Embodying decisions on work shifts into strategic manufacturing capacity planning

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In spite of the obvious impact that decisions on workforce planning have on the capacity of a production system, they are rarely mentioned in strategic capacity planning literature. This paper analyses the implications of embodying decisions on work shifts in strategic capacity planning and proposes a mathematical programming model that allows treating any type of relationship between the intensity in the use of the production equipment and maintenance and variable production costs. A computational experiment shows that the model can be solved in acceptable computing times for moderate values of the number of types of equipment and of work shifts. Using the model, the optimal solutions for diverse scenarios are presented and compared with those obtained under the assumption that the work shifts must be kept invariable over the entire horizon.

Keywords: strategic capacity planning; work shifts; manufacturing industry; mixed integer linear programming; optimal decision model

1. Introduction

This paper has two main purposes, namely, to highlight the relevance of decisions concerning the number of work shifts in the strategic manufacturing capacity planning process and to suggest and test a way to incorporate these decisions in the corresponding mathematical programming models.

Strategic capacity planning is concerned with the availability of facilities in the long term, typically some years. Therefore, it involves decisions having generally a strong influence on the company's results and that, once implemented, can only be reversed incurring in high costs.

Work shifts have been used for a long time in the manufacturing industry (24.8% of workers in the manufacturing industry in the USA had flexible work schedules in 2004: McMenamin, 2007) and in service industry, as a modality of

numerous papers dealing with work shift scheduling has been published (e. g. Buffa, Cosgrove and, Luce, 1976; Glover and McMillan, 1986; Lau, 1996), and with the many and important effects of work shift on the workers and how to manage them (Pati, Chandrawanshi, and Reinberg, 2001), only a little number of the works, either textbooks or research papers, take into account work shift when dealing with strategic capacity planning, in spite of the obvious impact that decisions on workforce planning have on the capacity of the production system, as it was already pointed out, more than one-half of a century ago, by Marris (1964), who related work shift implementation with strategic capacity planning, via the rate of capital utilisation. The persistence of the work shifts in the manufacturing industry is driven by economic factors such as the need for maximizing the return on capital, especially in capital-intensive industry (Pati, Chandrawanshi, and Reinberg, 2001) and in technological industry.

A recent review of mathematical programming models for strategic manufacturing planning in manufacturing (Martínez-Costa *et al.* 2014) identifies working planning decisions as one of nine groups of decisions that have to be addressed in the strategic capacity planning process. However, the review mentions only two papers (Fleischmann, Ferber, and Henrich 2006; Bihlmaier, Koberstein, and Obst 2009) that deal with them explicitly.

Fleischmann, Ferber, and Henrich (2006) consider overtime as a way to meeting demand. Bihlmaier, Koberstein, and Obst (2009) introduce the interesting distinction between technical and organisational capacities. Technical capacity is inherent to the available facilities, while organisational capacity is determined, given a technical capacity, by the adopted working time model. Demand changes should be addressed by managing properly both kinds of capacities.

Both papers, especially Fleischmann, Ferber, and Henrich (2006), do not consider decisions concerning working time management as strategic decisions, but as operational or tactical ones.

Instead, Fernandes, Gouveia, and Pinho (2013) points out that the decision of implementing a new work shift may imply a significant investment and is not fully reversible without incurring in an important cost. Therefore, this decision, concerning its economic implications, has the same main characteristics that an investment in a new equipment.

According to the point of view of Fernandes, Gouveia, and Pinho (2013), which we share, decisions on the number of work shifts, in a manufacturing setting, has to be dealt with at the same hierarchic level than those concerning buying and selling equipment. Since, for instance, starting with a facility of a given capacity and two work shifts or with a facility with double capacity and a single work shift will have different consequences thorough the planning horizon, both kind of decisions should be incorporated in the strategic capacity planning decision making process and in the models used to support it.

