log y_h\, A_i\,(p) + B_i\,(p) \qquad (3.1.2)$$

and Muellbauer calls them PIGLOG (price independent, generalised linear and logarithmic) and defines the minimum expenditure necessary to attain a specific level of utility u, at given prices p:

$$\log C(u, p) = (1-u)\log\{A(p)\} + u\log\{B(p)\} \tag{3.1.3}$$

where A (p) and B (p) are linear homogeneous concave functions.

In (3.1.3) u lies between 0 (subsistence) and 1 (bliss) so that the positive, linear and homogeneous function $A(p)$ can be regarded as the cost of subsistence, and $B(p)$ the cost of bliss (see Deaton and Muellbauer, 1980, p:313).

By taking specific functional forms for log $A(p)$, and log $B(p)$, we can derive the AIDS empirical cost function. Deaton and Muellbauer (1980b) suggested the following expressions:

$$\log A(p) = \alpha_0 + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j \tag{3.1.4a}$$

$$\log B(p) = \log A(p) + \beta_0 \prod_k p_k{}^{\beta k} \tag{3.1.4b}$$

which generate flexible cost functions with a large enough number of parameters so that, at any single point, their first, and second derivatives in prices and utility can be set equal to those of an arbitrary cost function. By substituting (3.1.4) into (3.1.3), we obtain the AIDS cost function:

$$\log C(u, p) = \alpha_0 + \sum{}_k \alpha_k \log p_k + \tfrac{1}{2}\sum{}_k \sum{}_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod{}_k p_k^{\beta_k} \tag{3.1.5}$$

Equation (3.1.5) is linearly homogeneous in prices provided that its parameters satisfy the conditions:

$$\sum_i \alpha_i = 1, \quad \sum{}_j \overset{*}{\gamma_{kj}} = \sum{}_k \overset{*}{\gamma_{kj}} = \sum{}_j \beta_j = 0 \tag{3.1.6}$$

The demand functions for each commodity can be derived from the cost function (3.1.6) by computing its price derivatives (the so-called Shephard's Lemma, see Shephard, 1953, p.17) which are the quantities demanded. The demand functions in budget share form, and as functions of prices, and utility (see Deaton, and Muellbauer 1980, p. 313), thus, are:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_0 \prod_k p_k^{\beta_k} \qquad (3.1.7)$$

where $\gamma_{ij} = 1/2 \left(\gamma^*_{ij} + \gamma^*_{ji} \right)$, w_i is the budget share of commodity i (i=1, ..., n), and pj is the price of the jth commodity (j=1, ..., n). For a utility maximising consumer, total expenditure y is equal to $C(u,p)$, and, by reversing this equality, we can obtain utility as a function of prices, and expenditure. Reversing (3.1.5) in this way, and substituting it in (3.1.7) we finally obtain the AIDS demand functions in prices, and expenditure:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(y / P) \qquad (3.1.8)$$

where P is a price index defined as

$$\log P = \alpha_0 + \sum_i \alpha_i \log p_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j \qquad (3.1.9)$$

Substituting (3.1.9) into (3.1.8) gives the non-linear equation we can use to estimate the model parameters:

$$\begin{aligned} w_i = (\alpha_i + \alpha_0 \beta_i) + \sum_j \gamma_{ij} \log p_j + \beta_i \Big\{ & \log y - \sum_k \alpha_k \log p_k - \\ & - \tfrac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j \Big\} \end{aligned} \qquad (3.1.10)$$

The model's economic properties of additivity, homogeneity, and Slutsky symmetry (see Deaton - Muellbauer, 1980, p.314), for all i, j = 1,2,..., n, respectively imply the following parameter restrictions :

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0, \quad \sum_i \beta_i = 0, \quad \gamma_{ij} = \gamma_{ji} \qquad (3.1.11)$$

Under the constraints (3.1.11) the model represents a system of share demand functions which adds up to total expenditure (i.e. $\Sigma_i w_i = 1$), and in the absence of changes in relative prices, and total real expenditure, maintains constant expenditure shares.

The model is made non-linear by the form of the price index P but can be easily linearised by substituting P with some proportional approximation. Deaton, and Muellbauer (1980, p.316) suggest the Stone index

$$\text{Log } P^* = \sum_i w_i \log p_i \qquad (3.1.12)$$

which transforms (3.1.10) into the simpler form:

$$w_i = \alpha_i^* + \sum_j \gamma_{ij} \log p_j + \beta_i \log(y / P^*) \qquad (3.1.13)$$

where $\alpha_i^* = (\alpha_i - \beta_i \log d)$ for $d = P/P^*$.

Equation (3.1.13) has proved to be a very good approximation[1] of (3.1.10), (see Deaton and Muellbauer, 1980), but if we use single equation *constrained* OLS to estimate it under constraints (3.1.11), then we obtain estimates that are not ML efficient, and do not satisfy adding-up any more. Therefore Deaton and Muellbauer (1980, p. 317) suggest that (3.1.13) should be used only to select the appropriate parameter restrictions, and then the whole system should be re-estimated simultaneously in its non-linear form (3.1.10).

3.2 The Estimation of the AIDS Model

3.2.1 The Empirical Model Assumptions

The budget share consumption equations (3.1.10) are non-linear in the parameters, and can be represented as:

$$w_{iht} = f_i(X_{ht}, \theta) + \varepsilon_{iht} \quad (i = 1,\ldots,n) \qquad (3.2.1)$$

where w_{iht} is the observed consumption of commodity i (budget share of total expenditure) by household h in period t, X_{ht} is the matrix of explanatory variables consisting of Total Expenditure, the price vector p_i and the household size vector[2] s_{ht}; θ is the parameter vector consisting of the AIDS parameters, and the household size parameters δ_i, and finally ε_{iht} is a vector of stochastic errors, $N(0,\Omega)$ distributed[3] .

We also assume that ε_{iht} is independent of X_{ht}, and that, for each h , it is:

$$\mathrm{Cov}(\varepsilon_{iht}, \varepsilon_{ihs}) = 0 \qquad \text{for } t \neq s \qquad (3.2.2)$$

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[1] On the possible introduction of bias in the estimates of the AIDS parameters when using this approximation see Pashardes, 1993. A demographically augmented linear version of the AIDS model has also been used by Rossi, 1988 to analyse the consumption behaviour of Italian households from aggregated data.

[2] As shown in Chapter 2, household size enters model (3.2.1) linearly. A non-linear form did not perform any better. For an Equivalence Scales approach to introducing demographic effects into an AIDS type demand system, see Ray, 1986.

[3] For an application of the AIDS model under an assumption of autoregressive errors, see Xepapadeas and Habib, 1995.

The parameters were estimated by a ML procedure[4] based on a modified Newton optimisation algorithm that approximates the inverse of the matrix of second derivatives, the Hessian, of the objective function at each iteration step by adding to it a correction matrix. This modified matrix provides the change to the parameter estimates at each successive iterative step (see Judge et al., 1985, App. B.2.4). As the model converges, the latest approximation of the Hessian is used to estimate the covariance matrix of the estimates. This method is fast, and adaptable to most types of functions, but sometimes when the surface of the objective function - in our case the log-likelihood (LL) function for the sample data - is irregular, or "lumpy", with many local maxima, it might converge to saddle points, and then follow the ridge[5] never reaching convergence (See Cossarini and Michelini, 1971).

A drawback of this method is that there is no certainty that the estimation procedure will reach convergence, and even when it does, there is no certainty that the parameter estimates correspond to a global maximum of the LL function. Therefore, it is imperative to re-estimate the model a few times with different sets of parameter starting values to verify that a global maximum has been attained, and that the estimates are effectively ML. If different starting values regularly generate different final estimates, the estimation iterative process is inherently unstable, and its

[4] Part of the SHAZAM econometric package in its "Power Mac Version 7". All computations were performed on a Power Macintosh 6100/60 computer.

[5] We found an interesting example of such a likelihood function when estimating the Linear Expenditure System for the Italian data by the Kmenta procedure in Chapter 2. As explained there the estimation proceeds in steps by successively estimating the variance-covariance matrix from the OLS residuals and then using it for a GLS estimation of the model parameters. The procedure can be repeated over and over again until two successive GLS estimates of the parameters are close enough to be accepted as identical and therefore to represent the "true" estimates. The solution is not found by maximising an objective function but relies on the fact that successive GLS covariance matrices will become more and more similar as they succesively generate one another. In the case of the Italian data the procedure needed 188 iterations to converge (instead of the usual 20-25) and the likelihood function kept increasing from one set of estimates to another up to the 49th iteration, hit a ridge between the 50th and the 58th, decreased up to the 165th and then increased again up to its maximum at the 188th in the following pattern:

Iteration	Value of the LL	Iteration	Value of the LL
42	2793.82	75	2793.90
49	2793.92	100	2793.86
50	2793.93 **	125	2793.82
53	2793.93	150	2793.75
55	2793.93	165	2793.87
58	2793.93 **	175	2797.24
59	2793.92	180	2802.54
65	2793.91	188	2802.80

An attempt to find a ML estimator by a gradient method in a situation like the one described above would be likely to generate local maximum estimates only, possibly in the ridge region marked by the asterisks.

"chaotic"[6] behaviour must cast serious doubts on the suitability of the model to fit, and explain the data.

We initially estimated the full AIDS system of four equations without imposing any of the constraints in (3.1.11), and tested all of them. Firstly, we tested all the constraints together with a joint Wald test; then we tested for the homogeneity and symmetry constraints separately. Both constraints were clearly rejected at any level of probability.

As a consequence of the adding-up condition, which is an essential part of any budget share model like (3.2.1), Ω is singular. To overcome this difficulty, we have two alternatives: either to constrain Ω itself (see for example Winters, 1984), or to delete one equation from the demand system (see Barten, 1969). We chose the latter solution, as we felt that it was less arbitrary than to impose ad hoc restrictions on Ω, and during estimation we deleted Equation 3 (Housing Operations). All the parameters of Equation 3 appear in the other equations as well, and can therefore be obtained by constrained estimation; but β_3 and δ_3 will have to be computed separately as residuals. From condition (3.1.11) it follows that $\beta_3 = (1 - \Sigma_i \beta_i)$, and to satisfy adding-up it must be $\delta_3 = (1 - \Sigma_i \delta_i)$.

3.2.2 The Estimation Results for New Zealand

For the New Zealand data, the estimation of the AIDS model with only the adding-up constraint proved almost impossible. For the few times convergence was achieved, either the parameter values were unacceptable (too large or too small), or the standard errors were meaningless (again too large or too small) or both. In most cases, the procedure did not converge, even after thousands of iterations[7].

The situation improved substantially after we imposed the symmetry condition $\gamma_{ij} = \gamma_{ij}$ (for all i, j = 1,...,n), and estimation became much easier with the iterative procedure converging in a reasonable number of iterations from most sets of starting values[8].

[6] For a definition of chaotic systems as crucially dependent on initial conditions see Devaney R L, 1992, Ch 10. A discussion of the non-convergence of the Newton-Raphson iterative method from the perspective of chaotic dynamic systems can be found in Ch 13.

[7] Nelson (1988, p.1305) reports a similar lack of convergence in estimating AIDS parameters for US data when more than three commodities were considered.

[8] The sets of starting parameter values we used more often were zeros and ones, or the parameter estimates obtained from the linearised version of the AIDS model (3.1.13).

However, even after imposing symmetry, the Maximum Likelihood (ML) estimates for the New Zealand data were extremely sensitive to the set of parameter values used to start the iterative estimation procedure, different sets of starting values often generating totally different ML estimates. Although the values of the log-likelihood function (LL) corresponding to different sets of estimates were usually different, a fact allowing us to identify the set of estimates with the highest likelihood, most values of the maximised LL function were extremely close to one another, and there does not appear to be a well defined maximum, but lots of local maxima. An equally unsatisfactory result of the estimation procedure is that sometimes changes in the convergence criterion generate different sets of final estimates.

In spite of the above difficulties, we are fairly confident that the parameter estimates we report in Table 3.1 are indeed ML estimates, as the LL function values associated with them are the highest we have obtained, within the parameters' domain, over a very large number of trial estimation runs, in which we have used different sets of parameter values to start the iterative procedure, and different convergence criteria.

The parameters which proved most unstable, and difficult to estimate were the price parameters, $\gamma_{i,}$, and the α_i intercepts. Contrary to what is sometimes stated in the literature (e.g. Deaton-Muellbauer, 1980, p.316, also Winters 1984, p. 248), the α_0 parameter - the intercept of the price index equation (3.1.9) - proved relatively easy

Table 3.1

Parameter Estimates for the Symmetric AIDS Model for New Zealand and Italy [a b]

	New Zealand		Italy	
α_0	−185.53	(53.547)	-224.66	(94.082)
α_1	19.734	(5.4891)	36.373	(14.6130)
β_1	− .1051	(.00432)	-.1560	(.00057)
γ_{11}	−1.9981	(.59973)	-5.3531	(2.4284)
γ_{13}	−.6307	(.17977)	-.5920	(.29378)
γ_{14}	.2018	(.11063)	1.7540	(.78837)
γ_{15}	2.4270	(.68468)	4.1910	(1.7340)
δ_1	.0450	(.00163)	.0445	(.00175)
α_3	7.9826	(1.81941)	1.6920	(1.3668)
β_3	−.0379		-.0087	
γ_{33}	.0915	(.10908)	-.2158	(.10197)
γ_{34}	.2284	(.11553)	.2081	(.07550)
γ_{35}	.3108	(.20042)	.5997	(.24252)
δ_3	−.0075		-.0326	
α_4	−2.9206	(.99240)	-11.584	(4.7929)
β_4	.0147	(.00203)	.0511	(.00143)
γ_{44}	−.1499	(.10591)	-.4098	(.26057)
γ_{45}	−.2803	(.11887)	-1.5522	(.58534)
δ_4	.0051	(.00114)	-.0085	(.00066)
α_5	−23.796	(6.3989)	-25.481	(10.3841)
β_5	.1283	(.00781)	.1136	(.00205)
γ_{55}	−2.4575	(.79063)	-3.2385	(1.34901)
δ_5	−.0426	(.00270)	-.0034	(.00104)
LL [c]	1341.844		2290.260	
R1 [d]	.8139		.8638	
R4 [d]	.3956		.8074	
R5 [d]	.6688		.8733	

a δ_i are the demographic parameters
b Parameters β_3 and δ_3 computed as residuals
c LL is the value of the Log-Likelihood function
d R square between observed and predicted.

Figures in brackets are standard errors. There are no equation 2 parameters as the Housing commodity is not considered. See chapter 1, on data.

to identify. We tried to set α_0 to some a-priori value, as suggested in the literature, but, as a result of this, estimation became even more difficult.

The β.s, and δ.s, the total expenditure, household size, and the time trend[9] parameters were quite robust, and their values did not change much among estimation runs, whatever the starting points, and the convergence criterion. However, all the time trend parameters, except the one in the equation for apparel, were insignificant.

3.2.3 The Estimation Results for Italy

For the Italian data, the estimation procedure was marginally better, and we are confident to have achieved a maximum of the LL function, but once again the price parameter estimates proved to be the most difficult to estimate. As in the New Zealand case, the total expenditure, household size, and time trend parameters were quite robust, and had very small standard errors of estimate; with the exception of the time trend parameters for Food and Housing Operations which were insignificant.

The parameter estimates corresponding to the highest value of the LL function attained over a large number of trials are shown in Table 3.1 for both countries. We must remember, however, that the estimates of the price parameters for both countries, especially for New Zealand, must be treated with great caution. The "chaotic" characteristics of the estimation procedure are an indication of the possible inadequacy of the AIDS model to describe household consumption behaviour.

3.3 The Elasticities

3.3.1 The Elasticity Computations

From the parameter estimates shown in Table 3.1 we have computed, for both Italy and New Zealand, the elasticity of consumption with respect to its own price, the prices of the other commodities (the cross-price elasticities), the household size, and total expenditure. All the elasticities were obtained by numerical differentiation of the

[9] As we did for the LES model we included in all equations a trend variable with values from 1 to 9 for New Zealand and 1 to 13 for Italy. The inclusion of a time trend is suggested to eliminate possible bias in the estimation of β_i, see Blundell,1988, p.28.

functional relationships defining the point elasticities [10] of consumption with respect to the above explanatory variables.

The elasticity of the share expenditure on the ith commodity w_i versus the price of the jth commodity p_j, for example, can be obtained from :

$$e_{ij} = [\delta\ (\log w_i) / \delta\ (\log p_j)] \quad (3.3.1)$$

where the derivative are calculated numerically at the variables' sample means, and other selected values.

From the share expenditure elasticities (3.3.1), it is possible to obtain the familiar own-price elasticities, which are defined as the change in the quantity demanded of a commodity which follows a marginal change in its price, other things being the same. A simple relationship exists between share and quantity elasticities. In symbolic terms if we define the quantity elasticity as

$$E_{ii} = [\delta\ (\log q_i) / \delta\ (\log p_i)] \quad (3.3.2)$$

then it is: $E_{ii} = e_{ii} - 1$.

To obtain the above relationship, between the quantity and share price elasticities, consider the definitions of the share variables appearing in (3.2.1):

$$w = (pq)/y \quad (3.3.3)$$

take the logarithms of both sides of (3.3.3), and differentiate with respect to log p to obtain the expression:

$$[\delta(\log w) / \delta(\log p)] = 1 + \delta(\log q) / \delta(\log p) - \delta(\log y) / \delta(\log p) \quad (3.3.4)$$

where the left hand side term is e, the second term on the right hand side is E, and that $\delta(\log y) / \delta(\log p) = 0$ because income is independent of prices [11].

[10] Given a function y = f(x), the point elasticity of y with respect to x is : $E_{y,x} = \delta \log y / \delta \log x$, Chiang, 1984, p.305.

For the cross price, household size and income elasticities it is $E = e$ because all these variables, as they appear in model (3.1.8), and in its demographically extended version (3.2.1), are independent among themselves and therefore their cross derivatives are zero. Only the quantity derivatives remain[12].

One of the advantages of computing the elasticities by numerical differentiation is that, as a by-product of the elasticity computations, we can obtain the Slutsky matrix - with the generic element $S_{ij} = (\partial q_i / \partial p_j)$ - from the matrix of the first derivatives of the share expenditures $s_{ij} = (\partial w_i / \partial z_j)$ because these two matrices are linked by the simple relation: [13]

$$S_{ii} = (y\, s_{ii} - q_i) / p_i \quad \text{and} \quad S_{ij} = (s_{ij}\, y) / p_i \tag{3.3.5}$$

For all the variable values at which we computed the elasticities we also computed the Slutsky matrices but, neither for Italy nor for New Zealand were the Slutsky matrices negative semi-definite,[14] thus pointing to a violation, by the empirical demand systems, of the customary assumption of concavity of the utility function.

In Part A of Table 3.2, and Table 3.3, we show the price elasticities - for New Zealand and Italy - computed at the means, for the four commodities considered. In Part B of Table 3.2, and Table 3.3, we report the household size, and total expenditure elasticities computed first at the sample means, and then for two other sets of consumption values: one taken midway between the lowest observation and the mean, the other midway between the mean and the highest observation. The first set of such values is defined as:

$$Q_1 = \text{lowest observation} + 0.25\ (\text{Range}) \tag{3.3.6}$$

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11 I am grateful to Ranjan Ray for discussing with me the relationship between share and quantity elasticities.

12 For household size for example it would be: $[\delta (\log w) / \delta (\log h)] = \delta (\log p) / \delta (\log h) - \delta (\log y) / \delta (\log h) + \delta (\log q) / \delta (\log h) = \delta (\log q) / \delta (\log h)$ because household size enters the demand equations as an additive and linear element, independent of income and prices.

13 If we substitute into the relationship $E = (e - 1)$ the expressions $E = (\delta q / \delta p)(p / q)$, and $e = (\delta w / \delta p)(p / w)$ we obtain : $S (p / q) = [\, s (p / w) - 1]$. Substituting into this last expression the share variable definition $w = pq/y$, we obtain (3.3.5).

14 The non-negativity of the Slutsky matrices was also observed by Rossi,1988, p.1312.

where Range = (highest observation - lowest observation). The second set is defined as:

$$Q_3 = \text{lowest observation} + 0.75 \text{ (Range)}. \qquad (3.3.7)$$

Finally, in Part C, and D of Table 3.2, and Table 3.3, we show the household size, and total expenditure elasticities separately for all the different household sizes considered in our data. These elasticities have been computed at the means of the sub-samples obtained by grouping together the households of equal size.

3.3.2 The New Zealand Elasticities

There are several interesting features of the estimated elasticities. To consider the price elasticities first. For Food and Housing Operations the own-price elasticities have the "right" (negative) sign, and they are all less than unity. The consumption of these two commodities is price inelastic: a marginal increase (decrease) in their prices will generate a less than proportional fall (rise) in their consumption. When the prices of these two commodities increase (decrease) the consumers will decrease (increase) the amounts purchased less than proportionately and therefore will spend a larger (smaller) share of their budget on them. This is a logical conclusion as the amounts of food and housing needed by a household cannot be easily and rapidly changed.

Transport has negative a own-price elasticity which is very close to unity. This implies that marginal changes in the cost of transport will induce almost proportional changes in the demand for transport. Because Transport is an almost *unit elastic* commodity its total expenditure will remain more or less constant when its price changes.

For Apparel both the own-price, and all the cross-price elasticities, are negative, and larger than unity. The expenditure on Apparel is highly elastic, i.e. it is very sensitive to price changes, both with respect to its own price and to the prices of the other three commodities considered here - New Zealanders give a very low priority to clothing expenditure.