However, this is not straightforward, because of the complexity of the relations between the usage of the equipment, which depends directly on the characteristics of the work shift organisation, and the costs of operation and maintenance and the residual value of the equipment as well. These relations are non-linear and specific of the production system. Likewise, the costs of changing the work shift organisation depend on the cause of the change (the acquiring of a new equipment or the introduction of a new work shift, for instance) also in a specific and often complex way.

The consideration of the states of the system, allows taking into account all these complex relations, by means of associating the costs to the states and the transitions between states. This is one contribution of the present paper, in which it is shown, at the same time that the adopted modelling approach is computationally tractable within reasonable type numbers of equipment and work shifts.

We propose, for a typical industrial setting, a mathematical programming model which includes the decisions on the technical equipment and on the number of shifts in every period as well. The model is used in a numerical experiment to illustrate the impact of taking into account the number of work shifts as a decision variable when planning assets acquisition.

The layout of the rest of this paper is as follows. Section 2 describes the industrial setting assumed for building the model and the numerical experiment, which are the object, respectively, of sections 3 and 4. The paper ends up with the conclusions and future prospects of research.

2. Problem statement

We consider a company that must satisfy the demand of a product during a discrete finite time horizon. To this effect, the company has to determine, for each period of the given planning horizon, the capacity of the equipment and the active work shifts, with the aim to optimise the global performance of the company throughout the plannig horizon.

The following assumptions are adopted:

Demand assumption

The demand in each period is known, has to be completely satisfied by the production system and cannot be deferred.

Production system assumptions

Given the production equipment (which, from now on, is simply referred to as equipment) and the work shift organisation at the beginning of the planning horizon, the company decides, at the end of each period, to acquire new equipment and hold or sell the old one.

When the company acquires equipment can opt between different types. The price of acquisition depends on the type of equipment and on the period, is paid in the period in which the equipment is acquired, and is considered an investment with a linear amortization plan in subsequent years. The equipment suffers deterioration that depends on its age and usage. Therefore, the selling price of the equipment depends on its type, age, and usage.

The cost of maintenance is paid each period and depends on the type, age and usage of the equipment.

The variable cost of production depends on the period and the type of the equipment and, maybe, on its age and usage.

To operate the equipment, the company has a workforce that is organized in work shifts. For example, the company could have two shifts: one shift working from Monday to Friday and the other on Saturday and Sunday. The company may decide in each period if activates, deactivates, or maintains one or several work shifts. The number of workers of a work shift depends on the type of the equipment. For example, a shift working from Monday to Friday may require 100 workers with a certain equipment, while may require 200 with another equipment.

The production capacity of the system in a period depends on the type of the equipment used and on the active work shifts.

Each period, the company pays the labour costs corresponding to the active work shifts. These costs depend on the type of equipment, the active work shifts and the period.

A change in the type of equipment may require changing (by hiring or dismissing) the number of workers necessary to operate the equipment. The cost of hiring and dismissing workers depends on both, the old and the new types of equipment, the active work shifts and the period. Additionally, the cost of dismissing a worker is proportional to the time elapsed since the worker was hired.

Activating and deactivating a work shift require doing changes in the functioning of the system. For example, the company must hire or dismiss workers when, respectively, activates or deactivates a work shift. Additionally, the company has to do other organizational or material changes. The cost of these changes depends on the work shift and the type of equipment.

Financial and tax assumptions

The company has a bank account to make and receive payments. At the end of each period, the account generates an interest income or an interest charge depending on whether the account balance is positive or negative.

Borrowing capacity is limited. Flows into and out of bank account are the following: collections such as bank interest, product and equipment sales; and payments such as bank interest, operating costs, equipment and work shift costs, and taxes.

Taxable income is calculated annually from the profit and loss account, which includes the most significant revenues and expenses, such as revenues, bank interest incomes and charges, fixed costs, fixed and variable operating costs, depreciation of assets, and profit or loss from equipment sales. Negative taxable income for a year can be carried forward to reduce future taxable income as long as the company has future positive taxable profits to offset losses. VAT repercussions are not considered in the cash flow model.

Optimisation criteria and decision variables assumptions

The objective is to maximise the cash balance at the end of the planning horizon after selling all assets for their residual value. Therefore, the problem consists in determining, for each period, the equipment and the work shifts that maximise the final cash balance.

3. Mathematical model

Let W be the number of work shifts that can operate the equipment of the company; E, the number of types of equipment, and T, the number of periods of the planning horizon.

The system defined in the former section can be described in each period with the values of the system state variables, α, τ and $\boldsymbol{\beta}$, where α is and integer that indicates the type of equipment used by the company, τ is the period when the equipment used by the company was acquired and $\boldsymbol{\beta}$ is an array of W binary components (one component for each work shift), which indicates the work shifts that are active during that period.

The elements of the set of system states, S, are defined by the combinations of values of α, τ, $\boldsymbol{\beta}$. We have, then, that $|\, S \,| = E \cdot T \cdot 2^{W}$.

The state transitions are allowed only at the end of each period. Therefore, each period begins with the system in a given state and this state is not altered throughout the whole period.

A state transition in a period is defined by an ordered pair of system states, (i, j) where $i = (\alpha_1, \tau_1, \boldsymbol{\beta}_1)$ is the state of the system at the beginning of the period and $j = (\alpha_2, \tau_2, \boldsymbol{\beta}_2)$ is the state of the system at the beginning of the following period.

In a specific period *t*, only a subset U_t of the state transitions is allowed. Let $i = (\alpha_1, \tau_1, \boldsymbol{\beta}_1)$ and $j = (\alpha_2, \tau_2, \boldsymbol{\beta}_2)$. The elements of U_t are the state transitions (i, j), such that:

- The production capacities corresponding to the two states, *i* and *j*, are sufficient to satisfy the demands of periods *t* and *t*+1, respectively.

- $\tau_1 \leq t$ (i.e. the period in which was acquired the equipment in *i* is not greater than t).
- Either $\tau_2 = t + 1$ or $\tau_2 = \tau_1$and $\alpha_1 = \alpha_2$ (i.e either the equipment is renewed at the end of period *t* or it is not).

Not all state transitions in the set U_t can be actually performed because of the limited borrowing capacity of the company and also because of the cash balance of the company in each period *t* is not completely determined by the system state. Indeed, although most of cash inflows and outflows depend only on the state of the system (as for example, the collection of selling the product and the payments of maintenance and acquisition of the equipment), there are some payments, as those related with the cost of dismissing workers when the equipment has changed or a work shift is deactivated, that not only depends on the state but also on the time elapsed since the workers were hired. For this reason, several addicional variables are needed in order to model the system.

Data

The data of the model are shown in table 1.

[Table 1 near here]

Variables

Table 2 includes the variables of the model.

[Table 2 near here]

The cost of dismissing workers at the end of a period is calculated by using two sets of variables. One, m_{tk} (t = 1,…,T; k = 1,…, W), is needed for determining the cost of dismissing all workers of each work shift k if they were dismissed at the end of each period t (as the cost of dismissing a single worker is assumed to be proportional to the time elapsed since the worker was hired, the cost of firing all workers must be increased each period by an amount, A_{tki}, t = 1,…,T, k = 1,…, W, $i \in S$, that depends on the work shift and on the state of the system at the beginning of the period, as well). The other set of variables, f_{tk} (t = 1,…,T; k = 1,…, W), is used for calculating the actual cost of dismissing workers of each work shift k, at the end of each period t. This cost results from multiplying the cost of dismissing all workers of the work shift k by the proportion of workers dismissed at the end of the period t.

Objective function

The objective function is the cash balance at the end of the planning horizon considering final cash balance before tax and corporate tax payment for the last year.

Maximise

$$Z = i_T - R \cdot b_T$$

where $R \cdot b_T$ is the corporate tax payment at the end of planning horizon.

Constraints

Bank account balance constraints

Determine the cash account balance at the end of period *t* taking into account the cash balance at the beginning of the period, the cost of dismissing workers, the corporate tax of the previous period, the interest income or charge, and the rest of the collections and payments that depend on the system transition at the end of the period.