TABLE 3.2

AIDS Model, Price, Total Expenditure and Household Size Elasticities for New Zealand

Part A: Whole sample of 180 observations, price elasticities computed at the means[(a)]. Mean household size FS = 2.5

Prices of:	Food	Housing	Apparel	Transport
Com.dities:				
Food	-.794	.554	-.280	-.200
Housing	.118	-.605	.117	-.593
Apparel	-1.626	-2.041	-1.965	-1.281
Transport	-.899	-1.644	.309	-.965

Part B: Whole sample of 180 observations, household size and total expenditure elasticities computed at the means [(a)]and the two "quartiles"Q_1, and Q_3[(b)]. Mean household size FS = 2.5

	Household Size Elasticities			Total Expenditure Elasticities		
Com.dities	Means	Q_1	Q_3	Means	Q_1	Q_3
Food	.370	.535	.316	-.346	-.500	-.295
Housing	-.121	-.126	-.090	-.140	-.146	-.104
Apparel	.145	.208	.113	.168	.241	.130
Transport	-.315	-.384	-.252	.378	.462	.303

Part C: Household size elasticities for the four sub-samples of 45 observations each, obtained by grouping households according to size[(a)]

Com.dities	FS=1	FS=2	FS=3	FS=4
Food	.1668	.3135	.4320	.5208
Housing	-.0446	-.0947	-.1485	-.2103
Apparel	.0652	.1261	.1689	.2016
Transport	-.1185	-.2390	-.3833	-.5585

Part D: Total Expenditure elaticities for the four sub-samples of 45 observations each, obtained by grouping households according to size[(a)]

Com.dities	FS=1	FS=2	FS=3	FS=4
Food	-.3898	-.3662	-.3364	-.3042
Housing	-.1295	-.1375	-.1438	-.1527
Apparel	.1879	.1818	.1623	.1453
Transport	.3566	.3595	.3844	.4200

(a) The consumption mean values used to compute the elasticities in Part A are those of the whole sample of 180 observations. In Parts B, C and D the means are those of the four sub-samples, with 45 observations each, grouping the households of equal size .

(b) Q_1 = lowest observation + 0.25 Range, Q_3 = lowest observation + 0.75 Range

It is interesting to note here that while the cross-elasticity between Apparel and the price of Food is negative and elastic, the cross-elasticity between Food and the price of Apparel is negative but inelastic. This asymmetry in consumer reactions to price changes reflects the more essential nature of food compared to apparel, it is easier to delay or reduce the purchase of clothing than the purchase of food.

A similar asymmetry in consumer reactions to price changes also exists between Housing and Transport, on the contrary the cross-elasticity between Transport and the price of Housing is negative and elastic, the cross-elasticity between Housing and the price of Transport is negative and inelastic. An increase (decrease) in the cost of housing will generate a more than proportionate reduction (increase) in the demand for transport, on the contrary an increase (decrease) in the cost of transport will generate a less than proportionate reduction in the demand for housing services and durables. Consumers, when allocating their budget resources, give priority to their housing needs, rather than to their transportation needs.

The Household Size elasticities (Table 3.3, first three columns of Part B, and Part C) are all less then unity (i.e.consumers' reactions to marginal household size changes are inelastic), those for Food and Apparel are positive, those for Housing Operations and Transport are negative.

The demand for Food moves in the same direction as household size, but much less than proportionately. For households at the lower levels of expenditure (Q_1 variable values) the quantity of food purchased changes by more than half as much as the marginal increase in household size; at higher levels of expenditure (Q_3 variable values), the change is about a third as much as that in household size.

This less than proportionate change in the purchase of food, with respect to marginal changes in household size, seems to indicate the existence of some economies of scale in food expenditure - as the expenditure on food grows less than proportionately than the household size, larger households will tend to spend a smaller proportion of their budgets on food. The economies of scale decrease as the household gets larger (Table 3.2 Part C). The smaller the household, the greater the savings achievable for a marginal increase in size: one-member households will increase their food consumption by about a fifth of their size increase, four-member households will increase their food consumption by about a half of their size increase. This, once again, is a reasonable finding as every householder experiences that the food cost per person decreases as the household gets larger, but at a decreasing rate.

The consumption of Apparel behaves in a way similar to the consumption of Food, but the absolute values of the Household Size elasticities for Apparel are half as large as those for Food: the changes in the consumption of clothing items, due to marginal changes in household size, are much less than the corresponding changes in food consumption. The demand for apparel is very inelastic with respect to changes in household size.

For Housing Operations and Transport, the Household Size elasticities are negative; those for housing being much smaller than those for transport. These indicate that the quantities of these two commodities consumed by the households decrease as household sizes increase, but less than proportionately. In the case of Transport, the larger the household the greater the marginal decrease in consumption as the household size increases (see Table 3.2 Part C). A possible reason why smaller households do not reduce their expenditure on transport as much as larger households is that while larger households might have either a large car or more than one car, and can find many ways to reduce their expenditure on transport, smaller households are likely to have only one small car and might find it difficult to reduce their transportation costs at all.

For all the commodities considered here, as the household size increases (see Table 3.2 Part C) the household size elasticities get larger in absolute value, the consumers' reactions to changes in the size of their households get more responsive as the household size increases: larger households react more strongly to changes in their size than do smaller households.

Looking now at the Total Expenditure elasticities (Table 3.3, last three columns of Part B, and Part D) we find that Food is negative and inelastic, thus confirming the Engel's Law which predicts that expenditure on food will decrease as income increases. In the case of New Zealand the proportionate decreases are larger at the lower levels of expenditure (Q_1 variable values) than at the higher levels of expenditure (Q_3 variable values).

Housing Operations too has a negative Total Expenditure elasticity, but quite small in absolute value. The expenditure on this commodity decreases as the total expenditure increases, but much less than proportionately, and it decreases in a similar way at all levels of expenditure considered (Q_1's, the means, and Q_3's).

The Total Expenditure elasticities for Apparel and Transport are positive and less than unity, and their absolute values decrease as the level of total expenditure increases. The households' response to increasing total expenditure gets less as the level of total expenditure gets higher.

Finally the Total Expenditure elasticities remain almost constant as the household size increases (see Table 3.2 Part D). For the New Zealand households the most important factor in deciding what amounts to purchase of the four commodities considered here, is not so much their size, but their level of total expenditure.

3.3.3 The Italian Elasticities

The price elasticities for Italy are difficult to explain and justify as they appear to have little economic rationale. All own-price elasticities, except Transport, are positive, with the elasticity for Housing Operations almost as large as five.. All the cross-elasticities are negative but most of them are substantially larger than unity in absolute value. Such highly elastic responses to price changes seem unlikely for essential commodities like those considered here.

TABLE 3.3

AIDS Model, Price, Total Expenditure and Household Size Elasticities for Italy

Part A: Whole sample of 325 observations, price elasticities computed at the means [(a)]. Mean household size FS = 2.9845.

Prices of:	Food	Housing	Apparel	Transport
Com.dities:				
Food	1.208	-1.660	-1.567	-1.252
Housing	-5.137	4.689	-.568	-.999
Apparel	-2.081	-.731	.350	-2.040
Transport	-3.361	-1.330	-3.212	-2.362

Part B: Household size and total expenditure elasticities for the whole sample of 325 observations. Computed at the means[a] and the two "quartiles" Q_1 and Q_3[b]. Mean household size FS = 2.9845

	Hous. Size Elasticities			Tot. Exp. Elasticities		
Com.dities	Means	Q_1	Q_3	Means	Q_1	Q_3
Food	.336	.538	.231	-.395	-.632	-.272
Housing	.111	.378	.201	-.024	.035	-.019
Apparel	-.266	-.431	-.171	.536	.866	.343
Transport	-.067	-.096	-.037	.749	1.078	.414

Part C: Household size elasticities for the five sub-samples of 65 observations each, obtained by grouping households according to size.

Com.dities	FS=1	FS=2	FS=3	FS=4	FS=5
Food	.145	.236	.329	.415	.480
Housing	.072	.163	.277	.386	.523
Apparel	-.075	-.181	-.278	-.380	-.485
Transport	-.024	-.047	-.062	-.086	-.113

Part D: Total Expenditure elasticities for the five sub-samples of 65 observations each, obtained by grouping households according to size.

Com.dities	FS=1	FS=2	FS=3	FS=4	FS=5
Food	-.508	-.414	-.384	-.364	-.337
Housing	-.020	-.023	-.026	-.027	-.029
Apparel	.449	.541	.556	.570	.583
Transport	.791	.782	.697	.719	.761

(a) The consumption mean values used to compute the elasticities in Part A were those for the whole sample of 325 observations. In Parts B the means were those of the four sub-samples, with 65 observations each, grouping the households of equal size .

(b) Q_1 = lowest observation + 0.25 Range, Q_3 = lowest observation + 0.75 Range

The elasticities for Transport in particular are unrealistically high and, as we have already pointed out in Section 2.4.3, they are in total contrast to available empirical evidence. It is a well recognised fact that in Italy increases in the cost of transport - a large proportion of which consist of excise taxes on automotive fuel - tend to have very limited and temporary effects on its consumption. This well known inelastic price response[15] is one of the reasons why increasing fuel taxes is one of the easiest ways for the Italian treasury to collect extra revenue.

The unsatisfactory nature of the price elasticities derived from the parameters of the AIDS model for Italy, and the computational problems involving its estimation, raise questions about how suitable this model is in capturing, with any accuracy, the consumption behaviour of Italian households with respect to changes in the prices of the commodities included in their budgets. Considering that the AIDS model is widely used in the analysis of consumer demand, our findings prompt caution when applying

[15] Rossi, 1988, Table 2, reports for Transport and Communications a compensated price elasticity of -.05

it to budget consumption data and suggest the need for a very careful scrutiny of the statistical characteristics of the data, and their suitability[16] to estimate this type of sophisticated econometric models.

The household size elasticities (Table 3.3, first three columns of Part B, and Part C) are more interesting: they are all less then unity (i.e. consumers' reactions to marginal household size changes are inelastic) and show some interesting differences in behaviour between Italian and New Zealand households.

For Italy, as for New Zealand, the Food household size elasticity is small, and positive - consumption of food changes in the same direction as household size but much less than proportionately. The expenditure on Housing Operations is inelastic for both countries, but it is positive for Italy and negative for New Zealand - expenditure on housing changes in the same direction as household size in Italy, and in the opposite direction in New Zealand, but much less than proportionately in both countries.

Apparel too is inelastic in both countries, but it is negative in Italy and positive in New Zealand. The Transport household size elasticity is negative and inelastic in both countries, but its absolute value is almost five times as large for New Zealand as it is for Italy, where this commodity appears to be totally inelastic with respect to the size of the household.

For the Italian households, as it was for the New Zealand ones, there appears to be some economies of scale in the expenditure for Food. As the household size grows marginally, the quantity of food consumed increases less than proportionately. For Italian households the economies of scale on food expenditure are smaller at high levels of expenditure (Q_3 values), and larger at lower levels (Q_1 values). At the means, the economies of scale achieved when the household size increases, are almost identical for the two countries.

16 A critical evaluation of the quality of Italian household consumption survey data appears in Rossi (1988) where, in an application of the AIDS model to Italy, he rejects expenditure survey data - based on consumption categories almost identical to ours - in favour of macro-economic data. On page 1307 he says: ... *the methodology of the available official consumers' expenditure survey does not allow a straight-forward comparison of observations in different time periods. This is especially true at the level of disaggregation adopted in this paper.*

Also for Housing Operations, for which the increase in consumption is less than proportional to the increase in household size, there appear to be the possibility of some economies of scale. An increase in consumption, less than proportional to the increase in the size of the household, may be explained by a more intensive use by the larger family unit of *public* items like heating, white-wares or television sets. The sharing of this type of goods by more household members will then result in economies of scale (see Nelson, 1988, p. 1301-02).

It might be interesting here to compare the reactions of Italian and New Zealand households to increases in the household size. As household size increases both Italian and New Zealand households increase consumption of Food and decrease consumption of Transport, but their reactions are the opposite with respect to Housing (decreasing in New Zealand and increasing in Italy), and to Apparel (increasing in New Zealand and decreasing in Italy). The consumption behaviour of Italian and New Zealand households is similar but not identical.

To summarise, we have found that for Italy all Household Size elasticities are less than one in absolute value, and this finding accords with expectations for the type of commodities considered in this study. A doubling of the household size, for example, would not normally double the expenditure on food or housing; particularly when the additional household members are often children, which they are in the New Zealand HEIS data, and are likely to be so in most of the Italian households. We have also found that as the household size increases the consumption responses to changes in the size of the household becomes more and more elastic: in Italy, like in New Zealand, larger households react more strongly to changes in their size than smaller households.

Coming now to the Total Expenditure elasticities (Table 3.3, last three columns of Part B, and Part D) we find that the Food and Housing elasticities are negative and less than unity, and that Apparel and Housing are positive, and also less than unity: the consumption of all four commodities is inelastic with respect to total expenditure.

The fact that food consumption is negative and inelastic with respect to total expenditure shows that Engel's Law is at work in Italy as well as in New Zealand. The elasticities' absolute values are much larger for "poor" households (Q1 values) than they are for "rich" households (Q_3 values): the decrease in food consumption as total expenditure increases, is much faster for households with low levels of expenditure than for households with high levels of expenditure.

The Total Expenditure elasticity for Housing Operations is extremely low, and it remains low at all levels of expenditure, but it is negative at the means and at high levels of total expenditure (the Q_3 variable values), and positive at low levels of total expenditure (the Q_1 variable values). However, as shown by the results reported in Part D of Table 3.3, the Total Expenditure elasticity for housing remains almost the same at all household sizes: spending decisions on housing do not seems to depend on household size but on the household's level of total expenditure.

Apparel, and Transport have positive Total Expenditure elasticities, with higher absolute values at low levels of expenditure, so much so that at Q_1 the demand for Transport becomes elastic. The consumption of these two commodities - more discretionary in character than food or housing - grows strongly when total expenditure grows. This reflects quite accurately social attitudes in Italy, where to be smartly dressed, and drive a smart car, are almost considered social "requirements",

As households get larger the Total Expenditure elasticity for Apparel gets slightly larger while the elasticity for Transport remains almost constant, also for these two commodities, as was the case for Housing, spending decisions do not seem to depend on the size of the household, but rather on the level of total expenditure.

As a general observation we might point out that both the Household Size and Total Expenditure elasticities decrease in absolute value as total expenditure increases from Q_1, to its mean value, to Q_3 (see Part B of Table 3.3): the consumption of the four commodities considered in this study, which in a modern society must be considered as essential, tends to react less and less strongly to changes in total expenditure, as total expenditure increases.

The effects of household size are different on the Household Size elasticities and the Total Expenditure elasticities. The Household Size elasticities increase in absolute value as the households get bigger, but still remain much less than one: whatever the household size, a marginal change in it results in a less-than-proportionate change in the household's consumption of all commodities (see Part C of Table 3.3).

On the contrary the Total Expenditure elasticities show almost no change as the family size increases from one member to five (see Part D of Table 3.3). Also for Italy, as it was the case for New Zealand, the Total Expenditure elasticities are not affected by household size.

Chapter 4

Demographically Extended Demand Models

4.1 Introduction

Barten (1964) pioneered the literature on the estimation of demographically extended 'complete demand systems' on family budget data pooled across different survey periods to contain price and family size variations. Most of these studies have been conducted on UK data (see for example Muellbauer, 1977, Pollak and Wales, 1981, Ray, 1983, 1993a and 1993b) since the UK, through its Family Expenditure Surveys, is one of the few countries to have a consistent times series of family budget data containing information on family size and composition.

Until a few years ago, there was, to our knowledge, no literature on preference-consistent demographic demand estimation on household budget data either for New Zealand or for Italy. More recently, studies on household consumption patterns, taking into account demographic effects by estimating preference consistent "complete demand systems", have appeared for both countries (see Chatterjee, Michelini and Ray, 1994, Filiberti, 1994 and Bollino, Perali and Rossi, 1995).

In this chapter, we analyse a few families of such preference-consistent complete demand systems, which we derive from alternative forms of utility functions and estimate their parameters under various restrictive assumptions. We test for linear Engel curves, *and provide evidence on non-linearity in income.* We use information on the number of children in the household to test for demographic effects on demand and check whether similar economic conclusions and model acceptance decisions are supported by different data. We also aim at verifying the effects on empirical results of the "quality" of the data employed.

The plan of this chapter is as follows: Section 4.2 describes the class of non linear demand functional forms (NLPS) and their demographic extensions that are estimated in this study. In Section 4.3, we describe the estimation of the models discussed in Section 4.2, and in Section 4.4, we discuss the tests used in choosing among them. In Section 4.5, we report alternative parameter estimates - obtained by the Generalised Method of Moments - for the preferred models, selected by the testing done in Section 4.4. In Section 4.6, we compute and show the demand elasticities for the preferred models. Finally, in Section 4.7, we discuss the problem of separability, and test for it.

4.2 The Theoretical Framework

4.2.1 Non-linear Preferences Demand System (NLPS)

In a paper on community preferences, Muellbauer (1976, p. 985) suggested a family of expenditure functions he called "PIGL", as their basic assumption is Price Independent Generalised Linearity. These functions assume that income is independent of prices p, and have the underlying cost function:

$$C(u,p) = \left[(A(p))^{\alpha} + u(B(p))^{\alpha}\right]^{\frac{1}{\alpha}} \tag{4.2.1}$$

where A and B are assumed linear, homogeneous and concave in p. In (4.2.1) for cost to be increasing with utility for $\alpha < 0$ as well as for $\alpha > 0$, it is enough to make u an increasing function of α, e.g. if u is replaced by αu (see Muellbauer 1976, p. 985).

Blundell and Ray (1984) have proposed a non-linear preferences demand system (NLPS) based on an expenditure function, which is a variant of (4.2.1) but still a member of the PIGL family of functions:

$$E(u,p) = \left[A(p,\alpha) + B(p,\alpha)u\right]^{\frac{1}{\alpha}} \qquad (0 < \alpha \leq 1) \tag{4.2.2}$$

where $A(p,\alpha)$ and $B(p,\alpha)$ are concave and homogeneous of degree α in prices p, and u is some utility index. The parameter α, if different from unity, measures the non-linearity of the Engel curve, and also allows for non-separable behaviour. It is apparent that, for $\alpha = 1$, (4.2.2) specialises to the class of Linear Preference Systems (LPS) underlying the Gorman Polar Form family of expenditure systems:

$$E(u, p) = A(p) + B(p)u \tag{4.2.3}$$

of which the Linear Expenditure System (LES) is the most well known. When (4.2.2) is multiplicative in p, then it reduces to (4.2.1), otherwise it will generate different demand systems (for an example see, Gorman, 1976).

For $A(p,\alpha)$ Blundell and Ray choose a functional form concave in p, which is a variant of Diewert's Generalised Leontief form (Diewert, 1971, p. 495)[1]:

$$A(p,\alpha) = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} p_i^{\frac{\alpha}{2}} p_j^{\frac{\alpha}{2}} \tag{4.2.4}$$

where the β_i are commodity specific price parameters.

For B(p,α) Blundell and Ray (1984) choose a variant of the well known Cobb-Douglas form:

$$B(p,\alpha) = \prod_i p_i^{\alpha\beta_i} \tag{4.2.5}$$

where the β_i are commodity-specific price parameters.

---

1 $D(z,p) = h(z)\sum_i \sum_j b_{ij} p_i^{\frac{1}{2}} p_j^{\frac{1}{2}}$ for $p_i, p_j, z \geq 0$, where h is a continuous, monotonically increasing function of output z which tends to $+\infty$ as z tends to $+\infty$ and such that $h(0) = 0$ and $B = (b_{ij})$ is a symmetric (n×n) matrix with nonnegative elements.

concave, and they will be increasing in p only under certain conditions (see Blundell-Ray 1984, p. 802).

If we substitute now (4.2.4) and (4.2.5) in (4.2.2), we can eliminate the unobservable utility variable u through use of the indirect utility function implied by (4.2.2), (4.2.4) and (4.2.5), and substitute it with:

$$u^* = \frac{y^{\alpha} - \sum_i \sum_j \gamma_{ij} p_i^{\frac{\alpha}{2}} p_j^{\frac{\alpha}{2}}}{\prod_i p_i^{\beta_i \alpha}} \tag{4.2.6}$$

where y is total expenditure. Applying Shephard's Lemma[2] we obtain (see Blundel - Ray, 1984, p.803) the following uncompensated expenditure shares system:

$$w_i = \sum_{j=1}^{n} \gamma_{ij} z_i^{\frac{\alpha}{2}} z_j^{\frac{\alpha}{2}} + \beta_i \left(1 - \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} z_i^{\frac{\alpha}{2}} z_j^{\frac{\alpha}{2}}\right), \quad \sum_{i=1}^{n} \beta_i = 1, \quad \gamma_{ij} = \gamma_{ji} \tag{4.2.7}$$

where subscripts $i, j = 1, ..., n$ refer to commodities in the household budget, and $z_i = p_i/y$ is 'normalised price', and y is aggregate expenditure. NLPS, besides being a general functional form homogeneous of degree zero in prices and total expenditure and admitting non linear, non-separable behaviour, allows a very simple nested test of linearity by testing for unit α. The linear demand system to which NLPS specialises for $\alpha = 1$ is the Linear Preferences Systems (LPS) which allows for non separability via all $\gamma_{ij} = 0$ for $i \neq j$. A further nested test of separability is carried out by testing H_0: $\gamma_{ij} = 0$ for all $i \neq$ j. The LPS, then, specialises to the restrictive LES.

Moreover, being a member of the PIGL family, NPLS satisfies "consistent aggregation" over individuals (see Muellbauer, 1976) and is therefore well suited for application on the type of grouped data used in this study.

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2 Briefly Shephard Lemma states: if a cost function c(y,p) satisfies a set of general conditions (positive real valued, non-decreasing left continuous in y, positive linear homogeneous, concave in p and differentiable w.r.t. factor prices) then it is:

$$\delta c/\delta p_i\,(y,p) = x_i\,(y,p)$$

where $x_i\,(y,p)$ is a cost minimising bundle of inputs i needed to produce output $y > 0$ for positive factor prices $p_i > 0$. See Shepard, 1953, p.17.

For $\gamma_{ij} = 0$ (all $i \neq j$), NLPS specialises to the Restricted Non-Linear Preference System (RNLPS) functional form, which is non-nested to LPS:

$$w_i = \gamma_{ii} z_i^{\alpha} + \beta_i \left(1 - \sum_k \gamma_{kk} z_k^{\alpha}\right) \tag{4.2.8}$$

The RNLPS functional form allows for both non-linearity and non-separability (see Chatterjee et al., 1994 p.280), but because of the parameter constraints $\gamma_{ij} = 0$ (all $i \neq j$), has limited price flexibility. It is possible to test for separability under the null hypothesis H_0: $\gamma_{ij} = 0$ for all $i \neq j$. The RNLPS in its turn, reduces to the restrictive Linear Expenditure System (LES) under the testable restriction $\alpha = 1$, which provides an alternative nesting link to test for linearity of the demand system.

For this family of models, price flexibility is successively reduced by the increasingly restrictive constraints on the γ parameters as we move from NLPS to RNLPS to LES. The nesting sequence for the NLPS family is shown in Figure 1.

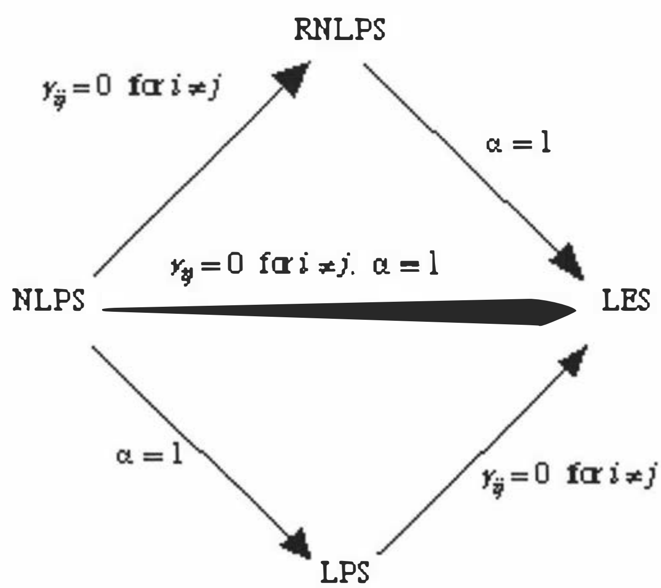

FIGURE 4.1: Nesting Sequence for the NLPS model

4.2.2 Demographically Extended NLPS and RNLPS

In empirical work on pooled household budget data of the type considered here, the demand systems described in Section (4.2.1) need to be demographically extended for data sets containing information on expenditure behaviour by households of different composition for each income group. The demographic extension of demand systems can take many forms (see Browning, 1992 for an extensive review and discussion of this topic), in the present paper, we will limit our analysis to three methods only: Demographic Scaling (DS) proposed by Barten (1964), Demographic Translation (DT) due to Pollak and Wales (1981), and Demographic Cost Scaling (DCS) proposed by Ray (1983)[3].

Demographic Scaling transforms the original cost function (4.2.2) into

$$y \equiv c(u,P,h) = \hat{c}(u, p_1m_1,\dots,p_nm_n) \tag{4.2.9}$$

where P denotes the price vector, and the m's are scaling parameters that depend on demographic variables: $m_i = M^i(h)$, where h represents the household composition. The DS procedure assumes quasi-price demographic effects, it modifies the traditional demand system in the following manner:

$$q_i = m_i\hat{q}_i(y, p_1m_1,\dots,p_nm_n) \tag{4.2.10}$$

Demographic Translation replaces (4.2.2) with

$$y \equiv c(u,P,h) = \sum_i p_i d_i(h) + \hat{c}(u,P) \tag{4.2.11}$$

where the $d_i = D^i(h)$ are the translation parameters. In behavioural terms, DT[4] demographically extends the demand system thus:

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[3] A non-parametric approach to demand analysis is suggested by Lewbel, 1991 and Rimmer and Powell, 1994.

[4] The DT method specializes the Gorman (1975) method which in turn is a generalisation of Barten's DS (See Muellbauer, 1977, p.464). Gorman adds a fixed cost element to Barten's cost function $c(u, p, m) = c(u, p_1m_1, p_2m_2, \dots, p_nm_n)$, where m is the commodity-specific equivalent scale and $u = u(q_i/m_i)$ for $i = 1, \dots, n$. Thus household composition has both a fixed cost effect and a quasi-price effect.

$$q_i(P,y,h) = d_i(h) + \hat{q}_i\left(P, y - \sum_k p_k d_k\right) \qquad (4.2.12)$$

Demographic Cost Scaling replaces (4.2.2) by:

$$y \equiv c(u,P,h) = m(P,h)\hat{c}(u,p_1,\ldots,p_n) \qquad (4.2.13)$$

where m, the general equivalence scale, is dependent on prices and households composition. Note that economic theory requires that m must be homogeneous of degree zero in prices. In behavioural terms, DCS generates the demand system:

$$q_i = \frac{\partial m}{\partial p_i}\frac{y}{m} + m\hat{q}_i\left(\frac{y}{m}, p_1,\ldots,p_n\right) \qquad (4.2.14)$$

DCS implicitly maintains the assumption that the general equivalence scale m, a parameter with considerable policy significance, is independent of reference utility or welfare. As Blackorby and Donaldson (1991) have recently pointed out, such an assumption is needed to interpret m as the "cost of a child".

From equations (4.2.10), (4.2.12) and (4.2.14), omitting the household and time subscripts to simplify notation, we obtain[5], the demand equations for the demographically extended systems DS-RNLPS DT-RNLPS and DCS-RNLPS :

DS-RNLPS: $$w_i = \gamma_{ii} m_i^{\alpha} z_i^{\alpha} + \beta_i\left(1 - \sum_k \gamma_{kk} m_k^{\alpha} z_k^{\alpha}\right) \qquad (4.2.15)$$

DT-RNLPS: $$w_i = \overset{*}{m}_i z_i^{\alpha} + \beta_i\left(1 - \sum_k \overset{*}{m}_k z_k^{\alpha}\right) \qquad (4.2.16)$$

DCS-RNLPS: $$w_i = \delta_i h + \gamma_{ii} m_0^{\alpha} z_i^{\alpha} P + \beta_i\left(1 - m_0^{\alpha} \sum_i \gamma_{ii} z_i^{\alpha} P\right) \qquad (4.2.17)$$

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[5] See Chatterjee et al. p.282-83.The restrictiveness of the DS method can be seen from the fact that (4.2.15) implies identical own-price ($\delta \log q_i / \delta \log p_i$) and own-specific commodity scale elasticities ($\delta \log q_i / \delta \log m_i$), while the cross elasticities differ by unity .

$$\sum_i \beta_i = 1, \qquad \sum_i \delta_i = 0, \qquad m_0 = \left(1+\theta_0 h\right) \tag{4.2.18}$$

with m_0 representing the general equivalence scale, defined as the ratio of the cost of obtaining a reference utility level u, at a given price vector p, for a household of type h, to the cost of obtaining the same utility level for the reference household, in our case a couple with no children (see Chatterjee et al. 1994, p. 282). Furthermore it is

$$m_i = \left(1+\theta_i h\right), \qquad m_i^* = \left(\gamma_{ii}^* + \theta_i h\right), \qquad P = \prod_k p_k^{\alpha\delta_k h} \tag{4.2.19}$$

and in (4.2.16) we have introduced demographic variables in the demand system by making γ_{ii}, a sub-set of the demand parameters, a linear function of the number of children in the household:

$$\gamma_{ii,h} = \gamma_{ii}^* + \theta_i h \tag{4.2.20}$$

In this paper, we treat a couple without children (h = 0) as our reference household, and because there is no distinction between young and old children in the New Zealand data, we ignore child age effects.[6]

As for the NLPS family of models, the imposition of parameter restrictions on the above demographic extensions of the RNLPS model gives us a variety of alternative demand systems, which have been tested in our empirical analysis of New Zealand and Italian data.

The testable restrictions $\delta_i = 0$ imposed on the DCS demographic procedure yield the "Naive Scaling" NS-RNLPS:

$$w_i = \gamma_i m_0^\alpha z_i^\alpha + \beta_i \left(1 - m_0^\alpha \sum_k \gamma_k z_k^\alpha\right) \tag{4.2.21}$$

6 For evidence of child age effects in the context of the above demand systems, see the studies on UK budget data by R. Ray (1993a, 1993b).

Similarly, by imposing on the DCS-RNLPS system the $\alpha = 1$ restriction, which allows income linearity in the Engel consumption function, we obtain the DCS-LES system:

$$w_i = \partial_i h + \gamma_i m_0 z_i P + \beta_i \left(1 - m_0 \sum_i \gamma_{ii} z_i P\right) \qquad (4.2.22)$$

The NS-LES system can be obtained either directly from DCS-RNLPS by imposing both the $\alpha = 1$ and $\delta_i = 0$ restrictions, or from NS-RNLPS, by imposing $\alpha = 1$ or from DCS-LES by imposing $\delta_i = 0$:

$$w_i = \gamma_i m_0 z_i + \beta_i \left(1 - m_0 \sum_i \gamma_{ii} z_i\right) \qquad (4.2.23)$$

The nesting sequence for the DCS demographically augmented demand systems is shown in Figure 2, where $i = 1,2,\ldots,n$.

Thus, models (4.2.17), (4.2.21), (4.2.22) and (4.2.23) are all nested together, and can be submitted to standard testing procedures. By contrasts, DT, DS and DCS are non-nested demographic procedures. In the DCS formulation, a test for $\delta_i = 0$ for all i constitutes a test for price invariance of the general equivalence scale and, indirectly, of the validity of the NS-RNLPS model.

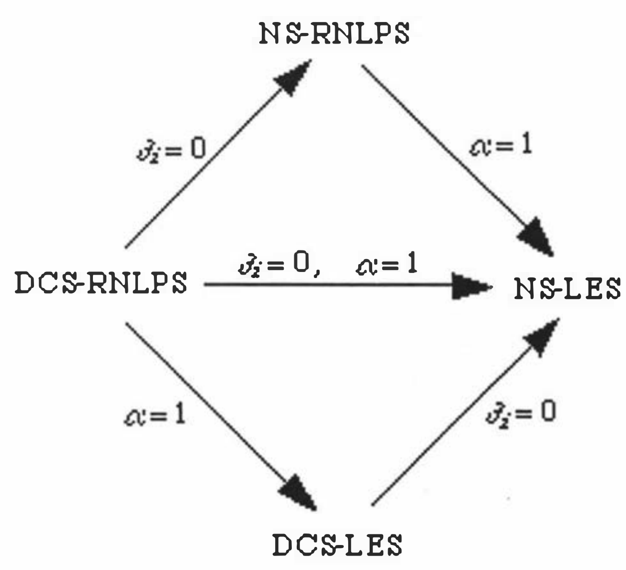

FIGURE 4.2: Nesting Sequence for the DCS-RNLPS model

4.3 The Models Estimation

4.3.1 The Estimation Process

The budget share consumption equations are generally non-linear in the parameters and can be represented as:

$$w_{iht} = f_i(X_{ht}, \theta) + \varepsilon_{iht}; \quad (\text{for } i = 1,\dots,n) \qquad (4.3.1)$$

where w_{iht} is the observed consumption on commodity i (budget share of total expenditure) by household h in period t, X_{ht} is a matrix of explanatory variables - total expenditure, prices and household size - and ε_{iht} is a vector of stochastic errors, $N(0,\Omega)$ distributed. We also assume that ε_{iht} is independent of X_{ht} and that for each household of type h it is $\mathrm{Cov}(\varepsilon_{iht}, \varepsilon_{ihs}) = 0$, for $t \neq s$.[7]

As a consequence of the adding up condition Ω is singular. To overcome this difficulty we have two alternatives: either to constrain Ω itself (see for example Winters, 1984) or to delete one equation (see Barten, 1969). We chose the latter solution as we felt it was less arbitrary than restricting Ω, and because deleting one equation entails no loss of information since Ω is singular by construction (see Nelson, 1988, p. 1306). During estimation we deleted Equation 3 whose parameters can however be obtained as residuals from condition (4.4.18):

$$\beta_3 = 1 - \sum \beta_i \quad \text{and} \quad \delta_3 = -\sum \delta_i \quad (\text{for } i \neq 3) \qquad (4.3.2)$$

The parameters were estimated by the Maximum Likelihood procedure described in Chapter 3.

[7] This implies zero correlation among the error terms for households of different size.

4.3.2 The Estimation Results

We have analysed ten models: NLPS, DCS-RNLPS, DT-RNLPS, DS-RNLPS and the nested sub-models derived from the first two: RNLPS, NS-RNLPS, LPS, LES, DCS-LES and NS-LES. For the first six, we checked the possibility of income linearity in the Engel function by imposing the constraint $\alpha = 1$. It appears from our experiments that the hypothesis of income linearity must be rejected for all models.

An interesting general result of the estimation process has been that model performance was very similar for both countries. Models which performed poorly, producing either absurd parameter estimates or showing convergence problems, did so consistently for both data sets. Similarly, models which seemed to fit the data well and had no problems to converge to either absolute or well defined local maxima of the Likelihood function, did so for both countries.

4.3.2.1 The Results for the NLPS, LPS, RNLPS, and LES Models

For the NLPS model (4.2.7), the Maximum Likelihood (ML) estimates are very sensitive to the set of parameter values used to start the iterative estimation procedure as different sets of starting values generate totally different "ML" estimates. Although the respective values of the LL function tend to be different, a fact allowing us to select the more likely set of estimates, the LL function appears to be extremely lumpy without a well defined maximum but with lots of local maxima of similar LL values generating different sets of estimates. Besides the uncertainty of having attained ML estimates, the data do not seem fine enough to allow the estimation of models as complex as NLPS. The estimates corresponding to the highest value of the LL function that we could attain are shown in the first column of Table 4.1, part A for New Zealand and part B for Italy.

The imposition of the $\alpha = 1$ constraint, generating the LPS model, makes the estimation easier, by giving convergence, in a reasonably small number of iterations, from most sets of starting values. But the linear constraint itself is clearly rejected, for both countries, by the Likelihood Ratio (LR) test reported in Table 4.4, and the standard errors of estimate are very large for most of the γ parameters; thus these estimates are insignificantly different from zero. The LPS estimates are shown in Table 4.1, second columns of parts A and B.

TABLE 4.1

Parameter Estimates[a] for the NLPS and its nested models

Part A: New Zealand

Model	NLPS		LPS		RNLPS		LES	
γ_1	24.062	(11.1341)	447.73	(252.40)	1.091	(.4514)	22.009	(9.2423)
α	.025	(.0135)	-	-	.093	(.0195)	-	-
γ_{13}	−7.733	(6.1734)	-401.92	(239.94)	-	-	-	-
γ_{14}	−19.652	(14.4661)	-232.95	(140.58)	-	-	-	-
γ_{15}	−1.851	(1.9132)	-750.28	(504.43)	-	-	-	-
β_1	−.737	(.4052)	.279	(.009)	−.088	(.0795)	.281	(.0090)
γ_3	−3.964	(16.1170)	378.03	(226.61)	2.169	(.4883)	22.881	(7.6439)
γ_{34}	23.359	(15.3882)	−119.10	(97.14)	-	-	-	-
γ_{35}	19.171	(8.5619)	−590.01	(404.54)	-	-	-	-
β_3	−1.194	(--------)	.222	(-------)	-.251	(------)	.223	(------)
γ_{45}	7.625	(2.8957)	−272.49	(192.16)	-	-	-	-
γ_5	−9.189	(3.9425)	277.93	(192.09)	−6.470	(1.377)	18.727	(13.548)
β_4	.771	(.3243)	.105	(.0035)	.322	(.0732)	.104	(.0039)
β_5	2.160	(.7548)	.394	(.0098)	1.017	(.1005)	.392	(.0101)
LL[b]	1190.319		1177.461		1180.289		1154.945	
k[c]	14		13		8		7	

a There are no parameters with subscript 2 as such subscript would refer to the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameter β_3 has been obtained as a residual from the restriction in (4.2.7). Figures in brackets are standard errors.

b Value of the log-likelihood function. c Number of parameters to be estimated

TABLE 4.1

Parameter Estimates[a] for the NLPS and its nested models

Part B: Italy

Model	NLPS		LPS		RNLPS [b]	LES	
γ_1	6.3180	(11.084)	213.50	(211.03)	-	89.370	(19.402)
α	.04161	(.04761)			-	-	
γ_{13}	-21.986	(12.831)	22.901	(192.79)	-	-	
γ_{14}	17.388	(9.0887)	88.340	(241.25)	-	-	
γ_{15}	.3189	(1.6461)	-152.07	(427.37)	-	-	
β_1	-3.0743	(3.9780)	.3579	(.01064)	-	.3588	(.01073)
γ_3	37.559	(20.830)	-52.030	(178.55)	-	-7.0141	(6.2238)
γ_{34}	-16.931	(9.5913)	-54.549	(90.778)	-		
γ_{35}	-.26207	(6.1543)	157.73	(169.43)	-	-	
β_3	-.0445	(--------)	.1402	(-------)	-	.2048	(-------)
γ_4	5.0757	(4.3311)	42.930	(203.50)	-	-12.160	(8.5294)
γ_{45}	-5.2780	(4.8027)	-51.371	(204.81)	-	-	
γ_5	5.0665	(6.4709)	71.469	(204.17)	-	-31.357	(15.295)
β_4	1.2694	(1.3387)	.1798	(.00364)	-	.1807	(.00353)
β_5	2.8494	(3.0891)	.3221	(.00525)	-	.2557	(.00406)
LL[c]	2059.549		1932.066		-	1867.991	
k[d]	14		13		-	7	

a There are no parameters with subscript 2 as such subscript would refer to the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameter β_3 has been obtained as a residual from the restriction in (4.2.7). Figures in brackets are standard errors.

b RNLPS did not converge. c Value of the log-likelihood function. d Number of parameters to be estimated

In the New Zealand case, the RNLPS model easily reached convergence to the same set of estimates from most sets of starting values. When the iterative procedure converges to different estimates, it does so for much lower values of the LL function, which clearly correspond to some local maximum. Most parameter estimates are significant, and the linear restriction $\alpha = 1$ (giving the LES model) is clearly rejected by the LR test. The restrictions $\gamma_{ij}=0$, for $i \neq j$, nesting the RNLPS model within the

NLPS, are also rejected by the LR test. The parameter estimates are shown in the third column of Table 4.1 part A.

For Italy, the RNLPS model did not converge. Whatever sets of starting values or convergence criteria we tried, convergence was never reached, even after thousands of iterations. The parameters either oscillated in a seemingly random way from one value to another or kept steadily increasing at each iteration while others kept decreasing, with the LL values hardly changing at all.

The LES model converged to the same set of final parameter estimates[8] from most sets of starting values, but once again all the linear restrictions nesting it into NLPS were rejected by LR tests, thus the model itself is rejected.

4.3.2.2 The Results for the DT-RNLPS, and DS-RNLPS Models

The estimation process for the DT-RNLPS model appears well behaved as it always converges to the same final set of estimates from a variety of starting values. We are confident that the final estimates correspond to a global maximum of the LL function or at least to a well defined local maximum within the parameters' domain. All the parameter estimates for Italy, reported in the fourth columns of Table 4.5, and more than half of those for New Zealand, reported in the third column, are significantly different from zero. The worst estimates appear to be those for γ_4 and γ_5 which have standard errors much larger than the values of the parameters themselves. The income linearity constraint $\alpha = 1$ and the hypothesis of no demographic effects, $\theta_i = 0$, are both clearly rejected by the LR test.

Model DS-RNLPS, on the other hand, proved extremely difficult to estimate. It was the model showing the most chaotic behaviour during estimation as it was the most sensitive to the initial values given to the parameters at the beginning of the iterative procedure. Different starting parameter values *always* generated different final sets of parameter estimates, even for very small initial differences. Sometimes, even a

[8] The parameter estimates we obtain here differ from those obtained in Chapter 2 because of the different structure of the error term.

change in the value of the convergence criterion, resulted in the same set of initial parameter values generating a different set of final estimates.

TABLE 4.2

Parameter Estimates[a] for the DS-RNLPS model for Italy and New Zealand

	Italy		New Zealand	
γ_1	.9130	(.87162)	1.714	(.58904)
θ_1	.1762	(.98590)	.6041	(.41813)
α	.1125	(.02870)	.1421	(.18655)
β_1	.2677	(.18999)	–.4637	(1.0138)
γ_3	.7337	(.42321)	.6940	(.30617)
θ_3	-.1985	(.92421)	–.1176	(.11003)
β_3	.3738	(--------)	-.0350	(-------)
γ_4	.1457	(.15944)	–.5082	(.83250)
θ_4	.1003	(.99811)	–.1716	(.18480)
γ_5	-.0379	(.42165)	–2.8780	(4.64261)
θ_5	.0499	(.99879)	.0533	(.09687)
β_4	.1058	(.05586)	.2277	(.21540)
β_5	.2527	(.07378)	1.2710	(1.1671)
LL[b]	1622.974		1328.774	
k[c]	12		12	

a *There are no parameters with subscript 2 as such subscript would refer to a the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameter β_3 has been obtained as a residual from the restriction in (4.2.7). Figures in brackets are standard errors.*

b *Value of the log-likelihood function*

c *Number of parameters to be estimated*

We report in Table 4.2 the sets of estimates with the highest LL we could obtain over a large number of experiments. They were obtained by using as starting values for the iterative process the parameter estimates for model RNLPS, with all $\theta_{\cdot s} = 0$.

Perhaps the most disturbing feature of the "chaotic" behaviour of DS-RNLPS was the experiment where, having used as starting values the set of final estimates for New Zealand reported in Table 4.2, but rounded to the second decimal, we ended up with a completely different set of estimates with a much lower LL. The imposition of the $\alpha = 1$ constraint made convergence impossible.

4.3.2.3 The Results for the DCS-RNLPS, NS-RNLPS, DCS-LES and NS-LES Models

The estimation procedure for DCS-RNLPS works well as it always converges to the same parameter estimates from most sets of starting values. Therefore, we feel confident that the estimation process has reached a global maximum of the LL function, and that the parameter estimates we have so obtained are indeed ML estimates.