The cash account balance at the end of period *t* is computed by adding the collection of interest or subtracting the interest to the interest charge to the cash balance before interests, depending on whether this balance is positive or negative.

$$i'_t - j'_t = \begin{cases} H + \sum_{(i,j)\in U_1} C_{1ij} \cdot x_{1ij} - \sum_{k=1}^{W} f_{1k} & t = 1 \\ i_{t-1} + \sum_{(i,j)\in U_t} C_{tij} \cdot x_{tij} - \sum_{k=1}^{W} f_{tk} - R \cdot b_{t-1} & t = 2, \dots, T \end{cases}$$

$$i_t = (1 + I_1) \cdot i'_1 - (1 + J_1) \cdot j'_1$$

Financial capacity constraints

Ensure that the negative bank account balance does not exceed the established limit:

$$d_t \leq D \quad t = 1, \dots, T$$

Profit/loss before tax calculation:

$$p_t - l_t = I_t \cdot i'_t - J_t \cdot j'_t + \sum_{(i,j)\in U_t} B_{tij} \cdot x_{tij} - \sum_{k=1}^{W} f_{tk}$$

$$p_t \le P_t \cdot z_t$$
$$l_t \le L_t \cdot (1 - z_t)$$

where t = 1,…,T. The last two sets of constraints ensure that either p_t or l_t equals zero, because otherwise cannot be assured, as can be seen in the following example: suppose that in period t - N there were losses that can be offset in period t (i.e., period t is the last period to compensate them) and suppose that the profit of year t is less than the losses to compensate. In this case p_t will be increased to compensate all the losses and l_t will be increased for the amount p_t - l_t so it equals the profit of year t.

Income tax base calculation constraints

Determine the corporate tax base in period t taking into account the results at the end of the year and the offsettings of tax liabilities from previous years.

$$l_t \ge \sum_{n=1}^{N} u_{tn} \qquad t = 1, \dots, T$$
$$b_t = p_t - \sum_{n=1}^{t-1} u_{t-n,n} \quad t = 1, \dots, N$$
$$b_t = p_t - \sum_{n=1}^{N} u_{t-n,n} \quad t = N + 1, \dots, T$$

State transition constraints

Guarantee that a single state transition is selected every period and that the state of the system at the end of period t and at the beginning of the period t + 1 is the same.

$$\sum_{(i,j)\in U_1} x_{1ij} = 1$$
$$\sum_{i|(i,j)\in U_{t-1}} x_{t-1,ij} = \sum_{k|(j,k)\in U_t} x_{tjk} \quad j \in S \quad t = 2, \dots, T$$

Cost of dismissing workforce of a work shift constraints

Impose that the cost of firing workforce of the work shift *k* in period *t* is proportional to both the number of workers dismissed at the end of period *t* and the cost of dismissing all the workers of the work shift *k* if they were dismissed at the end of period *t*.

$$f_{1k} = \sum_{(g,h)\in V_{1k}} F_{kgh} \cdot A_{1kg} \cdot x_{1gh}$$

$$f_{tk} \leq \sum_{(g,h)\in V_{tk}} F_{kgh} \cdot A_{tkg} \cdot x_{tgh}$$

$$+ F_{kij} \cdot m_{t-1,k} + \max\{F_{kgh} \cdot M_{t-1,kg} | \ (g,h) \in V_{tk}\} \cdot (1 - x_{tij})$$

$$f_{tk} \geq \sum_{(g,h)\in V_{tk}} F_{kgh} \cdot A_{tkg} \cdot x_{tgh} + F_{kij} \cdot m_{t-1,k}$$

$$- \max\{F_{kgh} \cdot M_{t-1,kg} | \ (g,h) \in V_{tk}\} \cdot (1 - x_{tij}$$

$$f_{tk} \leq \max\{F_{kgh} \cdot M_{tkg} | \ (g,h) \in V_{tk}\} \cdot (1 - \sum_{(g,h)\in U_t \setminus V_{tk}} x_{tgh})$$

where $(i,j) \in V_{tk}$, $k = 1, ..., W$ and $t = 2, \ldots, T$.