The set of starting values producing faster convergence are the parameter values obtained from the DCS-LES model with α = 0.5. Considering that the DCS-LES model is nested within the DCS-RNLPS, as shown in Figure 4.2, and should therefore approximate it well, this seems to confirm that for this whole family of models the LL function is well behaved and grows smoothly up to its maximum. We show the ML estimates of the DCS-RNLPS model for New Zealand, and for Italy, in Table 4.5.

The estimates for the DCS-LES model are shown in the first column of Table 4. 3, Parts A and B. Convergence is easily reached for both countries and all parameters are significantly different from zero with the only exception of γ_4, γ_5 and δ_5 for New Zealand. The best set of starting values is given by the NS-LES estimates. The $\alpha = 1$ restriction is however rejected by the LR test against the DCS-RNLPS model nesting it.

As for all the other models in the DCS-RNLPS nesting sequence, the estimation procedure for NS-RNLPS performs very well for New Zealand. It converges to the same estimates for any set of starting values we tried. Most parameters are significant, but the $\delta_i = 0$ restrictions, which characterise the NS-RNLPS model, are rejected by the LR test against the DCS-RNLPS model negating this model as a valid explanation of New Zealand household consumption patterns.

TABLE 4.3

Parameter estimates[a] for the NS-RNLPS, DCS-LES and NS-LES Models

Model	DCS-LES		NS-RNLPS		NS-LES	
			Part A: New Zealand			
δ_1	.0136	(.00402)	-	-	-	-
β_1	.1554	(.00953)	–.2032	(.08051)	.1768	(.00593)
γ_1	9.0610	(1.93522)	1.6139	(.17602)	5.6195	(1.44042)
α	-	-	.2389	(.03838)	-	-
δ_3	-.0318	(--------)	-	-	-	-
β_3	.2732	(--------)	.2233	(-------)	.2611	(-------)
γ_3	8.4448	(2.61701)	–.4324	(.29727)	3.3881	(1.43601)
γ_4	.9071	(.68728)	–.2690	(.15261)	1.0992	(.45212)
γ_5	3.6903	(3.98760)	–.2690	(.15261)	1.9825	(2.15950)
θ_0	.9215	(.28811)	2.0476	(.46416)	2.0143	(.48622)
δ_4	.0117	(.00147)	-	-	-	-
δ_5	.0065	(.00546)	-	-	-	-
β_4	.0804	(.00469)	.1005	(.01636)	.0859	(.00496)
β_5	.4910	(.01376)	.8794	(.08472)	.4712	(.00956)
LL[b]	1312.390		1267.587		1253.353	
k[c]	11		9		8	

[a] *There are no parameters with subscript 2 as such subscript would refer to a the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameters δ_3 and β_3 have been obtained as residuals from the restrictions in (4.2.18). Figures in brackets are standard errors.*

[b] *Value of the log-likelihood function.* [c] *Number of parameters to be estimated.*

Like RNLPS, the NS-RNLPS model for Italy did not converge. Whatever sets of starting values we tried, convergence was never reached, even after thousands of iterations. Some parameters kept increasing at each iteration while others kept decreasing - the iterative procedure was clearly diverging .

TABLE 4.3

Parameter estimates[a] for the NS-RNLPS, DCS-LES and NS-LES Models

Model	DCS-LES		NS-RNLPS	NS-LES	
			Part B: Italy[d]		
δ_1	.0347	(.00511)	-	-	
β_1	.2239	(.01679)	-	.3215	(.00828)
γ_1	69.565	(8.06780)	-	63.229	(11.23501)
α	-	-	-	-	-
δ_3	-.0162	-	-	-	
β_3	.1994	-	-	.1556	-
γ_5	-33.085	(8.60620)	-	-12.614	(8.88881)
θ_0	.1146	(.03217)	-	.2899	(.04953)
δ_4	-.0136	(.00148)	-	-	
δ_5	-.0049	(.00301)	-	-	
β_4	.2331	(.00488)	-	.1963	(.00265)
β_5	.3436	(.01056)	-	.3266	(.00622)
LL[b]	2060.303			1943.960	
k[c]	11		-	8	

[a] *There are no parameters with subscript 2 as such subscript would refer to a the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameters δ_3 and β_3 have been obtained as residuals from the restrictions in (4.2.18). Figures in brackets are standard errors.*

[b] *Value of the log-likelihood function.* [c] *Number of parameters to be estimated.* [d] *NS-RNLPS did not converge*

The last model nested in this sequence is NS-LES and, once again, it proves to be a well behaved model with all parameters highly significant with the exception of γ_5 for New Zealand and all γ.s, except γ_1, for Italy. In the DCS-RNLPS nesting sequence, the linear restriction on α and the zero restrictions on the δ.s are always rejected in favour of the nesting model.

The demographically important income scale parameter θ_0 is highly significant for all models in the DCS-RNLPS nesting sequence.

4.3.3 General Considerations on the Estimation Procedure

The NLPS and DS-RNLPS models perform very poorly during the estimation procedure, the final values of the parameter estimates being totally dependent on the initial values of the parameters in the iterative procedure[9]. Although different sets of estimates tend to correspond to different LL values, often small differences in the final value of the LL function correspond to totally different sets of estimates. The extreme lumpiness of the LL function, and the consequent instability of the ML estimates, do not seem to recommend these models as suitable to fit and represent the data.

The bad performance of the NLPS model must also cast doubts on its nested sub-models, even considering that their estimation procedures run smoothly, as all the restrictions nesting them into NLPS were rejected by Likelihood Ratio (LR) tests shown in Table 4.4 for both Italy and New Zealand data.

In contrast to the behaviour of the NLPS and DS-RNLPS models, both the DT-RNLPS and the DCS-RNLPS models, including all models in the DCS-RNLPS nesting sequence (with the exception of NS-RNLPS for Italy), seem to have very smooth and continuously increasing LL functions giving convergence to the same set of estimates from almost all starting points[10].

What we found most gratifying was how the data clearly rejected, during estimation, some of the models in favour of others, instead of allowing with equal ease the estimation of all of them. Data sets on households consumption as extensive as those used here, with clearly defined although highly aggregated expenditure categories, cannot support varied forms of economic behaviour and hypotheses without casting doubts on all of them. The way some of our models proved almost impossible to estimate shows how inadequate were the economic foundations on which those models stood in representing and explaining the consumption behaviour of New Zealand and Italian households.

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[9] This extreme sensitivity to initial conditions seems to identify the NLPS and DS-RNLPS models as "chaotic", see Kellert, 1993, p.12.

[10] The starting values we tried most often were zeros or ones (+ and -) or the parameter estimates obtained from a nested model. We also checked that small variations in the values of some (or all) parameter starting values, obtained in the manner described above, had no effect on the final estimates.

Equally interesting is the fact that the two 'data-preferred' models - DCS-RNLPS and DT-RNLPS - are also two of the richest in economic content, and take into account demographic factors in very similar ways. It is also worth noting that all nested sub-models implying simpler economic behaviour and/or linear assumptions were rejected by statistical tests in favour of their unrestricted parent models nesting them. Of particular economic interest is the rejection by all models of the linearity of the Engel function, implied by the α=1 restriction.

The fact that the comments made above on model behaviour during estimation, and their nesting hierarchy, apply equally well to both the New Zealand and the Italian case is a final confirmation that DCS-RNLPS, and to a lesser extent DT-RNLPS, are the models best suited to describe household consumption behaviour, at least when applied to the type of data we used, which are the most commonly available to researchers working in the field of empirical demand analysis.

We show in Table 4.5 the parameter estimates for DCS-RNLPS and DT-RNLPS for the two countries together for ease of comparison.

4.4 Model Testing

To choose a preferred model structure we performed an extensive testing exercise, the results for both New Zealand and Italy are shown in Table 4.4 which reports the values of the Likelihood Ratio test used for the nested models:

$$LR = 2\log L - 2\log L(H_0) \qquad (4.4.1)$$

(see Harvey, 1963, p.63) and the values of the Akaike Information Criterion used for the non-nested models (see Judge et al., 1985, p.870):

$$AIC = -2/T * \log L + 2k/T \qquad (4.4.2)$$

where k is the number of parameters and T the number of observations. In (4.4.1) and (4.4.2) log L is the value of the LL function for the maintained hypothesis and log $L(H_0)$ is the value of the LL function for the null hypothesis, ie, the nested model.

From the values of the LR tests it appears that all the models nested by both the NLPS and the DCS-RNLPS must be rejected[11]. In fact, all the models along both sequences are rejected by the ones preceding them (eg LES is rejected in favour of LPS).

Looking at the Akaike Information Criterion for all nested and non-nested models together, it appears that for New Zealand the DCS-RNLPS model seems to be the best, followed by DS-RNLPS , DT-RNLPS and DCS-LES.

For Italy, the DT-RNLPS seems to be the best followed by DCS-RNLPS, DCS-LES and NLPS. However, considering the poor performance during estimation of the DS-RNLPS and NLPS models, we think their high AIC values do not mean much and can be disregarded because the value of the likelihood function from which they are derived is little more than a "mathematical accident"; without any economic theoretic significance.

Based on this evidence, we consider the DCS-RNLPS model to be the 'preferred' one, and will submit it to further econometric analysis before using it in the empirical study of the New Zealand household consumption data. We will not consider for further analysis either DCS-LES, because it is nested within DCS-RNLPS, or DS-RNLPS, because of its poor estimation performance. However, we will submit to further analysis the DT-RNLPS model to use it as a comparison to the empirical performance of the non-nested DCS-RNLPS[8].

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[11] The null hypotheses $\alpha = 1$, $\gamma_{ij} = 0$ and $\delta_i = 0$ are rejected at all levels of probability considering the respective LR tests have distributions with 1, 4 and 7 d.f. for the NLPS sequence and 1, 3 and 4 d.f for the DCS-RNLPS sequence.

[8] For a discussion of "encompassing" as a criterion for model selection see Hendry, 1993, p.412-414. For a selection procedure based on likelihood dominance see Pollak and Wales, 1991.

TABLE 4.4

Likelihood Ratio Tests[a] for the Nested Models and Akaike[b] (AIC) Information Criterion for the Non-nested Models

Part A: New Zealand

Models	k^c	AIC[b]	LPS	RNLPS	LES	DCS-LES	NS-RNLPS	NS-LES
NLPS	14	13.07	25.72 (1)	20.04 (4)	70.74 (7)			
LPS	13	12.94			45.02 (4)			
RNLPS	8	13.06			50.7 (1)			
DCS-RNLPS	12	14.66				18.75 (1)	127.1 (3)	155.58 (4)
DCS-LES	11	14.46						118.08 (3)
NS-RNLPS	9	13.95						28.48 (1)
NS-LES	8	13.83						
DT-RNLPS	12	14.62	59.92[e] (1)					
DS-RNLPS	12	14.63	f					
LES	7	12.76						

Part B: Italy

Models	k^c	AIC[b]	LPS	RNLPS[d]	LES	DCS-LES	NS-RNLPS[d]	NS-LES
NLPS	14	12.79	254.96 (1)		383.12 (7)			
LPS	13	12.00			128.16 (4)			
RNLPS[d]	8							
DCS-RNLPS	12	13.50				184.20 (1)		610.34 (4)
DCS-LES	11	12.93						426.14 (3)
NS-RNLPS[d]	9							
NS-LES	8	11.61						
DT-RNLPS	12	15.59	132.58[e] (1)					
DS-RNLPS	12	10.08	f					
LES	7	11.60						

[a] *The number of d.f. is in bracket.* . [b] *The minus sign is omitted, the highest AIC indicates the "best" model.*

[c] *Number of parameters* . [d] *This model does not converge.* [e] *LR test for hypothesis: $\alpha=1$.*

[f] *DS-RNLPS with the a=1 constraint does not converge.*

4.5 The GMM Estimates

Because our data are time-series observations of cross-sectional units of household consumption, we have to consider the possibility of both auto-correlation and heteroskedasticity. While for linear models there are established techniques to deal with similar problems (see Kmenta, 1986, Ch 12.2), for our case of a multi-equation, non-linear demand system we will have to resort to more ad-hoc solutions.

We re-estimated the DCS-RNLPS and the DT-RNLPS models by the Generalised Method of Moments (GMM). Given a linear[9] over-identified model, with more instrumental variables than regressors, it can be shown that, under certain conditions, including homoskedasticity, a consistent and asymptotically efficient estimator is defined by:

$$\hat{\beta} = (X'WX)^{-1}X'Wy \qquad (4.3.3)$$

where X is the matrix of regressors, y the vector of regressands, and W a matrix of instrumental variables that projects orthogonally onto $S(W)$, a sub-space of X, uncorrelated with the error term u (see Davidson, Mackinnon, 1993, Ch. 7.4). However, if heteroskedasticity is present, $\hat{\beta}$ is no more efficient, and we need to construct another estimator which takes into account the characteristics of the covariance matrix and satisfies the instrumental variables moment conditions:

$$E(W_t \cdot u_t) = 0; \quad u_t = y_t - X_t\beta \qquad (4.3.4)$$

If we have a diagonal matrix Ω with element $\Omega_{tt} = E(u_t)^2$ then the covariance matrix satisfying (4.3.3) is simply $W'\Omega W$, sometimes called the weighting matrix, and the resulting GMM estimator is:

$$\beta^* = \left(X'W(W'\Omega W)^{-1}W'X\right)^{-1}X'W(W'\Omega W)^{-1}W'y \qquad (4.3.5)$$

---

[9] This is a very simplified exposition of GMM. For a full treatment and an extension to non-linear models see Davidson, Mackinnon, 1993, Ch. 17.

In most cases Ω is unknown, and has to be estimated. We did this by using the non-linear version of the heteroskedastic-consistent covariance matrix estimator suggested by White (1980a and 1980b) which is obtained as the matrix of residuals at the 2SLS stage of the GMM estimation procedure. In this way, we took into account the possibility of any unknown form of heteroskedasticity in our data.

To try and improve our parameter estimates we introduced two instrumental variables: total household expenditure as a proxy for income; and a time trend variable, with values from 1 to 9, to represent the nine successive two-year periods spanned by the New Zealand data, and from 1 to 13 for the Italian data. The income proxy takes care of any income effects on consumption, and the time trend might take care of some of the auto-correlation effects[12].

For New Zealand, the GMM estimation procedure, which is a very demanding one computationally because of the White procedure, showed good convergence characteristics for DCS-RNLPS as well as for DT-RNLPS.

For DCS-RNLPS the starting values had to be chosen with some care to actually obtain convergence and the best set of starting values for the estimation process was the set of DCS-RNLPS Maximum Likelihood parameters estimates. The GMM and ML estimates for New Zealand were very similar. Only for θ_0, the GMM estimate was substantially different, and a definite improvement over the ML estimate as its size was more according to economic expectations. The GMM estimates for New Zealand are shown in the first column of Table 4.5.

For DT-RNLPS, the convergence characteristics were even better than those for DCS-RNLPS. Convergence was reached from a variety of starting values, and the best set was the ML parameter estimates for the RNLPS model with all θ parameters initially set to zero. Once again, the GMM estimates were very similar to the ML estimates. They are shown in the third column of Table 4.5.

[12] With only nine time series units we felt a rigorous attempt at estimating auto-correlation would have been exceedingly difficult and produce dubious results.

Table 4.5

Parameter Estimates[a] for the DCS-RNLPS and DT-RNLPS models for New Zealand and Italy data

	DCS-RNLPS				DT-RNLPS			
	New Zealand[b]		Italy[c]		New Zealand[b]		Italy[c]	
δ_1	.0201	(.01150)	.0682	(.00558)				
β_1	-.3062	(.12112)	-.7785	(.24842)	-.1971	(.07464)	-.3619	(.10246)
γ_1	1.8115	(.20217)	-1.3955	(1.54083)	.7031	(.71832)	-14.2696	(3.25951)
θ_1					.6321	(.31061)	5.1902	(1.14123)
α	.1878	(.04229)	.1606	(.04003)	.2089	(.03392)	.2083	(.02883)
δ_3	-.0163	(---------)	-.0317	(--------)				
β_3	.0630	(---------)	.3990	(--------)	.0570	(--------)	.3246	(---------)
γ_3	.4449	(.22196)	5.6153	(3.49710)	.7918	(.24454)	23.3605	(9.60701)
θ_3					-.1657	(.18402)	-4.4140	(1.87862)
γ_4	-.5335	(.16673)	3.1721	(1.98491)	1076	. (.60412)	19.4304	(6.37541)
θ_4					-.4094	(.29085)	-4.3508	(1.45993)
γ_5	-3.5637	(1.06203)	5.5539	(3.49822)	.6230	(3.60523)	37.3953	(11.97805)
θ_0	.4975	(.47816)	-.0979	(.01530)				
δ_4	.0084	(.00220)	-.0163	(.00192)				
δ_5	-.0122	(.00150)	-.0202	(.00388)				
θ_5					-2.6517	(1.75004)	-8.4315	(2.76722)
β_4	.1677	(.03314)	.4696	(.08308)	.1643	(.02457)	.3477	(.03528)
β_5	1.0755	(.15002)	.9099	(.17741)	.9758	(.09272)	.6896	(.07470)
J[d]	42.94		-		38.49		-	
LL[e]	-		2173.92		-		2187.72	
k[f]	12		12		12		12	
R^2	.8155		.8252		.8150		.8199	
R^2	.3329		.7488		.3517		.7498	
R^2	.6233		.8447		.6210		.8317	

a *There are no parameters with subscript 2 as such subscript would refer to a the Housing commodity which has not been considered in the present study, see chapter 1 on data. Parameters δ_3 and β_3 have been obtained as residuals from constraint (4.2.18). Figures in brackets are standard errors.*

b *GMM estimates.* [c] *ML estimates.* [d] *Hansen's test of over-identifying restrictions, chi square distributed with k d.f. See Davidson - MacKinnon, 1993, p. 235 and p. 616.*

e *Value of the log-likelihood function.* [f] *Number of parameters to be estimated*

The R^2 are between the observed and projected values of the dependent variables.

In the case of Italy, the GMM estimation procedure for both DCS-RNLPS and DT-RNLPS showed signs of chaotic behaviour - it either did not converge or converged to economically unacceptable values (eg we obtained the estimate $\theta_0 = 19{,}602$ when we used the ML parameter estimates as starting values), or it converged to different final estimates from different starting values, or converged to different sets of final parameter estimates for almost identical values of the maximised objective function. Therefore, for Italy, we retained the ML estimates, which are shown in the second and fourth columns of Table 4.5.

The main difference between the Italian and New Zealand estimation results has been that while the best estimation method for Italy was ML, for New Zealand it seemed to be GMM. For both countries, the final decision to prefer DCS-RNLPS over DT-RNLPS was that the price elasticities, shown in Tables 4.7 and Table 4.8, computed from it, proved more self-consistent and in accordance with economic expectations than those obtained from DT-RNLPS[13].

We feel comfortable in our decision to choose DCS-RNLPS and DT-RNLPS by their very similar estimation behaviour for both countries, and by the way all models nested within them and all linear restrictions on income (the Engel curve) came to be equally rejected.

An interesting comparison is between the values of m_0 - the reference equivalence scale, sometimes interpreted as the "cost of a child" - which, for Italian and New Zealand households, can be derived from equation (4.2.18) for the corresponding estimated value of parameter θ_0.

For the New Zealand households, the value of m_0 increases as the size of the household increases, but slightly less than proportionately, thus showing some small economy of scale in the cost of children. Whereas for Italy[14], the value of m_0 decreases

[13] Although most elasticities were of similar size and sign, whether computed from DCS-RNLPS or DT-RNLPS, some of the family size elasticities computed from the latter were unacceptably large.

[14] Because our data do not specify the age of the household components, m_0 does not specifically refer to children but more generally to any additional household member.

slightly as the household size increases: an increase in household size seems to reduce expenditure in Italian households[15].

This apparently perverse behaviour of the Italian general equivalence scale obtained from the DCS-RNLPS model is however reflected in the original data that, as household size increases, show substantial decreases in average household expenditure on housing, almost constant expenditure on apparel, and very small increases for transport.[16] Only food shows a clear increase in expenditure as household size increases.[17]

This appears to be a data induced problem which, as we shall see in Chapter 6, makes computation of household equivalence scales very difficult. In fact, in the Italian surveys, to compute the cell consumption averages, households are grouped according to their size, and without taking into consideration their composition. This averaging out of all households characteristics, but their size, loses too much information on the variations in consumption behaviour among households of different composition, to allow a meaningful estimation of consumption scales.

The computed values of m_0 for the household sizes common to the two countries are reported in Table 4.6.

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[15] Similar results which imply that large households need *less* money to be as well off as small households are reported in Muellbauer, 1977, footnote 1, p.464.

[16] Average weekly household expenditure in thousands of lire. V1=Food, V3=Household Operations, V4=Apparell and V5=Transport.

H. Size	h = 1	h = 2	h = 3	h = 4	h = 5
Mean V1	293.80	411.41	467.04	514.14	594.62
Mean V3	522.77	511.85	460.37	451.66	430.18
Mean V4	147.57	141.44	143.46	144.73	143.65
Mean V5	233.20	245.17	274.56	268.89	258.18

[17] It must be pointed out in this context that, given the high propensity to save of Italian households, a possible explanation for the decrease in expenditure as household size increases might be due to an increase in savings to improve the financial security of the larger family unit.

TABLE 4.6
Estimates of m_0 obtained from the DCS-RNLPS models

Household size	h = 1	h = 2	h = 3	h = 4
Italy	1.098	1.0	0.902	.804
New Zealand	0.503 -	1.0	1.497	1.995

4.6 The Demand Elasticities

4.6.1 The Elasticities Computation

Based on the results in sections 4.4 and 4.5, we have computed the own and cross-price elasticities, the household size elasticities and the total expenditure (i.e. the aggregate expenditure for the four commodities considered) elasticities for models DCS-RNLPS and DT-RNLPS for both countries - using the ML parameter estimates for Italy and the GMM estimates for New Zealand. As in Section 3, all the elasticities were obtained by numerical differentiation of the functional relationships expressing the point elasticities of the *consumption shares* of the four commodities considered with respect to the above explanatory variables. The own-prices *share elasticities* then work out to be:

$$e_{ii} = (\delta w_i / \delta z_i)\,(z_i / w_i) \qquad (4.6.1)$$

where $z = p/y$ represent the "normalized" price.