The first set of constraints applies for period 1, the second and third sets apply when some or all the workers of the work shift *k* are dismissed, and the fourth set of constraints applies when no workers of the work shift *k* are dismissed.

Potential cost of dismissing the whole workforce of a work shift constraints

Enforce that the cost of dismissing all the workforce of the work shift *k* if it was dismissed at the end of period *t* takes into account the time elapsed since the workers were hired.

$$m_{1k} = \sum_{(i,j)\in U_1} A_{1ki} \cdot x_{1ij} - f_{1k}$$
$$m_{tk} = m_{t-1,k} + \sum_{(i,j)\in U_t} A_{tki} \cdot x_{tij} - f_{tk}$$

where $k = 1, \ldots, W$ and $t = 2, \ldots, T$.

4. Computational experiments

We implemented and solved the model with IBM ILOG CPLEX 12.6 in Intel Core i5-6500 workstation with 16 Gigabytes of RAM operating under Windows 10 (64-bits).

Two computational experiments were performed. The first one, to check that the model can be solved in a reasonable amount of computing time and to explore its practical size limits to do this. The aim of the second experiment was to analyse the behaviour of the model in different scenarios and to compare the optimal solutions that it provided to those obtained under the assumption that the work shifts must be kept invariable over the entire horizon.

The size of each instance depends on three parameters: T (number of periods), W (number of work shifts) and E (number of types of equipment). The complete definition of the instances requires a great amount of information, which is generated from some primary data and parameters, as for example:

- The set S of system states.
- The demand in each period and the sale price.
- For each work shift operating each type of equipment, the number of workers, the production capacity, the labour costs during the activity of the work shift, the

cost of hiring and dismissing a worker, the costs of activating and deactivating the work shift.

- For each type of equipment and each period, the investment required to acquire the equipment, the cost of maintenance, non-amortisable cost required to set up the equipment, the variable cost of production and the selling price of equipment acquired in previous periods.

The input of the instances was calculated according to expressions and the values of parameters that can be found in https://ioc.upc.edu/ca/EOLI/research/SCPWS. For example:

- The sets U_t (allowed state transitions in period t) were defined, according to the rules explained in section 3, given the set S, the production capacity of each work shift working with each type of equipment, and the demand in period t.
- The sets V_{tk} (state transitions in which some or all workers of a work shift are dismissed) were calculated from the set of state transitions, U_t, and the number of workers working at the beginning and at the end of period t.
- B_{tij} (profits of transitions from state i to state j in period t) were computed taking into account the product sales, the selling price of the equipment (if any equipment is sold), the equipment maintenance cost, the variable cost of production, the depreciation of the equipment, the cost of acquisition (if any equipment is acquired), and the costs associated to the workforce (as the cost of activating or deactivating work shifts, the cost of hiring workers and the salaries of the workforce).

- The values of C_{tij} (cash account collections and payments when the system transitions from state i to state j in period t) were calculated in a manner analogous to those of B_{tij}.

4.1. Maximum instance sizes

The aim of the first computational experiment was determining the size of the instances solved in less than 3,600 seconds. We designed 20 cases with a number of work shifts ranged from 2 to 5 and that of types of equipment, from 3 to 7. Table 3 shows the production capacity of each combination of work shift and equipment types. The cost of acquiring the equipment in the first period was set from 450,000 (type 1) to 1,033,306 (type 7). The labour costs of the work shifts in the first period were set to 1 monetary unit per production capacity units for work shifts 1 and 2, and to 1.2, 1.4 and 1.6, respectively, for work shifts 3, 4 and 5. The demand during the planning horizon was set between 9,000 and 11,000 units. We were able to solve 11 out of the 20 cases (Table 4 shows their sizes and the corresponding computing times). The remaining 9 cases exceeded the capacity o the used computing system.