The relationship between the own-price share elasticities in (4.6.1), and the quantity elasticities E, as defined in (3.3.2), is once again[18] $E_{ii} = e_{ii} - 1$.

For the cross-price elasticities it is simply $E_{ij} = e_{ij}$ because, as already explained in Section 3.3, all prices are independent among themselves, therefore their cross-

[18] If we consider the variable definitions $w = (p*q)/y = z*q$ and then differentiate their logarithms with respect to log(z) the result follows, see Section 3.3.

derivatives vanish and the quantity derivatives only remain in the elasticity expressions.

The share elasticities e will be negative only when a commodity has a quantity elasticity which is negative, and less than unity, i.e. when it is $E < -1$. This is so because, if for a specific commodity the quantity purchased decreases less than proportionately to the increase in price, then the amount spent on that commodity will increase. As a consequence, most share elasticities are bound to be positive and small.

Share elasticities measure how changes in prices, or in other explanatory variables, will change the share allocation of the householders' total expenditure among the commodities included in the households' budgets. A positive share elasticity (equivalent to quantity inelasticity) shows that an increase in price will result in an increased expenditure on that commodity, therefore an increase in its share of the household budget, even if the quantity purchased has decreased. We show the own-price share elasticitiesfor New Zealand and Italy, obtained from the DCS-RNLPS model, in Part A.2 of Tables 4.7 and 4.8 .

While all the price elasticities we present in Tables 4.7 and 4.8, with the exception of those presented in Part A2, are the familiar quantity elasticities, the household size and total expenditure elasticities are *share elasticities.*

The household size and total expenditure elasticities measure the change in the households' *share expenditure* on a specific commodity when the size of that household, or its total expenditure, changes at the margin. This makes it easier to evaluate the effect of a change in household size, or total expenditure, on the way households *divide their resources among the commodities included in their budgets.*

Let us consider food as an example: it is quite natural that a larger household will consume more food, and spend more on it than a smaller household. Therefore, rather than measuring the quantity changes consequent to marginal changes in household size, it seems more interesting to analyse the changes in the food's *share* of total household expenditure.

We are convinced that in an expenditure share model, not only does it make sense to measure expenditure share elasticities, but also that the share-expenditure elasticities are easier to interpret, when the commodities in question are as highly aggregated and heterogeneous as the ones we are dealing with in this study.

Once again, for all the variable values at which we have computed the elasticities, we have also computed[19] the Slutsky matrices; but neither for Italy nor for New Zealand, the Slutsky matrices were negative semi-definite. The non-negativity of the Slutsky matrices points to a violation, by both the DCS-RNLPS and the DT-RNLPS demand systems, of the customary assumption of concavity of the utility function: a result consistent with what we have already observed for the LES and AIDS models.

Alternatively, it is the assumption that when price change the consumer is held to the same indifference surface, or the same level of utility, by a compensating income variation (Benavie, 1972, p. 100-102) which is violated, so that the negativity condition of the Slutsky matrix does not apply anymore.

The violation of the fundamental (but often unreasonable, see Blundell, 1988, p.19) assumptions of concavity seem to be the normal state of affairs in most empirical studies (see for example Deaton-Muellbauer, 1980, p.321 and Chatterjee-Michelini-Ray, 1994, p.280), and the only way to make certain to have negative Slutsky matrices appears to be to constrain them accordingly during estimation (see Brenton and Winters, 1992, p.268).

A notable exception to the empirical failure of the negativity of the Slutsky matrix is Nelson (1988, p. 1310-11) who reports a negative semi-definite Slutsky matrix, but it is interesting to note that her consumption data refers to a very homogeneous group of adult only households, with "heads" aged 35 to 55 and mostly (96%) having only one or two members. However, when she estimates her model separately for households with three or more members the Slutsky negativity condition is not satisfied anymore. This seems to indicate that, in empirical applications, the negativity condition of the

[19] The relationship between the Slutsky matrix and the matrix of the share expenditure derivatives is even simpler in this case than it was for the AIDS model, as it is: $S_{ii} = (1/p_i)(s_{ii}-q_i)$ and $S_{ij} = s_{ij}/p_i$.

Slutsky matrix does apply only when very homogeneous groups of consumers are considered, which is not the case in most applications, including the present one.

The elasticities we obtained from the DT-RNLPS model, although acceptable for prices, were unacceptably high (in the range $1 < e < 4$) for the household sizes. Unrealistic household size elasticities seem to suggest that the model fails to capture the effects of household size on consumption. A conclusion strengthened by the consideration that for the New Zealand data four out of the six parameters entering the demographic variable m_i^*, which represents the effects of household size, were insignificant. Therefore, we do not present the DT-RNLPS elasticities, as they are unreliable and of dubious interpretation. They are however available on request.[13]

4.6.2 Presentation and Discussion of the New Zealand Elasticities

The consumption elasticities with respect to price, household size and total expenditure for model DCS-RNLPS are shown in Table 4.7 for New Zealand and in Table 4.8 for Italy.

In part A.1 of Table 4.7 we report the New Zealand quantity versus price elasticities, and in Part A.2 the share elasticities, computed at the sample means, for the whole sample of 180 observations and a mean household size of 2.5.

In part B and C, we show the elasticities computed at the two variable values Q_1 and Q_3, which we defined in (3.3.2) and (3.3.3), again for the whole sample of 180 observations.

[13] During estimation, we tried for both DCS-RNLPS and DT-RNLPS models the introduction of time trends, mostly in linear and logarithmic form. For both models, the best results were obtained from linear time trends: for DCS-RNLPS they were needed in all equations in the system, for DT-RNLPS the best results were obtained by introducing a time trend only in Equation 3 (Housing Operations). The parameter estimates and the elasticities obtained from the time trend augmented models were very similar to those obtained without any time trends.

In parts D,E, F and G we show the quantity elasticities for the four types of household sizes considered in our data. Here, the elasticities have been computed at the means of the four sub-samples, of 45 observations each, obtained by grouping together the households of equal size.

The reason for computing the elasticities at so many different values of the dependent and explanatory variables is that, for a highly non-linear model, as the one under consideration, the consumption function might have very different elasticities at different loci along it, and may therefore prove unsuitable to analyse consumers' response away from the means. This does not seem to be the case for the DCS-RNLPS model as the consumption function seems well behaved with high elasticities for low levels of consumption and prices and then smoothly decreasing elasticities as consumption and prices increase.

There are several interesting features of the estimated elasticities. To consider the household size elasticities first. There is an almost unique consistency in the magnitudes and signs of these elasticities relating to the different household sizes. They are all inelastic (ie elasticity of absolute values less than unity) and, apart from housing and transport, are all positive in sign. The low values of the elasticities indicate that, whatever the average household size, a marginal change in it results in a less-than-proportionate change in the household's share expenditures on the consumption items in question. Only in respect of Housing Operations and Transport the changes in the expenditures are in the opposite direction to the changes in the household size. This probably indicates a degree of scale economy in the use of these two durable consumer items. As the household size grows marginally, expenditure on these items decreases in proportional terms, as the items are used more intensively by the larger household unit.

The positive signs of the other household size elasticities indicate that expenditures change in the same direction as household size. For items such as Food and Apparel, this would seem to make sense - a doubling of the household size for example would not normally double the expenditure on food, particularly when the additional household members are children, as they are in the HEIS data.

An interesting feature of the household size elasticities concerns the almost totally inelastic response of both housing and apparel to size changes, when the household consists of a single adult only (see part D of Table 4.7). Housing in particular remains very inelastic up to households of size three (see parts E, F and G of Table 4.7) indicating that for small families the housing expenditure remains very much the same until their size is large enough to require big changes in their housing requirements. For household of size one to three there appear to be substantial economies of scale.

The Total Expenditure elasticities are all smaller than unity in absolute value, indicating "inelasticity" again. The elasticities for Apparel and Transport are positive: the proportion of total expenditure spent on these commodities increases as total expenditure increases as can be expected, in a modern society, for these types of commodities; they are negative for Food and Housing.

TABLE 4.7

DCS-RNLPS Model: Price, Household Size and Expenditure Elasticities for New Zealand, GMM parameter Estimates.

(All elasticities computed at the means[a])

Part A.1: Whole sample of 180 observations. Average household size FS=2.5

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.320	.023	-.038	-.313	.362	–.352
Housing	.028	-.877	.014	.065	–.054	–.175
Apparel	–.136	.043	-1.432	-.421	.117	.220
Transport	-.296	.052	.153	-.921	–.312	.398

Part A.2: Whole sample of 180 observations. Average household size FS=2.5. Own-price share expenditure elasticities

Food	Housing	Apparel	Transport
.680	.123	-.432	.079

Part B: Whole sample of 180 observations. Household size FS = 2.5
Elasticities computed at the Q_1 variable values[b]

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.019	.034	-.055	-.455	.520	–.505
Housing	.029	-.872	.015	.068	–.056	–.183
Apparel	–.194	.062	–.620	-.162	.173	.310
Transport	–.349	.062	.181	-.903	–.380	.485

Part C: Whole sample of 180 observations. Household size FS = 2.5
Elasticities computed at the Q_3 variable values[c]

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.337	.023	-.037	-.310	.329	–.339
Housing	.024	-.895	.012	.056	–.030	–.150
Apparel	–.122	.039	–1.384	-.476	.083	.188
Transport	–.252	.045	.132	-.927	–.273	.363

Part D: Households of Size 1. Sub-sample of 45 observations

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.253	.036	-.047	-.342	.206	–.393
Housing	.036	-.883	.011	.062	–.003	–.160
Apparel	–.168	.040	–1.477	-.360	.026	.245
Transport	–.271	.051	.142	-.933	–.158	.372

Part E: Households of Size 2. Sub-sample of 45 observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.306	.027	-.040	-.319	.321	–.361
Housing	.029	-.882	.013	.062	–.036	–.166
Apparel	–.148	.043	–1.453	-.392	.092	.232
Transport	–.271	.049	.141	-.930	–.252	.366

Part F: Households of Size 3. Sub-sample of 45 observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.347	.020	-.035	-.301	.396	–.337
Housing	.024	-.877	.015	.064	–.080	–.176
Apparel	–.035	.043	–1.412	-.447	.153	.210
Transport	–.298	.051-	.154	-.919	–.349	.397

Part G: Households of Size 4. Sub-sample of 45 observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.388	.013	-.030	-.283	.442	–.312
Housing	.019	-.870	.018	.067	–.135	–.192
Apparel	–.105	.044	–1.380	-.489	.204	.193
Transport	–.338	.055	.173	-.904	–.458	.445

a The consumption mean values used to compute the elasticities in Part A, B and C were those for the whole sample of 180 observations. In Parts D, E, F and G the means were those of the four sub-samples, with 45 observations each, grouping the households of equal size .

b Q_1 = lowest observation + 0.25 Range

c Q_3 = lowest observation + 0.75 Range

Food becomes less elastic as average household expenditure increases - from Q_1 to the means, and from the means to Q_3 - showing that "poorer" families (with expenditure twenty five percent below the mean) react more strongly than "rich" families (with expenditure twenty five percent above the mean) to changes in total expenditure, reducing their expenditure on food as total expenditure increases faster than rich families. Richer households do not need to do this, the price of food has little effect on their food consumption. This seems to confirm for New Zealand the validity of Engel's Law.

The Housing elasticity increases slightly in absolute value as the level of expenditure increases - from Q_1, to the means, to Q_3 - and as the household size increases from one to four, households react more strongly to changes in prices reducing the amount they spend. This ability to reduce the impact of price rises seems to confirm the existence of small "economies of scale" in the expenditure for housing, an interesting and quite logical conclusion.

In general, the Total Expenditure elasticities do not seem to vary much among households of different size (see parts D to G of Table 4.7).

It is interesting at this point to look at the own-price share expenditure elasticities, shown in Part A2 of Table 4.7, which are all positive, with the exception of Apparel. An increase in prices will decrease the quantities purchased of these commodities (see the quantity elasticities shown in Part A1 of Table 4.7, which are all negative but less than unity in absolute value), but not enough to absorb completely the price increase, and households need to spend more on them than they did before the price increases. A higher proportion of the households' total expenditure will have to be spent on those commodities which have become dearer: their consumption shares will increase together with prices, but less than proportionately. The share elasticities have an immediate and useful interpretation as they show how changes in prices affect the proportional allocation of a household's consumption budget. As these three commodities can be considered 'necessities', their low price elasticities (indicating a relative insensitivity of demand to changes in price) are perhaps not unexpected.

Apparel's own-price share expenditure elasticity is negative and smaller than unity in absolute value: an increase (decrease) in price will reduce (increase) the share of total expenditure for Apparel less than proportionately, but will reduce (increase) the "quantity" of this commodity more than proportionately. Apparel, unlike the other three commodities, seems to be quite sensitive to changes in its own price.

Most cross-price elasticities (eight out of twelve) are rather small in absolute value, pointing to a substantial level of independence between the consumption of some of these four commodities and the prices of the other ones - a ten percent change in the price of Food, for example, will change the consumption of Housing by only 0.28 percent. Seven cross-elasticities out of twelve are positive, such a large number of positive elasticities is likely to be the reason for the non-negativity of the Slutsky matrix.

The remaining four cross-price elasticities are negative and smaller than unity, but large enough in absolute value to suggest some inter-dependence between the price of Food and the demand for Apparel and Transport; and the price of Transport and the demand for Food and Apparel. In these four cases consumption moves in the opposite direction to a change in price, but in a less than proportionate way.

At low levels of expenditure - corresponding to the Q_1 variable values - Food is almost completely inelastic (see part B of Table 4.7): changes in price leave the quantity of food purchased by the households practically the same. Food becomes more elastic as consumption increases to the mean and the Q_3 expenditure levels (part A.1 and C of the Table 4.7). This increase in the price elasticity of Food, as income (total expenditure) increases, seems to confirm our observation above, when discussing the Total Expenditure elasticities, of the apparent validity for New Zealand of Engel's Law.

For Housing and Transport the own-price elasticities do not change much whether taken at low variable values (i.e. Q_1), or at mean values, or at high values (i.e. Q_3). For Apparel the own-price elasticity more than doubles when consumption increases from Q_1 to mean values, then it decreases slightly at the higher levels of expenditure represented by Q_3.

For most cross-price elasticities the differences at low and high levels of expenditure are small, with the exception of the cross-elasticity between the demand for Apparel and the price of Transport which more than doubles when consumption increase from Q_1 to Q_3.

This relative constancy in the values of the elasticities, for different levels of consumption and different household sizes, seems to indicate that, on average, New Zealand households tend to react to price changes in a similar way, at all levels of total expenditure and for all household sizes.

4.6.2 Presentation and Discussion of the Italian Elasticities

The Italian elasticities, obtained from the DCS-RNLPS model, are shown in Table 4.8, and they are organised in a pattern similar to that of Table 4.7, the only difference being a larger number of observations in the sample and more household sizes.

To consider the household size elasticities first. They are all negative, except for Food, and all less than unity, except for Household Operations. The share expenditure on food increases together with household size but less than proportionately. Apparel and Transport share expenditures decrease when household size increases, but less than proportionately. Housing expenditure decreases more than proportionately, at the means, at the low variable values Q_1, and for households of size 4 and 5 (see parts A1, B, F and G of Table 4.8), but less than proportionately at the Q_3 values, and for households of size 1 and 2 (see Parts C, D and E of Table 4.8).

With respect to Housing expenditure the households' response to changes in their size is greatly affected by both the expenditure level at which they are (Q_1, means or Q_3), and their current size: larger households, and households at low expenditure levels, respond very strongly to changes in their size by decreasing their share expenditure on housing more than proportionately. The share of household expenditure going into housing gets smaller as household size gets larger, a fact pointing to the possibility of substantial economies of scale[20]. This might be explained in more than one way.

One possibility is that while small and large households have similar patterns and levels of housing expenditure - due to the existence of "public" goods like TV sets, refrigerator, telephone, and, most important, the maintenance of the house itself, the cost of which does not increase proportionately with size - larger households are likely to have more than one income earner, and therefore a higher income, a smaller portion of which will be spent on household operations.

[20] Substantial economies of scales in housing are reported by Nelson, 1988, Tables II and III.

Another possibility is that householders do not necessarily and instantly upgrade their housing standards as the size of the household increases. Therefore larger households will spend a substantially smaller share of their incomes on housing than smaller households, usually with lower incomes, do.

Finally the "public" goods available in the house will get used by more members in the larger households, therefore increasing their utilization rate and decreasing their unit cost per member.

Apparel expenditure is much less sensitive than housing operations to movements in household size, but it still reacts negatively to it. The share expenditure on Apparel

TABLE 4.8

DCS-RNLPS Model: Price, Household Size and Expenditure Elasticities for Italy. ML Parameter Estimates.

(All elasticities computed at the means[a])

Part A.1 : Whole sample of 322 observations. Mean household size S=3.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	−1.020	.097	.055	.101	.234	−.331
Housing	.048	-.525	-.197	-.341	−1.016	−.093
Apparel	.023	−.163	−.781	-.292	−.196	.456
Transport	.024	-.161	-.167	-.942	−.035	.552

Part A.2: Whole sample of 322 observations. Mean household size S=3. Own-price share expenditure elasticities.

Food	Housing	Apparel	Transport
-.020	.475	.219	.058

Part B: Whole sample of 322 observations. Mean household size. Elasticities computed at the Q_1 variable values [b]

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	–1.040	.185	.103	.192	.436	–.564
Housing	.062	-.350	-.244	-.426	–1.236	–.120
Apparel	.039	–.273	-.649	-.471	–.310	.729
Transport	.040	–.262	–.258	-.907	–.057	.865

Part C: Whole sample of 322 observations. Mean household size. Elasticities computed at the Q_3 variable values [c]

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	–1.023	.110	.062	.116	.180	–.296
Housing	.031	-.682	-.120	-.211	–.460	–.052
Apparel	.022	–.145	-.813	-.251	–.104	.339
Transport	.021	–.136	–.134	-.951	–.007	.395

Part D: Households of Size 1. Sub-sample of 65 observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	–1.067	.151	.084	.149	.099	–.386
Housing	.043	-.594	-.152	-.263	–.227	–.069
Apparel	.022	–.136	-.824	-.258	–.054	.364
Transport	.031	–.193	–.211	-.949	–.023	.626

Part E: Households of Size 2. Sub-sample of 65 observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	−1.038	.113	.064	.115	.168	−.322
Housing	.044	-.559	-.166	-.288	−.541	−.077
Apparel	.024	−.160	-.789	-.295	−.132	.442
Transport	.029	−.184	−.196	-.943	−.040	.618

Part F: Households of Size 4. Sub-sample of 64[d] observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	−1.005	.084	.048	.090	.282	−.279
Housing	.046	-.467	-.206	-.355	−1.451	−.097
Apparel	.021	−.166	-.772	-.288	−.242	.459
Transport	.023	−.165	−.166	-.932	−.011	.556

Part G: Households of Size 5. Sub-sample of 63[d] observations.

Prices of:	Food	Housing	Apparel	Transport	H. Size	Tot. Exp.
Comm.ties:						
Food	-.993	.068	.039	.076	.306	−.256
Housing	.043	-.464	-.209	-.361	−2.002	−.101
Apparel	.019	−.160	-.725	-.271	−.261	.458
Transport	.022	−.163	−.160	-.925	.053	.574

a *The consumption mean values used to compute the elasticities in Part A, B and C were those for the whole sample of 322 observations. In Parts D, E, F, G and H the means were those of the four sub-samples, grouping the households of equal size.*
b *Q1 = lowest observation + 0.25 Range*
c *Q3 = lowest observation + 0.75 Range*
d *Cells containing zero observations have been dropped*

decreases (increases) as the household size increases (decreases), but much less than proportionately. As households grow larger they spend a smaller proportion of their budgets on clothing.

Transport is totally inelastic with respect to household size, and its share expenditure moves in the opposite direction to changes in household size. For the five member households the Transport household size elasticity becomes positive and the share expenditure on transport moves in the same direction as household size, but much less than proportionately. However the Transport elasticities are all so small in absolute value that the share expenditure on this commodity seems almost independent of the size of the household, and tends to remain constant among all households.

In general, all household size elasticities are higher, in absolute values, at the low levels of expenditure, represented by Q_1, than at the higher levels, represented by Q_3. The model non-linearity clearly shows how households react more strongly to marginal changes in household size at low levels of consumption than at high levels of consumption.

The total expenditure elasticities for Food are negative, which means that as total expenditure increases (decreases) the households share expenditure on food decreases (increases), therefore confirming that for Italy too, as well as for New Zealand, the Engel's Law does indeed apply. Also, for Housing, the total expenditure elasticities are negative, but very small in absolute value. Only at the low expenditure levels represented by the Q_1 values there seems to be any noticeable relationship between the level of total expenditure and housing share expenditure.

For Apparel and Transport the total expenditure elasticities are positive, showing an opposite consumer behaviour than for food and housing: the budget shares of these two commodities increases as the household's total expenditure increase. Their absolute values decrease as total expenditure increases from Q_1 to Q_3. As households reach higher levels of expenditure, Apparel and Transport become more and more inelastic. As household size changes from one to five the Apparel total expenditure elasticity gets larger but only moderately so, the Transport elasticity, on the contrary, shows a slight decrease.

Coming now to the price elasticities, all commodities have negative own-price elasticites rather large in absolute values: those for Food, Apparel and Transport close to unity, that for Housing Operations quite smaller. The low own-price elasticity for Housing Operations is no surprise: given the stickiness of the Italian housing market - where a change of house is often difficult to achieve and very expensive - most households are likely to prefer to spend liberally to maintain in good condition the house they live in, and sometimes to upgrade it, rather then to buy a new one. Thus, the relative insensitivity of Italian households to the cost of furniture, furnishing and house appliances. This fact is shown even more clearly by the housing share elasticity (Part A2 of Table 4.8) which is positive: when the price of household goods increases the share expenditure on them increases, the amount spent on housing represents a larger share of the household budget. The decrease in quantity, caused by the increase in price, is less than proportionate. Therefore the amount spent on this commodity will increase following the price increase.

Transport has a higher elasticity (i.e. reacts more strongly to changes in price) than Apparel, showing that Italians rather walk than dress shabbily, an attitude easily confirmed by taking a stroll in any Italian city. The Transport elasticity remain almost the same at the three expenditure levels at which the elasticities have been computed (Q_1, means and Q_3, shown in Parts A1, B and C of Table 4.8) and for all household sizes (Parts D to G).

The Apparel own-price elasticity is lowest in absolute value at Q_1 (E = -.649), and highest for the households of size one (E = -.824), then increasing to E = -.725 for the households of size five.

The cross-price elasticities between Housing Operations, and the prices of Apparel and Transport are all negative and less than unity; their absolute values decreasing as total expenditure increases from Q_1 to Q_3, and decreasing as household size increases from one to five. Remembering that housing and transport are two of the major items of expenditure for most households, a decreasing (increasing) elasticity as expenditure increases (decreases), and/or household size decreases (increases), reflects rational economic behaviour.

Similar comments apply to the cross-elasticites between Transport and the price of Housing , although they remain almost constant with respect to household size. Most other cross-price elasticities tend not to vary very much among the different household sizes and for the three variable value levels (Q_1, means and Q_3) at which they have been computed.

Comparing now the New Zealand and Italian elasticities the most apparent differences are in the own-price elasticities for Food and Apparel, the former much more elastic in Italy than in New Zealand and the latter much more elastic in Italy than in New Zealand. New Zealanders react much more strongly than Italians to changes in the price of Apparel, Italians react more strongly than new Zealanders to the price of Food. This reflects quite well social attitudes in the two countries: while in Italy to be well dressed is almost a social obligation, in New Zealand there prevails a much more casual attitude.

Of some interest are the almost identical own-price the elasticities for Transport in the two countries: both negative and close to (but less than) unity. In both countries a marginal increase in the cost of transport reduces its consumption by an almost equal amount.

A final general observation is that in both countries most cross-price elasticities are extremely small in absolute value, showing that the demand for most commodities is almost independent of the prices of most of the others.

4.5 The Problem of Separability

To try and evaluate the effects of aggregation on price elasticities we can test whether the decision by consumers of how to allocate their budget among the four commodities considered in this study is independent of the prices of their components. If it is not, then the price elasticities for the aggregated commodities become much more difficult to interpret and there is a need for more disaggregated data. A solution to this problem might be found in the literature on Separability[21] in models of international trade.

The usual assumptions in international trade models, allocating a country's imports among suppliers, is that demand is separable over foreign and domestic sources and that import demands are homothetic and mutually separable (see Winters, 1984 and 1985). This implies a two-stage budgeting procedure whereby total imports are explained by one set of variables whilst their allocation among all the exporting countries producing them is explained by another. More specifically, once total imports have been decided, import allocation is independent of domestic prices.

To test for separability, Winters' approach (1985, p.337) was to estimate a very general but separable demand system for imports, and then test, by means of a Lagrange Multiplier test, whether the introduction in the model of domestic prices would significantly improve the model's explanatory power.

In our case, to check the effects of the prices of the main components of the four commodities considered on budget allocations, we can subdivide the main commodities into their components and then re-estimate the resulting more general model. Then, if the disaggregated model shows a clear improvement in its explanatory power over the more aggregated model, we must reject separability: the prices of the components of the aggregated commodities have an effect on budgeting decisions and the level of aggregation is too high to allow proper economic insight into the consumers' behaviour.

The Italian data we used in Section 4.3 to estimate the models considered, offer the possibility to subdivide the housing expenditure into two sub-groups: "Housing, Fuel and Electricity" and "Furnishings and Home Appliances". Therefore, following Winter, one way of testing for separability could be to re-estimate DCS-RNLPS with Housing disaggregated and then compare the results with those obtained from the aggregated model, with only one housing expenditure group. If the aggregated model is rejected in favour of the disaggregated one this would provide some evidence

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[21] For a rigorous definition and mathematical treatment of Separability see Chapter 3 of Blackorby et al. 1978.

against the assumption of separability, and, therefore, for the need of less aggregated consumption categories.

Because the aggregated demand model (with four commodities) and the disaggregated model (with five commodities) are not only parametrically non-nested, but also have different dependent variables, we cannot apply the above procedure, but instead we must test for Separability directly. This can be done following the procedure suggested by Blackorby et al. (1978, Chapter 8.2.2) which, under a set of assumptions about the form of the demand function and the possibility to approximate it by a Taylor's series expansion, reduces the testing for separability to a simple, and testable, set of parametric restrictions (Blackorby et al., 1978, p. 299).

In our case these parametric restrictions are:

$$\beta_j \gamma_{ik} = \beta_i \gamma_{jk} \quad \text{(for } i,j,k\text{=1,2,...,n and } i \neq j \neq k) \tag{4.5.1}$$

and they can be tested, for the estimated five equation model, by a joint Wald χ^2 test.

The results of the re-estimation of DCS-RNLPS with housing disaggregated into its two components are shown, in summary form, in Table 4.9, where we report the estimated values of the price and demographic parameters for the two disaggregated housing commodities, the corresponding parameter values for the aggregated model, the values of the log-likelihood functions, the number of estimated parameters, and the value of the Wald statistics, with the number of degrees of freedom in brackets. From the Wald tests it appears that the hypothesis of Separability cannot be rejected.

Interestingly enough, the sums of the disaggregated parameters shown in the second row ($\gamma_2 + \gamma_3$, $\delta_2 + \delta_3$ and $\beta_2 + \beta_3$) are almost identical to the respective values of the aggregated parameters shown in the first row of Table 4.8. This seems to exclude the possibility of model mis-specification.

To further evaluate the evidence supporting the assumption of separability we will re-estimate both DCS-RNLPS and DT-RNLPS, the two best performing models so far,

TABLE 4.9

Housing parameter estimates for the DCS-RNLPS aggregated and disaggregated models. Also reported the Log-Likelihood values (LL), the number of parameters (k), and the Wald statistic (W)

Aggregated Housing, V2 and V3 appear as a single variable: V3=V2+V3	γ_3=5.6153	δ_3=-.03164	β_3=.39899	LL=2173.9 k=12 W=1.238 (5)
Disaggregated Housing: V2 and V3 are estimated separately.	γ_2=4.1364 γ_3=1.6714	δ_2=-.01473 δ_3=-.01709	β_2=.04277 β_3=.36316	LL=3133.16 k=15 W= 1.683 (8)

on a set of less aggregated consumption data, which we had available for Italy only. This analysis will be the done in Chapter 5.

Chapter 5

Some Experiments with Disaggregation

5.1 Introduction

Following the results of Chapter 4, which seem to support the assumption of Separability, we attempt in this chapter to re-estimate the two best models tried so far, the DCS-RNLPS and the DT-RNLPS, using Italian data, in which two of the commodity groups considered in Chapter 4 - Food and Housing Operations- can be disaggregated into two sub-groups: Food and Drinks; and Housing Expenditure and Home Furnishings. Thus, this disaggregated demand system contains six equations instead of the four considered in Chapter 4.

One further difference between the data used in Chapter 4 and those we analyse here is that the disaggregated data are subdivided into fifteen income classes instead of five so that the number of observations increases from 325 to 900. This increase in the number of observations, together with the increase in the number of equations in the demand system, made the estimation procedure much slower and the convergence of the iterative process more difficult to achieve. The larger number of observations has improved the performance of the DT-RNLPS model, which has a simpler mathematical structure, but has worsened that of the more complex DCS-RNLPS.

The estimation of the DCS-RNLPS model became more difficult computationally and the resulting parameter estimates less reliable statistically. The choice of the starting values for the iterative procedure had a crucial effect both on the convergence characteristics of the iterative process and the resulting values of the final estimates obtained from it.

In some instances, we obtained different sets of final estimates with very similar LL values which made it difficult to decide which one to consider as the "true" estimates. In other instances, small changes in the starting values of the parameters at the beginning of the iterative process generated completely different sets of final estimates: the iteration process thus showed pronounced chaotic characteristics.

5.2 The Estimation Results

5.2.1 The DCS-RNLPS Model

The increase in the number of observations (from 325 to 900), and in the number of parameters in the demand system to be estimated (from eleven to eighteen), has made the estimation of the DCS-RNLPS model much more difficult.

For many sets of starting values, the estimation process did not converge, or converged to sets of estimates with a negative α, contrary to one of the basic assumptions - viz that α must be positive and less than one - of the consumption function (2.19) underlying the DCS-RNLPS model. In other cases, the process converged to estimates of no economic meaning (eg. parameter values larger than 10^6). Often different sets of starting values converged to different sets of final parameter estimates, among which it was not always possible to choose the "best" estimates (those with the highest LL) because the values of the LL corresponding to the different sets of estimates we had obtained were almost identical. The estimation procedure for DCS-RNLPS showed clear chaotic characteristics.

Computational times were often high: an estimation run of 100 iterations took approximately twenty five minutes, and in some cases the estimation process required between 400 and 500 iterations to converge to a solution. The longest computational time we required was eight hours and seventeen minutes for a run of almost 1500 iterations, and it produced nonsensical results !

From the behaviour of the estimation process, it appears that the increase in the number of observations has flattened the surface of the data generated DCS-RNLPS likelihood function (LF) making it "lumpy": there is a variety of very similar local maxima spread all over the LF hyper-surface among which it is rather difficult to identify any well defined maximum of the LF function, corresponding to economically meaningful parameter values. There is no guarantee that any set of estimates is, in fact, *the* Maximum Likelihood one, even if it corresponds to the highest value of the LF function found over a number of trials.

The flatness of the LF function probably depends on the type of data, based on cell averages and broad aggregates, which cannot show any great differences in the response of households to price changes irrespective of the number of income classes. Thus, the larger number of observations does not increase the amount of information available for the estimation of the model but it only makes the computations more burdensome.

After much experimentation, involving, inter-alia, many sets of starting values and convergence criteria, we are fairly confident to have obtained, within the sub-space of economically acceptable parameter values, the ML estimates, corresponding to a maximum of the LF function. These estimates are shown in the first column of Table 5.1, together with their standard errors, the value of the logarithm of the likelihood function (LL), the number of parameters, and the value of the Akaike Information Criterion (see Judge, Griffith et al. 1985, p.870):

$$AIC = -(2/N)\,LL + (2k/N) \qquad (5.2.1)$$

where N is the number of observations, LL the value of the Log-Likelihood function, and k the number of parameters in the model.

TABLE 5.1

Parameter Estimates for the DCS-RNLPS and DT-RNLPS models.[a]

	DCS-RNLPS		DT-RNLPS		DT-RNLPS (TT)	
δ_1	.0082	(.00472)	-		-	
β_1	.1058	(.00793)	-.5325	(.16440)	-.4740	(.00138)
γ_1	-.0785	(.01593)	5.9661	(.28612)	5.7741	(.98467)
θ_1	-		-.8292	(.03602)	-.9348	(.37690)
α	.0207	(.02036)	.0136	(.00053)	0152	(.00029)
γ_2	.3083	(.42358)	-1.5796	(.84912)	-1.4705	(.36527)
θ_2	-		.3074	(.11701)	.3355	(.12462)
γ_3	.2441	(.30671)	7.9298	(.61582)	6.7376	(1.07981)
θ_3	–		-1.0805	(.07793)	-1.0579	(.41364)
γ_4	.0108	(.01147)	-6.4796	(.42023)	-6.2562	(1.09160)
θ_4	-		.9196	(.05374)	1.0359	(.41909)
γ_5	.0191	(.01028)	-1.9177	(.21291)	-1.8355	(.33229)
θ_5	-		.2758	(.03129)	.3095	(12769)
γ_6	.0262	(.02273)	-13.859	(.41718)	-13.8330	(2.37830)
θ_0	-.1968	(.76779)	–	–	–	–
θ_6	-		1.9667	(.05458)	2.2861	(.91526)
δ_2	.0378	(.02710)	–	–	–	–
δ_4	-.0114	(.00627)	–	–	–	–
δ_3	.0321	(--------)	–	–	–	–
δ_5	-.0811	(.01175)	–	–	–	–
δ_6	.0144	(.09995)	–	–	–	–
β_2	.2494	(.18453)	.1716	(.07450)	.1495	(.01246)
β_3	.1413	(--------)	-.6762	(--------)	-.5224	(--------)
β_4	.1748	(.11241)	.5949	(.03017)	.5289	(.00472)
β_5	.1248	(.07188)	.1828	(.02106)	.1616	(.00355)
β_6	.2039	(.16947)	1.2594	(.03293)	1.1565	(.00781)
LL	9180.26		10,070.88		10,072.68	
k	20		19		20	
AIC	-20.356		-22.293		-22.339	

[a] *Parameters δ_3 and β_3 have been obtained as a residual from the restrictions in (4.2.18).*
Figures in brackets are standard errors.

All parameters have acceptable values, although α appears to be very small, much smaller than its estimates obtained in Section 4, with the exception of those obtained from the NLPS model. The value of θ_0 is negative, confirming the findings of Section 4.5 that the average household expenditure on non-food commodities decreases as household size increases (see footnote 16). Some of the standard errors are rather large, as might be expected, given the unstable behaviour of the estimation process.

Because of all the reasons given above we consider the parameter estimates for the DCS-RNLPS model to be unreliable and, therefore, they should be treated with caution.

5.2.2 The DT-RNLPS Model

For the DT-RNLPS Model, the increase in the number of observations has had a beneficial effect on the performance of the model during estimation. Convergence is reached from a variety of starting values and most of them give convergence to the same set of final estimates. When the estimation process converged to different sets of estimates, it was usually possible to distinguish among them those corresponding to local maxima, because the values of their LL functions were much lower than those associated with the ML estimates. However in a few instances the estimation process converged to different sets of final estimates with very similar LL values. Thus it appears that convergence problems may be present in the DT-RNLPS model too. The most likely cause of these convergence problems is likely to be the type of data we are using, which do not allow enough variation among the consumption patterns of different households, grouped only according to size and income, with all the other economic and social characteristics averaged out within the reporting cells.

The simpler mathematical structure of the DT-RNLPS Model seems to be able to take advantage of the increased number of observations to produce statistically better estimates than those obtained in Chapter 4. We show the estimates of the parameters of the DT-RNLPS model in the second column of Table 5.1, together with their standard errors, the value of the LL function, the number of parameters, and the value of the Akaike Information Criterion.

The values of the parameters are all economically acceptable, and the low standard errors confirm the good behaviour and the robustness of the estimation process. The LL value for the DT-RNLPS Model is higher, and the Akaike Information Criterion value lower, than those obtained for the DCS-RNLPS model, making DT-RNLPS the statistically preferred structure.

However when we computed the price elasticities for the DT-RNLPS we found that the own-price elasticity for Drinks was too high ($E_d > 5$) to be economically meaningful. We re-estimated DT-RNLPS, with a simple linear time trend added to the Drinks equation,[1] to try and take into account the possibility that time-related non-economic components, such as fashion or changing social habits (like eating out, or the growing use of mineral water, instead of tap water), are influencing the consumption of Drinks. These alternative estimates are shown in the third column of Table 5.1.

Although most of the standard errors of estimate are smaller for DT-RNLPS without a time trend than with a time trend, the Akaike Information Criterion is lower for the latter than for the former. Faced with such contradictory statistical evidence, we chose as our preferred structure the version of the model which generated more economically meaningful consumption elasticities. As this happened to be the model with a time trend added we took it as our preferred structure, and we present and discuss the elasticities computed from its parameters in Section 4.

5.3 The Elasticities

From the parameter estimates shown in Table 5.1, we have computed the price, household size and the total expenditure elasticities[2] for the DCS-RNLPS, the DT-

[1] To ensure adding-up a time trend is also implicitly added to equation 3.

[2] Also in this chapter the price elasticities refer to quantities but total expenditure and household size elasticities refer to share expenditures.

RNLPS model without a time trend in the Drinks equation, and the DT-RNLPS model with a time trend in the Drinks equation.

The elasticities for the DT-RNLPS model with a time trend in the Drinks equation are reported in Table 5.2; the elasticities reported in Part A are computed at the means, in Part B they are computed at the Q_1 variable values, and in Part C at the Q_3 variable values. To save space we do not report the elasticities for the DCS-RNLPS model and the DT-RNLPS model without a time trend, but they are available on request.

We have also computed the household size elasticities separately for the five household sizes, to quantify the effects of marginal changes in household size for households of different size. They are reported in Table 5.3.

All the own-price elasticities obtained from the DCS-RNLPS model were negative and had an absolute value very close to unity; they only differed in their second or third decimal point positions. Conversely, all the cross-price elasticities had absolute values very close to zero, but, once again, very similar to one another as they only differed in the third or fourth decimal point positions. Because it is most unlikely that Italian households react almost exactly in the same way to price changes in each of the six consumption categories analysed, we have to conclude that the DCS-RNLPS model seems unable to capture, with any degree of accuracy, the reactions of Italian households to price changes.

The poor performance of the DCS-RNLPS model is likely to be a consequence of the type of the data we had available which, consisting of large cell averages, are not the ideal ones to take into account the possibility of consumers shifting their resources among the components of the six broad consumption categories considered here, while leaving the overall allocation of their budgets among them almost unchanged. These results suggest that our data do not report the consumption behaviour of the Italian households in fine enough detail to measure accurately the inter-commodity price effects. The increase in the number of income classes instead of improving the quality of the data brings about "chaotic behaviour" into the estimation process, thus making the resulting parameter estimates unreliable. The consumers' behaviour, with respect to price movements, cannot be properly captured by the model.

Most of the elasticities computed from the DT-RNLPS model *without a time trend* seem to have the right sign and size. A notable exception is Drinks, which has not only a very large own-price elasticity ($E_d > 5$) with a positive sign, but also some very large cross-price elasticities, especially for Apparel and Transport. One possible reason for such large elasticities - and wrong sign for the own-price elasticity - is that non-price effects, like changing social attitudes and fashions, might dominate price effects. To test this assumption, we again experimented with the introduction of various types of time trends.

The most interesting results were obtained by adding to the Drinks equation a simple linear time trend. The resulting parameter estimates are shown in the last two columns of Table 5.1. These estimates are very similar to those obtained from DT-RNLPS without a time trend - a fact pointing to robustness of the estimates. In spite of the similarities between the parameter estimates, the elasticities obtained from the DT-RNLPS model without a time trend were rather different from those derived from it when a time trend was added.

The own-price elasticity of Drinks derived from the DT-RNLPS model with a time trend, is still positive, as it was in the absence of a time trend, but is now smaller and less than unity. The positive own-price elasticity for Drinks can be partially explained by the phenomenal rise in Italy, during the last fifteen years, in the consumption of mineral water which has almost completely replaced tap water for drinking, and sometimes also for cooking. This has happened regardless of substantial price increases.

More than half of the cross-price elasticities, as it was the case for the four equation model, are very small in absolute value ($E_{ij} < |.1|$), the quantities consumed of the six

Table 5.2

Price and total expenditure elasticities for the DT-RNLPS Model with a Time Trend in Equation 1. Italian data with six commodities.

Part A : Elasticities computed at the means.

Prices of:	Drinks	Food	Housing	Apparel	Furnish.	Transport	Hous. Size	Tot. Exp.
Drinks	**.870**	-.094	-.274	-.639	-.184	-1.414	.104	-.265
Food	-.007	**-1.015**	-.020	-.017	.005	.038	.298	-.010
Housing	.061	-.021	**-.728**	-.082	-.024	-.182	-.310	-.049
Apparel	-.185	.064	-.284	**-1.211**	.068	-.526	-.181	.076
Furnish.	-.073	.025	-.112	.094	**-1.141**	.209	-.335	.020
Transport	-.304	.106	-.467	.391	.113	**-1.108**	.096	.124

Part B: Elasticities computed at the Q1 variable values [a b]

Prices of:	Drinks	Food	Housing	Apparel	Furnish.	Transport	Hous. Size	Tot. Exp.
Drinks	**2.701**	-.186	-1.436	-1.264	-.364	-2.799	.203	-.519
Food	-.014	**-1.029**	-.039	.034	.010	.076	.590	-.020
Housing	.120	-.042	**-.461**	-.163	-.047	-.361	-.613	-.099
Apparel	-.366	.127	-.562	**-1.419**	.135	-1.041	-.357	.151
Furnish.	-.144	.050	-.222	.185	**-1.280**	.414	-.663	.041
Transport	-.602	.209	-.924	.773	.225	**-1.215**	.190	.241

Part C : Elasticities computed at the Q3 variable values [a c]

Prices of:	Drinks	Food	Housing	Apparel	Furnish.	Transport	Hous. Size	Tot. Exp.
Drinks	**-.518**	-.024	-.187	-.165	-.047	-.364	.018	-.072
Food	-.005	**-1.009**	-.012	.011	.003	.-24	.187	-.007
Housing	.035	-.013	**-.835**	-.050	-.014	-.110	-.194	.028
Apparel	-.109	.038	-.173	**-1.123**	.040	-.307	-.099	.047
Furnish.	-.034	.012	-.054	.043	**-1.065**	.096	-.152	.010
Transport	-.140	.050	-.224	.179	.052	**-1.050**	.051	.060

a *Household size = 3.*
b *Q1 = lowest observation + 0.25 Range*
c *Q3 = lowest observation + 0.75 Range*

commodities considered here are independent of the prices of most other commodities.[3]

Only the cross-elasticity between the consumption of Drinks and the price of Transport is larger than unity, may be because the recurrent large increases in the excise tax on petrol, which used not to be in the Consumers Price Index, have a contractionary effect on people's social and drinking habits.

Other large cross-elasticities are those between the consumption of Transport and the prices of the Drinks, Housing and Apparel, the consumption of Drinks and the price of Apparel, and the consumption of Apparel and the price of Transport; in all these cases the consumption of a specific commodity will be substantially reduced by a marginal increase in the price of the other commodity, but the reduction will be less than proportionate to the increase in price.

Among all the negative cross-elasticities we find very interesting those between Transport and Housing, which seem to refute the stereotype of Italians being more keen to spend money on their cars than on their homes. In fact, the opposite seems true, with the expenditure on transport being affected by the cost of housing more strongly than the expenditure on housing is affected by the cost of transport (the respective elasticities being E_{TH}=.467 and E_{HT}=.182).

The own-price elasticity for Housing is negative but smaller than unity, as might be expected for such a necessary commodity, the "quantity" of which cannot change very much in response to price changes. Housing has also a strong influence on the consumption of all the other commodities, except Food, as can be seen from their cross-price elasticities with respect to it, all rather large in absolute value. Considering the importance, in any household budget, of the cost of housing, its price is bound to have a sizable effect on the amounts of all other commodities purchased by the households.

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[3] In analyzing the cross-price elasticities it should be kept in mind that at high levels of commodity aggregation, such as the ones we are using, it might make little economic sense to talk about cross substitution between most of the commodities we consider here. Thus our results are only tentative

For the other four commodities the own-price elasticities are negative and marginally larger than unity in absolute value, their quantities change more than proportionately in response to price changes, and in the opposite direction.

The total expenditure share elasticities are negative for Drinks, Food and Housing, and positive for the other three commodities. The share of expenditures on these commodities decreases as total expenditure increases, but much less than proportionately. Only for Drinks the decrease is substantial, a ten percent increase in total expenditure corresponds to a two and a half percent decrease in its expenditure share. Out of the three remaining commodities with total expenditure positive share elasticities, those for Apparel and Furnishings are very small, and only the Transport expenditure share seems to depend, to some extent, on total expenditure - if total expenditure increase by ten percent then the expenditure share on Transport increases by one and a quarter percent.

In general, total expenditure share elasticities become smaller in absolute value as the households' total expenditure increases from the low Q_1 values (Part B of Table 5.2), to mean values, to the higher Q_3 values (Part C of Table 5.2), showing that for all the commodities considered, consumer reaction to increased total expenditure becomes weaker as total expenditure increases.

Also worth noting is how the Housing elasticity becomes positive at the higher levels of expenditure implied by Q_3. For the households already enjoying a higher than average standard of living, the expenditure share on Housing increases, but much less than proportionately, when total expenditure increases. This is a complete reversal of what happens at lower levels of expenditure, at which the expenditure share of housing decreases as total expenditure increase.

In Table 5.3, we show the household size elasticities for the six commodities, and the five household sizes considered, computed at the consumption mean values for the sub-samples of 180 observations, where the households are grouped according to size.

The household size elasticities at the consumption mean values for the whole sample of 900 observations are shown in the eight column of Table 5.2

Table 5.3

Household size elasticities[a] for the DT-RNLPS Model with a Time Trend in Equation 1. The mean variable values are for the specific sub-samples of 180 observations.

Comm.ies	One Comp.nt	Two Comp.nts	Three Comp.nts	Four Comp.nts	Five Comp.nts
Drinks	.048	.077	.117	.144	.148
Food	.115	.184	.257	.375	.433
Housing	-.071	-.161	-.266	-.445	-.621
Apparel	-.052	-.129	-.199	-.262	-.337
Furnishings	-.081	-.185	-.337	-.521	-.667
Transport	.024	.051	.066	.122	.170

[a] Computed at sub-samples' the means.

Two characteristics of these elasticities are rather striking. The first is their extreme low absolute values for the smaller households of only one component. The second is how changes in household size tend to generate much less than proportional changes in share expenditure.

The low values of the size elasticities for the households with only one component seem to indicate that the pattern of expenditure (the way total expenditure is shared among the six commodities) does not change much when there is a marginal increase in the household size. In view of the discrete character of household size, a marginal increase must be a straightforward change from one member to two, with the extra member most likely (but not necessarily) being an adult. It then follows that the expenditure patterns in households with one or two adults must be so similar that a change in their size, from one member to two, has no detectable effect on how the households' resources are shared among the commodities included in their budget.

For the larger households, the size elasticities indicate that, as the household size increases, there will be increases in the proportions of total expenditure spent on Food, Drinks and Transport, and decreases in the proportions for Housing, Apparel and Furnishing.

Chapter 6

Household Consumption Equivalence Scales

6.1 Introduction

Consumption equivalence scales are supposed to measure the relative levels of expenditure required by households of different size and composition to attain a comparable standard of living. They are usually expressed in numbers which are multiples of the scale value arbitrarily given, usually unity, to a chosen reference household, often comprising two adults with no children. Such scales can be used, inter alia, in the measurement of poverty and inequality, in setting the levels of welfare benefits, and in linking income tax rates to the taxpayers' ability to pay.

One of the basic assumptions in the computation of equivalence scales is that household well-being at a fixed level of income, or total expenditure, is an inverse function of the household size[1]. This implies that as the household size increases the household expenditure must increase too in order to maintain a constant standard of living.

Considering the difficulty in defining, let alone measuring, the "standard of living", a variety of methods can be devised to calculate equivalence scales. A rational way to resolve this issue is to equate the standard of living with the concept of "utility", and assume that two households having the same standard of living will enjoy the same level of utility (see Barten 1964 and Muellbauer 1974). Equivalence scales between households of different composition can then be found by equating their levels of utility and then comparing the incomes, or total expenditures, required by the different households to achieve that level of utility.

Following the logical development of the present work we first tried to estimate the consumption equivalence scales for the New Zealand and Italian households from the demographically extended, and preference-consistent, demand models discussed in Chapter 4. Unfortunately, when we tried to compute the equivalence scales from the estimated parameters of the DCS-RNLPS and DT-RNLPS models, we obtained results which were clearly unacceptable.

The general equivalence scale m_0 generated from the DCS-RNLPS model, which has been shown in Table 4.6, is unacceptable as it shows that for New Zealand the households consumption requirements increase in almost exact proportion to their size, and for Italy that they actually decrease with size. For the DT-RNLPS model the equivalence scales assume negative values for Italy, and decreasing values for New Zealand, when household size increases from three to four components.

As a consequence we had to resort to a variant of the LES model often utilized in empirical studies on equivalence scales and recently applied to Australia (see Binh and Whiteford 1990), and New Zealand (see Rutherford et al.,1990, and Smith, 1989)

The plan of this chapter is as follows: Section 6.2 describes the Extended Linear Expenditure System which we use to estimate the equivalence scales; in Section 6.3 we report and discuss the New Zealand results, and in Section 6.4, the Italian results.

6.2 Theoretical Framework

6.2.1 The ELES Equivalence Scales

The Extended Linear Expenditure System (ELES) derives from the Klein-Rubin utility function[2] we discussed in Chapter 2, but modified in a way to incorporate the scale effects in household consumption:

$$U(q) = \sum_{i=1}^{n} \beta_i \log[(q_{ij} / s_{ij}) - \gamma_i] \quad \text{for } j = 1, ..., M \tag{6.2.1}$$

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1 For a contrary view see Pollak and Wales, 1979.

2 This type of utility function is directly additive (see Deaton, 1974). This can be a very restrictive assumption when dealing with highly disaggregated commodities, which however is not the case in this study.

where q_{ij} is the quantity of the *ith* commodity consumed by households of composition type *j*, s_{ij} is the consumption equivalence scale[3] for the *jth* type household, β_i is the marginal budget share and γ_i is a parameter representing the "subsistence" consumption[4] for the *ith* commodity. The household type composition being defined by the number of adults and children in the household.

Equation (6.2.1) is subject to the following constraints:

$$\beta_i > 0, \gamma_i > 0 \text{ and } \sum \beta_i = 1, \text{ for } i = 1,\ldots,n \tag{6.2.2}$$

If we now maximise the consumers' utility as expressed by (6.2.1), subject to the budget constraint :

$$\sum p_i q_{ij} = v_j \tag{6.2.3}$$

where p_i is the price of the *ith* commodity and v_j the total expenditure by the *jth* type household on the commodities considered, then we obtain (see Lluch 1973 and Kakwani 1980, Ch 16) the modified LES demand system:

$$v_{ij} = \alpha_{ij} + \beta_i (v_j - \alpha_j) \tag{6.2.4}$$

where: $v_{ij} = p_i q_{ij}$ is the expenditure on the *ith* commodity by the *jth* household

$\alpha_{ij} = p_i\, s_{ij}\, \gamma_i$ is the "subsistence" expenditure on the ith commodity by the *jth* household and

$\alpha_j = \Sigma_i\, \alpha_{ij}$ is the *jth* household's total "subsistence" expenditure.

Equation (6.2.4) implies that only the intercepts α_{ij} are affected by household composition and it has been shown by Muellbauer (1974) that not all α_{ij} are identifiable.

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3 Equivalence scales are defined in relation to a Reference Household whose scales - s_{ir} - are set to unity.

4 For a discussion of the concept of subsistence expenditure see Kakwani, 1980, Ch. 9.

In order to identify the subsistence parameters, Kakwani (1977) utilised an extended version of the LES, suggested by Lluch (1973), which contains an additional linear aggregate micro-consumption function:

$$v_j = (1\text{-}\beta)\ \alpha_j + \beta\, y_j \tag{6.2.5}$$

where β is a common marginal propensity to consume and y_j represents the *jth* household's net income (or total expenditure). The n+1 equations system, obtained by combining (6.2.4) and (6.2.5), is identified in all of its parameters (for a proof see Kakwani, 1980, p.353-4) and it defines the ELES demand system modified for household composition.

For the reference household ***r***, the *ith* commodity-specific equivalence scale must, by definition, be s_{ir} = 1, for all n commodities in the budget; therefore for the jth household, the commodity-specific equivalence scale must be:

$$s_{ij} = \alpha_{ij}/\alpha_{ir} \tag{6.2.6}$$

and the household-type equivalence scale must be:

$$s_j = y_j / y_r \tag{6.2.7}$$

if y_j and y_r are the minimum expenditures required for a given utility level. For net income approaching subsistence expenditure, (6.2.7) becomes (see Binh and Whiteford, 1990, p.229):

$$s_j = \alpha_j/\alpha_r \tag{6.2.8}$$

We will estimate the equivalence scales described by (6.2.6) and (6.2.8) for New

Zealand and for Italy. For New Zealand, the reference household will be the two-adult household with no children; for Italy, the two member household[5].

6.2.2 The Estimation of the ELES Model

Let us transform equation (6.2.5) as:

$$v_j = \delta_j + \beta\, y_j \qquad (6.2.9)$$

and estimate its parameters; then from the estimated δ.s it will be possible to obtain the subsistence expenditure of household type j from the relationship: $\alpha_j = \delta_j /(1-\beta)$ and use it to compute a new explanatory variable $z_j = (v_j - \alpha_j)$ to substitute into equation (6.2.4) to obtain the new demand system:

$$v_{ij} = \alpha_{ij} + \beta_i\, z_j \qquad (6.2.10)$$

where all α_{ij} are fully identified.

If we now add to every equation in the demand system described by (6.2.9) and (6.2.10) an error term with zero mean, and uncorrelated across household types and over observations - that is if we assume that $E(\varepsilon_{ij}) = 0$ and that $\mathrm{Var}(\varepsilon_{ij})$ is a block variance- covariance matrix with σ_{jj} elements on the diagonals and 0 everywhere else - then we imply that the error term variance is not constant over households of different composition. It is well known that, under such assumptions, which imply heterosckedasticity, the ordinary least square estimators are no longer best linear unbiased (BLUE), but generalised least squares (GLS) estimators will be BLUE (see Kakwani,1980, p. 356-7).

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5 It must be remembered here that the Italian data give no information on household composition.

To estimate the above demand system from the Italian and New Zealand household consumption data we try three GLS type estimation procedures. The Kmenta procedure discussed in Chapter 2.2.2; a weighted least squares procedure (WLS) and a procedure suggested by White (1980a). All three procedures are commonly used to deal with heterosckedastic data. Because we have already discussed at length the Kmenta procedure, we summarise here only the other two.

The WLS procedure - often suggested in the literature[6], as a way to deal with the estimation of heteroskedastic models, such as for example household expenditure systems whose observations are cell averages, as in the present instance - essentially consists in weighting the original observations by some other variable[7] which is formally related to the variance of the model error component. Let us consider the regression model:

$$Y_t = \beta X_t + \varepsilon_t \quad \text{where } \mathrm{Var}(\varepsilon_t) = \sigma^2/W_t \quad \text{for } t=1,\ldots,T \tag{6.2.11}$$

where W represents the variance modifying variable, if we then transform the regression model as:

$$\sqrt{W_t}\; Y_t = \sqrt{W_t}\; \beta X_t + \eta_t \tag{6.2.12}$$

we obtain a new model which is homosckedastic, because it is $\mathrm{Var}(\eta_t) = \sigma^2$, and therefore it can be estimated by OLS.

The White procedure applies to a regression model like (6.2.9) when the errors are assumed to have an heteroskedastic structure such that: $\mathrm{Var}(\varepsilon_t \mid X_t) = g\,(X_t)$, with g a known, and possibly parametric, function. Then it can be shown (see White, 1980a, p. 818-21) that the matrix:

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6 As a general reference see Judge et al. 1985, Ch 11. For a specific application to the ELES model see Binh and Whiteford, 1990.

7 In the present instance we used as weights the number of households in the cells.

$$(X'X/T)^{-1}\, V^*(X'X/T)^{-1} \text{ with } V^* = [\Sigma\, e_t^2\; X'X]/T \qquad (6.2.13)$$

where $e_t = Y_t - b\,X_t$ represents the regression t residual and, b ,the OLS estimator of β, is a heteroskedastic-consistent estimator of the model's variance-covariance matrix.

Since $[(b - \beta) / \sqrt{T}\,]$ converges to normality, matrix (6.2.13) can also be used to construct asymptotic confidence intervals for the parameter estimates. These results can be extended to non-linear models - $Y = f(X, \beta) + \varepsilon$ - by replacing, in the above formulas, X with $[\delta f(X, \beta)/ \delta \beta]$.[8]

Because all household types must have a common marginal propensity to consume β, all the equations in system (6.2.9) must be estimated together. In a procedure already discussed in Chapter 2, and described in (2.2.11) and (2.2.12), system (6.2.9) was collapsed into one single equation[9] in which the v_j variables were stacked one after another to generate a new dependent variable V with $(M \times T)$ observations, and the δ_j parameters were associated with $[(M \times T) \text{ x } 1]$ vectors of dummy variables D_k containing only ones and zeros, in a pattern similar to the one shown in the first RHS matrix of equation (2.2.12).

6.3 The ELES Estimation results

6.3.1 The ELES Results for New Zealand

In the estimation of the aggregate micro-consumption system (6.2.9) the best results were obtained by the WLS procedure. The White procedure gave results similar to the WLS procedure but its goodness of fit was marginally worse. Also its estimate of

8 See White, 1980b.

9 For a similar procedure of collapsing a demand system into a single equation see Griffiths and Valenzuela , 1996.

the subsistence consumption level for the four-member households appeared to be an under-estimate as it was smaller than the estimated subsistence consumption level for the three-member households.

The Kmenta procedure did not perform well at all: the estimated subsistence consumption level for the four-member households not only was less than the estimated subsistence consumption level for the three-member households, but also almost the same as that for the two-member reference household.

The WLS estimates of the δ_j parameters appearing in (6.2.9), and the estimates of the subsistence expenditures levels a_j, derived from them, are shown in Table 6.1, together with the minimum observed total consumption for the household type.

Once the subsistence expenditure parameters α_j had been estimated, we plugged them into equation (6.2.4) to obtain equation (6.2.10) which was then estimated by a constrained[10] Seemingly Unrelated Regression (SUR) procedure[11] (see Zellner, 1962). The results are reported in Table 6.2.

Table 6.1

Estimates for New Zealand of the parameters of the micro-consumption function and of the household subsistence expenditures

Hous. Size	1	2[a]	3	4
δ_j	39.44	85.88	104.95	114.83
St. Errors	6.995	8.683	11.701	11.150
α_j	62.26	135.57	165.67	181.26
Min. Obs.[b]	70.95	124.26	141.89	169.98
β = .366[c] (.0143)	R^2= .876[d]	DW= 1.758	R^2= .905[e]	LL=-954.93

[a] Reference household. [b] Minimum observed total consumption. [c] Standard error in brackets. [d] Adjusted for d.f. [e] Between observed and predicted.

10 The constraints were $\Sigma\, \alpha_{ij} = \alpha_j$, see Binh and Whitford, 1993, p.228.

11 The four equations in the demand system had to be estimated together because of the across-equation restrictions.

From Table 6.1 and 6.2 we can now obtain the commodity-specific and the household equivalence scales based on equations (6.2.6) and (6.2.8). These are reported in Table 6.3.

The values of the subsistence expenditures, both household specific as reported in Table 6.1, and commodity specific as reported in Table 6.2, compare well with the

Table 6.2

Estimates for New Zealand of the commodity-specific subsistence expenditures

Hous. Size	**1**	**2[a]**	**3**	**4**
α_{1j} (Food)	18.27 (1.4511)	43.69 (1.4646)	60.36 (1.4921)	74.96 (1.4910)
α_{3j} (Housing)	24.67 (1.1823)	42.40 (1.1933)	47.18 (1.2157)	46.28 (1.2148)
α_{4j} (Apparel)	2.38 (.0828)	9.05 (.8304)	14.22 (.8460)	18.62 (.8454)
α_{5j} (Transport)	16.94 (1.6749)	40.44 (1.6905)	43.91 (1.7223)	41.40 (1.7211)
Com. dities	**1**	**3**	**4**	**5**
β_i	.235 (.0064)	.229 (.0053)	.099 (.0037)	.437 (.0074)
R^2	.916	.906	.819	.940
LL	162.145			

[a] Reference Household. *Standard Error in brackets.*

Table 6.3

New Zealand commodity-specific and household-type equivalence scales

Hous. Type[a]	**(1,0)**	**(2,0)[b]**	**(2,1)**	**(2,2)**
Food	.42	1.0	1.39	1.72
Hous. Ops.	.58	1.0	1.11	1.09
Apparel	.26	1.0	1.57	2.06
Transport	.42	1.0	1.09	1.02
Tot. Exp.	.46	1.0	1.22	1.34

[a] First digit in brackets refers to the number of adults in the household, the second to the number of children. [b]Reference Household.

observed minimum amounts that the households have spent in total - see the fifth row of Table 6.1 - and on the four commodities[12].

The estimated commodity specific subsistence expenditures are slightly above the observed minimum expenditures for all commodities and all household types, with the exception, for single member households, of the subsistence expenditures on Food and Apparel, which are substantially less than their observed minimum expenditure, and for Transport which is almost the same.

Although these findings might seem in contrast with one of the fundamental conditions of model (6.2.1), in which for $v_{ij}<\alpha_{ij}$ the underlying utility function is not defined, we feel they are acceptable on empirical grounds, as as it is not realistic for the subsistence level of expenditure to be much below the *observed minimum expenditure in the whole sample*. In fact, a subsistence expenditure close to the minimum observed expenditure does insure that for most observations in the sample the condition $v_{ij}<\alpha_{ij}$ is actually fulfilled. [13]

The equivalence scales reported in Table 6.3 look quite acceptable and in fact both the total expenditure scales, and the commodity specific scales, are similar to those obtained for Australia by Binh and Whiteford (1990, Table 4 and 5) for the corresponding commodity groups and household types.

As far as we are aware this is the first time that a set of commodity specific equivalence scales, derived from the constrained maximisation of a utility function, has been computed for New Zealand from actual household budget consumption data[14].

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[12] We did not report the minimum observed commodity expenditures in Table 6.2 to avoid excessive clutter, but they are available on request.

[13] Also, if we take into consideration the estimation error, we would find that the lower bound for the estimated subsistence expenditure is much smaller than the minimum sample observation.

[14] Because our data are time series of household budgets spanning a nine years period, the equivalence scales we derived from them implicitly take into account the effects of price changes.

Most of the other recent studies available on the literature on equivalence scales for New Zealand either have not been based on the analysis of expenditure data (see Easton, 1995, p.93) or, if computed on HEIS data (Rutherford et al., 1990), were not derived from utility based demand models.

The only comparable work is Smith (1989) who estimated household equivalence scales from the ELES model using unit record data from the 1985/86 HEIS organised in a much more disaggregated form then the data we worked with. Smith's data take into account up to twenty types of households, grouped not only according to the number of adults and children in the household but also their ages. Smith's work being an inhouse research report of Statistics New Zealand was able to use HEIS data in its unit record form - a privilege not available to outside users.

The total expenditure scales he obtained for the households with one adult member, two adults and a child, and two adults and two children match our total expenditure scales reported in Table 6.3 almost exactly. The Smith scales were: .43, 1.23 and 1.31; while ours are: .46, 1.22 and 1.34. Smith did not report any commodity specific equivalence scales.

6.3.2 The ELES Results for Italy

The application of the ELES method to the Italian data, to compute the equivalence consumption scales, proved very difficult due to the characteristics of the data themselves.

When estimating the expenditure subsistence levels α_j from equation (6.2.9) we obtained substantially different results from the three estimation methods we tried, a clear signal of non-robust estimates.

We choose the set of estimates obtained from the Kmenta procedure, reported in Table 6.4, because the other two procedures under-estimated the subsistence expenditures for the households with one component (they were almost zero), and over-estimated the subsistence expenditures for the households with four and five components (they were almost as large as the largest observation in the sample). The White procedure estimate for the subsistence expenditure level for the one component households was in fact negative.

In the fifth row of Table 6.4 we report the total expenditure equivalence scales for Italy derived from the α_j estimates. They seem to be in broad agreement with the specific scales for Food shown Table 6.5.

Our attempts to estimate, from (6.2.10), the α_{ij} parameters representing the commodity specific subsistence levels for each household type, proved unsuccessful as the α_{ij} estimates for all commodities except Food were too small to be realistic (once again almost zero), in some extreme cases they were even negative.

Table 6.4

Estimates for Italy of the parameters of the micro-consumption function, the household subsistence expenditures[a] and the total expenditure equivalence scales.

Hous. Size	1	2[b]	3	4	5
δ_j	14.330	19.799	29.357	33.155	35.260
St. Errors	4.894	1.331	2.395	2.645	3.011
α_j	235,300	325,110	482,050	544,420	578.98
Tot.Exp.Eq. Scales	.78	1.0	1.48	1.67	1.76
β = .939[a] (.00069)	R^2= .989[c]	DW= 1.63	R^2= .999[d]	LL=-608.84	

[a] Standard Error in brackets. [b] Reference Household [c] Raw-moment R-square
[d] R-square between observed and predicted.

The expenditure for Food, which was the only one growing steadily together with household size, dominated the estimation and captured most of the "available" households subsistence expenditure.

These findings seem to confirm what we found in Chapter 4, Table 4.5: in the case of Italy the household size parameter θ_0 in the DCS-RNLPS is negative, and as a consequence the general equivalence scale m_0, reported in Table 4.6, decreases as the household size increases. The peculiarity of the Italian data was also captured by the commodity specific household size parameter θ_i of model DT-RNLPS, the estimated values of which, as reported in Table 4.5, are negative. The only exception is θ_1, the household size parameter for Food, which is positive.

The consistency of these results clearly indicates that these apparently wrong model responses do not depend on the characteristics of the models themselves but rather on the peculiarities of the data (which we have discussed at length in Chapter 4, footnote 16) reporting decreasing household expenditures as the households size increases.

Table 6.5

Estimates for Italy of the commodity-specific subsistence expenditure for Food and the relative equivalence scale

Hous. Size	1	2[a]	3	4	5
m_{1j}	.64	1.0	1.36	1.73	2.09
α_{1j}	117,530	207,370	284,400	349,250	427,340
Stan.d Error	12.910	12.971	14.260	14.973	20.811
Food Equiv. Scale	.57	1.0	1.37	1.68	2.06
$\beta = .176$[b] (.0059)	$R^2 = .835$	$R^2 = .832$[b]	DW=1.699		

[a] Reference Household. [b] Adjusted for d.f.

In consideration of the fact that the estimates of the subsistence parameters for Food α_{1j}, obtained by the WLS method, appeared to be acceptable statistically as well as economically, we report them in Table 6.5, together with the resulting equivalence scales.

In the second row of Table 6.5 we also report the values of the specific household equivalence scales for Food m_{1j}, obtained from the parameters (reported in the fourth column of Table 4.5) of the DT-RNLPS model, by applying definition (4.2.19). We do not report the DT-RNLPS equivalence scales for the other commodities because, as is the case for the ELES estimates, they show decreasing household expenditures as the household size increases.

The ELES and the DT-RNLPS scales seem to agree exceptionally well, notwithstanding the totally different models used to estimate them. This consistency in values of the two scales gives some reassurance about their robustness and about the ability of the models to capture rational, and expected, household consumption behaviour when the data are structured in a way that allows such rational behaviour to become apparent.

In the case of the Italian data this does not generally happen because households are grouped together only according to size, so that most of the variation in behaviour we can expect from households with different demographic and social characteristics, is absorbed into the all inclusive cell averages.

Another factor which might hide the true consumers' reactions to price changes is the broad consumption groups considered here. When prices change consumers are more likely to switch their expenditures between goods included *within* the groups rather than *among* the groups.

Conclusions

In this study we have used time series of household consumption data for Italy and New Zealand to estimate a variety of preference consistent demand models directly derived from specific utility functions.

In Chapter 1, we briefly discussed and described in a comparative framework the household consumption data collected over a number of years for samples of New Zealand and Italian households.

In Chapter 2, we analysed the properties and characteristics of the Linear Expenditure System (LES), which derives from the addilog utility function suggested by Klein and Rubin (1947-48), and we estimated its parameters by repeated applications of the Kmenta (1986, p. 616-35) procedure for pooling time series and cross-sectional observations, and/or of Zellner's (1962) Seemingly Unrelated Regressions (SUR) estimation procedure.

Contrary to what was reported by other researchers working with the LES model (e.g. Theil 1975, p. 240), repeated estimates of the model parameters did not converge but instead kept getting more and more divergent at each iteration until the estimated values of the parameters became meaningless. Successive estimates failed to converge for Italy as well as for New Zealand. But, for Italy, the parameter estimates obtained at the second iteration - or second round, to use the terminology of Chapter 2 - generated price and family size elasticities that were more acceptable in an economic sense than those obtained from the first round estimates.

Based on the values of the elasticities, the best parameter estimates seem to be those obtained from the first round of the Kmenta procedure for New Zealand, and from the second round for Italy. The SUR estimates, or estimates obtained by successive applications of the Kmenta and the SUR procedures, were discarded because they generated price elasticities totally unacceptable according to economic principles (they often were positive and much larger than one).

One of the difficulties in interpreting the LES parameter estimates obtained from the different estimation methods we tried, and in choosing the "preferred structure" among them, was that they all seemed equally good on the basis of the usual statistical tests or goodness of fit criteria. Often, the only way to choose among alternative structures was a careful scrutiny and interpretation of the economic conclusions derived from them.

However, both estimation methods performed rather poorly. They did not converge from one estimation round to another, and converged with great difficulty within the estimation procedures themselves. These results, inevitably, cast some doubts on the suitability of the LES model for the present applications, and on the reliability of its parameter estimates, even though such estimates satisfied most statistical tests and goodness of fit criteria. This was especially true in the case of the application of the "extended" (Section 2.2.3) Kmenta procedure to New Zealand.

In Chapter 3, we analysed and estimated the Almost Ideal Demand System (AIDS) suggested by Deaton and Muellbauer (1980). The AIDS system, derived from an indirect utility function (for the definition of indirect utility functions see Phlips, 1974, p. 27-31) first suggested by Muellbauer (1975 and 1976), allows aggregation under conditions of homogeneity and concavity in the utility function with respect to prices.

The AIDS model proved to be very difficult to estimate, as the estimation procedure showed chaotic characteristics: the final estimates obtained at the end of the iterative process were dependent on values given to the parameters at the beginning of the iterative process itself. To make sure we had in fact found a maximum of the Likelihood function, and the corresponding Maximum Likelihood parameter estimates, we had to re-estimate the model many times, with different starting parameter values, to verify that we had achieved a global maximum, and not just a local one.

From the AIDS parameter estimates, we computed the price elasticities at the means, and the family size and total expenditure elasticities at the means and also at two other intermediate values, one smaller and one larger than the mean. For Italy, the price elasticities obtained from the AIDS model had unrealistically high absolute values, and were difficult to explain from an economic perspective; thus confirming the possibility, initially suggested by the chaotic behaviour of the estimation procedure, that the underlying demand model is inadequate to represent correctly the effects of prices on households' consumption. For New Zealand, the AIDS price elasticities were more according to economic expectations.

The family size and total expenditure elasticities look much more acceptable for both countries. However, if we compare the values of the elasticities obtained from the LES and the AIDS models, we can see they are quite different. Thus one or the other model (or possibly both) must be considered as inadequate to describe the consumption behaviour of New Zealand and Italian households when their total expenditures and/or household sizes change.

While for the LES and AIDS models the effects of demographic variables have been taken into account by introducing them directly into the demand equations, without reference to the underlying utility function, for the demographically augmented models considered in Chapter 4, the effects of demographic variables enter the demand models indirectly, via the utility function.

The first model considered in Chapter 4 is the Non-Linear Preference Demand System (NLPS), first proposed by Blundell and Ray (1984), which is based on a family of utility functions suggested by Muellbauer (1976 p. 985) who called it Price Independent Generalised Linearity (PIGL). The NLPS model assumes concavity and homogeneity in prices, the degree of homogeneity depending on α, an estimable parameter. The parameter α, if different from unity, measures the non-linearity of the Engel curve and also allows for non-separable behaviour. If $\alpha = 1$ then the NLPS specialises to the class of Linear Preference Systems (LPS) derived from the Gorman Polar Form of expenditure systems (see Gorman, 1976). The assumption of linearity of the Engel curve (i.e. of $\alpha = 1$) was clearly rejected by all models nested within the NLPS.

In its estimable form, the NLPS is a complex, non-linear, demand system with a large number of parameters (see equation 4.2.7 in Chapter 4), which describe households' consumption of the various commodities included in their budgets, as shares of total expenditure, and allows consistent aggregation over individuals. By imposing restrictions on some of its parameters, the NLPS model can nest a whole family of demand models of decreasing complexity (see Figure 4.1 in Chapter 4).

The NLPS model proved extremely difficult to estimate, and showed typically chaotic behaviour, for both Italy and New Zealand. Its iterative estimation process was sensitively dependent on initial conditions, that is, the values given to the parameters at the beginning of the iterative process. Different starting parameter values, very often generated different final sets of parameter estimates, even for very small initial differences.

The set of parameter estimates, shown in Table 4.1 of Chapter 4, correspond to the highest value of the likelihood function we could obtain over many estimation runs, starting from a variety of parameter values, some of them obtained from models nested within the NLPS family of models. Although we are reasonably confident to have achieved, within the parameters' domain, a maximum of the likelihood function, we cannot be absolutely certain to have done so, under chaotic conditions of total sensitivity to initial conditions there is always the possibility that we have reached a local maximum, instead of a global one.

By assuming $\alpha = 1$ in the NLPS model, we obtain the LPS model, and estimation becomes much easier, convergence is reached to the same set of parameters from most starting points, and for all convergence criteria we tried. The same ease of estimation was found for most of the other models nested into the NLPS (the exception being the RNLPS for Italy). However, all the models nested by NLPS, were rejected when tested against the model, or models, nesting them (see Table 4.4 in Chapter 4). Thus, this whole family of models was rejected as inappropriate to describe the consumption behaviour of New Zealand and Italian households.

Another family of models we considered was based on Demographic Cost Scaling (DCS), first suggested by Ray (1983), which is an extension of the Restricted Non-linear Preference System (RNLPS), and where the cost of children enter as a general equivalence scale parameter independent of reference utility. As for NLPS, the DCS-RNLPS model is homogeneous of degree α in prices, and α, if different from unity, measures the non-linearity of the Engel curve. By imposing restrictions on some of its parameters, the DCS-RNLPS model will nest a whole family of demand models of decreasing complexity (see Figure 4.2 in Chapter 4).

The estimation procedure for DCS-RNLPS works well as it always converges in a reasonable number of iterations to the same parameter estimates from most sets of starting values. This seems to indicate that a global maximum of the LL function exists, and was actually reached by the estimation procedure.

Of some interest is the fact that, unlike what happened for the NLPS model, for which we could not find any specific set of starting values, among the many we tried, giving better and faster convergence than any other, in the case of the DCS-RNLPS, we found that the set of starting values, consisting of the parameter estimates obtained from the DCS-LES model (with α set equal 0.5), gave the fastest convergence.

Because the DCS-LES model is nested within the DCS-RNLPS, and therefore should approximate it well, this finding seems to confirm that for this whole family of models the LL function is well behaved and grows smoothly up to its maximum. For all models in the DCS-RNLPS family, as was the case for NLPS, the assumption of income linearity, $\alpha=1$, was clearly rejected.

We also considered two more demographically augmented demand systems based on Restricted Non-linear Preferences: Demographic Scaling (DS-RNLPS), proposed by

Barten (1964), and Demographic Translation (DT-RNLPS), proposed by Pollak and Wales (1981).

In the DS-RNLPS model, the prices entering the utility function are modified by scaling parameters that depend on household composition (see equation 4.2.9 in Chapter 4). In the DT-RNLPS model, the demographic parameters, which once again depend on the household composition, enter the utility function directly and have both fixed effects and quasi-price effects on the demand functions (see equations 4.2.11 and 4.2.12 in Chapter 4). These last two models are not nested within, and do not nest, any other models.

The DS-RNLPS model proved extremely difficult to estimate. The model also showed chaotic behaviour during estimation, as it was sensitively dependent on initial conditions. Different starting values *always* generated different final sets of parameter estimates, even for very small initial differences. Sometimes, even for the same set of initial values, a simple change in the value of the convergence criterion (e.g. from c = 0.0001 to c = 0.001), was sufficient to produce completely different sets of final estimates.

After dozens of experimental estimation runs, we feel that most of the parameter estimates obtained for the DS-RNLPS model, even when statistical tests showed them to be acceptable or "good", were in fact completely meaningless semi-random numbers, generated by a mathematical optimisation process, and devoid of any statistical or economic interpretation.

The DT-RNLPS model on the contrary was easy to estimate, the estimation process always converged to the same final set of parameter estimates from almost all the sets of starting values we tried. Changes to the value of the convergence criterion had no effect. We are confident that the final estimates of the DT-RNLPS model we have obtained correspond to a global maximum of the LL function, and therefore are ML estimates. Income linearity in the Engel curve, $\alpha = 1$, and the hypothesis of no demographic effects were both clearly rejected.

Judging by their performance during the iterative estimation procedures the DCS-RNLPS and DT-RNLPS models appear to be the ones to be preferred on empirical grounds, their data-generated likelihood functions are smooth, and show evidence of well defined maxima.

The DS-RNLPS and the NLPS models appear to be totally rejected by the data, they generate "lumpy" likelihood functions with many, badly defined, local maxima. Likelihood functions, with shapes like these, make it very difficult, sometimes impossible, to obtain reliable parameter estimates from estimation methods based on the maximisation of the sample likelihood functions.

In Section 4 of Chapter 4, we tested all the models estimated in Section 3 to choose among them the preferred structure; the results of the tests are shown in Table 4.4. For New Zealand, the best model appeared to be DCS-RNLPS followed by DT-RNLPS; for Italy, the best model was DT-RNLPS followed by DCS-RNLPS.

The high values of the Akaike Information Criterion for the DS-RNLPS model for New Zealand and for the NLPS for Italy which, by themselves, might give the erroneous impression that these models are acceptable, are somehow misleading, given the poor performance of these models during estimation. This is a typical example of how the results of statistical testing should be complemented by information on the performance of the models during estimation. If a model proves difficult to estimate, and shows chaotic behaviour, this should cast serious doubts on its ability to describe and explain the data, and it should lead to its rejection.

In view of the information provided by the model testing results, together with the models' performance during estimation, it appears DCS-RNLPS and DT-RNLPS should be chosen as the preferred structures among all other models considered in this study. It is significant these two models, so clearly preferred by the data for both Italy and New Zealand, are two of the richest in economic content and also incorporate demographic effects in very similar ways.

It is also worth noting that all DCS-RNLPS nested sub-models, implying simpler economic behaviour and/or linear assumptions, were rejected in favour of their unrestricted "parent" nesting them. Of particular economic interest was the rejection by all models of the linearity of the Engel function implied by the α=1 restriction.

Once we were convinced the DCS-RNLPS and DT-RNLPS models were the models best able to represent and explain the data correctly, we proceeded to re-estimate them in a way that would allow a more complex error structure than the homosckedastic, uncorrelated one assumed in section 4.3.1. To this end, we chose a non-linear variation of the Generalised Method of Moments (see Davidson and MacKinnon, 1993, ch 17), which included the procedure suggested by White (1980), to estimate consistently the covariance matrix of a regression model with an error component affected by an unknown form of heterosckedasticity. We also introduced instrumental variables to take care of possible income effects on consumption, and auto-correlation in the residuals.

The GMM estimation method improved the parameter estimates for New Zealand, but not for Italy. Therefore, for Italy, we retained the ML estimates. The preferred parameter estimates were then used to compute the price, household size and total expenditure elasticities.

As a further check on the suitability of the DCS-RNLPS model to represent the available consumption data, we have computed for both countries (from the estimated values of the demographic parameter θ_0) the values of m_0, the general consumption equivalence scale, for the standard household of two components, in the base year, when prices are scaled to unity. This is sometimes interpreted as the "cost of a child".

We found that for New Zealand households the value of m_0 (reported in Table 4.6 of Chapter 4) increases as the size of the household increases, but slightly less than proportionately, thus showing some small economy of scale in the cost of children. For Italy, the value of m_0 decreases slightly as the household size increases; thus an increase in household size seems to correspond to a decrease in the expenditure required to maintain the same standard of living.

This apparently paradoxical result simply reflects the characteristics of the data, which actually record slightly decreasing total expenditure as household size increases (see the footnote to Table 4.6).

In the last section of Chapter 4, we tried to check whether or not the assumption of separability was supported by empirical evidence. Because in the Italian data the Housing commodity group could be split into two components: *Housing, Fuel and Electricity* and *Furnishings and Home Appliances* , we could re-estimate the DCS-RNLPS model with a disaggregated Housing group, obtaining a five-commodity demand model, instead of the four-commodity model used so far, and then test both models for separability. The tests did not reject the hypothesis of separability.

In Chapter 5, to confirm the results of Chapter 4 on separability, we utilised a much more detailed set of Italian data (with fifteen income classes instead of five and six commodity groups instead of four) and re-estimated the DCS-RNLPS and the DT-RNLPS models. The results were quite interesting. The behaviour of the two models during estimation had changed drastically: although both models were difficult to estimate, because convergence to a clearly defined maximum of the LL function was difficult to achieve, the DT-RNLPS model now performed better than DCS-RNLPS, which showed distinct chaotic characteristics.

These findings give some support to the notion that there are limits to the level of disaggregation we can impose on consumption groups if we want to gain any meaningful insights into consumers' behaviour. In empirical studies, although data aggregation should not proceed beyond the level at which it might contradict the essential conditions for aggregation: the commodity groups must be homogeneous, and all consumers grouped together must behave as a *single consumer*, a certain level of aggregation is still necessary to maintain enough variation in the consumption patterns of the different commodities to make the models estimable. Even more so when the data themselves refer to the average consumptions of very large cells, which tend to cancel out the behaviour of individual household.

In the Italian case, for example, it seems inappropriate to aggregate households only according to size, to the exclusion of all other demographic and social characteristics, because the resulting groups are unlikely to be homogeneous, and consumers within them will not behave as a "single consumer". Equally inappropriate is to aggregate commodities like food and drinks into a single consumption group, because their consumption is bound to react very differently to similar price movements.

In Chapter 5 we have computed the price, household size and total expenditure elasticities for the disaggregated, six-commodity, DT-RNLPS model. The results seem to be in keeping with economic expectations, and to represent correctly the attitudes of Italian consumers.

For all the models for which we have computed the price elasticities, the Slutsky matrices were generally not negative semi-definite. Only for the six equation DT-RNLPS model were some of the Slutsky matrices negative or approached negativity, with five negative eigenvalues out of six, instead of only one or two out of four as it was in the case of the aggregated models.

These results, although contrary to theory, seem to be a normal state of affairs in most empirical demand studies (for an exception see Nelson 1988, p. 1310-11), and the only way to make certain that the empirical Slutsky matrices be symmetric and negative, is to constrain them accordingly, during estimation (see Brenton and Winters, 1992, p.268).

In Chapter 6 we estimated household - and commodity - specific expenditure equivalence scales, for both New Zealand and Italy, to measure the relative levels of expenditure, required by households of different size and composition, to attain a comparable standard of living in "utility" or welfare sense.

As far as we are aware, this is the first time that a set of commodity specific equivalence scales, derived from the constrained maximisation of a utility function, has been computed for New Zealand from actual household budget consumption data. We feel that fills a glaring gap in the area of household budgets analysis for New Zealand.

Our results are comparable with similar consumption scales reported in the literature (e.g. Binh and Whiteford 1990, for Australia and Smith 1989, for New Zealand). They also compare well with other published equivalence scales for New Zealand based on different data sets or measurement techniques (see Easton 1994, Pashardes 1993 and Rutherford et al. 1990).

The application of the ELES method to the Italian data, to compute the equivalence consumption scales, proved very difficult, because of the characteristics of the data themselves, which show decreasing total household expenditures as household sizes increase, as noted earlier. We could only compute a set of household type specific equivalence scales for the consumption of Food, which is the only commodity whose expenditure grows together with household size, although less than proportionately.

We estimated the Food equivalence scale for Italy both from the ELES consumption model, and from the commodity specific household size parameter θ_1 of model DT-RNLPS. The two scales are almost identical, notwithstanding the totally different models used to estimate them. This strict similarity of the two scales gives some reassurance about their robustness, and about the ability of the two models to capture rational, and expected, household consumption behaviour when the data allow such rational behaviour to become apparent.

In the course of this study, we tried to compare, whenever possible, the consumption behaviour of New Zealand households with that of Italian households. We found some differences: the Transport and Apparel consumption patterns, for example; and some similarities: the decrease in the share expenditure on food as total expenditure increases, a very clear indication of the Engel's law at work; or the little variation, among households of different size, of the share expenditure elasticities of the four commodities with respect to household size. For New Zealand, as well as for Italian households, how total expenditure is shared among the four commodities considered, depends more on the level of total consumption than on the household size.

Finally, we believe that one important result of our work has been to highlight the requirement for better data in the area of household budget statistics. For example, in reporting the results of households budget surveys, households should be classified not only according to size, and/or to the number of adults and children, but also according to the age of the household members and other socio-economic variables. Most of all, instead of reporting cell averages, data should report detailed unit-records. If, to publish unit records is considered impossible, because of privacy requirements, then data could be based on small cells containing only a few households, instead of large cells containing dozens of households.

In view of the data inadequacies noted above, we feel that most of our findings and results must be viewed as tentative, to be confirmed, or otherwise, once better data become available. Meanwhile, applied econometric studies must continue addressing important policy issues in a range of areas involving the welfare of households with the help of the imperfect and incomplete data bases such as the ones used here.

This is a point we have made several times in this study. Given the nature of the data, and the difficulties inherent in the estimation of the kind of non-linear models of consumer behaviour being used here, and in other similar studies, there is not much one can do to resolve this problem. The consequence of this state of affairs is that our results, in many instances, must be treated, at best, as indicative only.

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