[Table 3 near here]

[Table 4 near here]

Afterwards, we solved the same cases considering that work shifts were hierarchized (i.e. the work shifts are completely ordered and a given work shift cannot be initiated unless all the preceding ones are active; in practice this is a common situation, because the hierarchy is implied by the different costs and productivities of the diverse work shifts). Under this assumption, which reduces the number of feasible state transitions, we were able to solve the 20 cases in less than 500 seconds (sizes and computing times are shown in Table 5).

[Table 5 near here]

4.2. Optimal solutions in different scenarios

The second computational experiment consisted in solving instances corresponding to 36 scenarios, all of them with $T = 10$, $W = 3$ and $E = 3$. The scenarios were defined by combining the features of three factors: demand evolution (constant, increasing, decreasing, and increasing during the first half of the planning horizon and declining during the second half; all the variations are linear), labour costs and economies of scale in the cost of equipment (it is considered that each of these two last factors may have low, medium and high levels).

The main data that define the instances are shown in Table 3 (production capacities of each work shift working with each type of equipment), Figure 1 (demand evolution), Table 6 (labour costs) and Table 7 (costs of acquiring each type of equipment in the first period).

[Figure 1 near here]

[Table 6 near here]

[Table 7 near here]

Table 8 shows the optimal capacity planning decisions. Of course, the shape of the optimal policies depends primarily on the evolution of demand. However, with the values considered, the economies of scale follow it closely in importance. Instead, the level of the considered labour costs (in spite of the great differences between the corresponding values) has a weak incidence on the configuration of the optimal solutions.

[Table 8 near here]

In all twelve scenarios corresponding to high economies of scale (i.e. regardless of the evolution of the demand) the optimal solution consists in acquire high capacity equipment and use, at any moment, the lower number of work shifts necessary to satisfy the demand. This policy is also optimal for constant demand, medium economies of scale and high labour costs. In all these scenarios, the deterioration of the equipment is not enough to justify its renewal. Conversely, in all other scenarios, in which is optimal the use of low or medium capacity equipment, this has to be renewed within the planning horizon, because of the elevated number of hours of use implied by the multiplicity of working shifts.

For all other five scenarios with constant demand, the optimal capacity planning consists in operate at three work shifts low capacity equipment, which has to be renewed in the middle of the planning horizon.

Concerning growing demand, in the six scenarios with low or medium economies of scale, the solutions involves medium capacity equipment, renewed at the beginning of period 8, with the minimum number of workshifts required to meet the demand at any moment. These policies are symmetrical of those corresponding to declining demand, under the same assumptions relative to the economies of scale.

It is also optimal, in case the demand increases first and then decreases, to work always with medium capacity equipment, renewed in the middle of the planning horizon, and adapt the number of turns at each moment to the volume of the demand.

A common characteristic of all the obtained optimal policies is that the equipment capacity is invariable throughout the planning horizon and, when the demand varies, the adjustment of the production capacity is achieved by means of the number of active work shifts.

Additionally, we solved, for each one of the 36 scenarios, 3 instances in which the number of working shifts was forced to be the same during all the planning horizon (equal to 1, 2 or 3, respectively, in the 3 instances). The aim of this part of the computational experiment was to compare the optimal solutions obtained when the number of work shifts is variable with the best solution from those of these 3 instances (see Table 8). In all scenarios (except for increasing demand, medium economies of scale, high labour costs), the type of equipment is the same, whether the number of work shifts is considered fixed or it is variable. Of course, however, the optimal value of the objective function may differ more or less from one assumption to the other. Clearly, there is no difference for constant demand scenarios. For the others, the relative differences range from values less than 1% for the scenarios with low labour costs to more than 40% in the case of increasing demand, low economies of scale and high labour costs (the same combination of economies of scale and labour costs gives values over 30% for decreasing and increasing-decreasing demands). Of course, the specific values of the relative differences depend on the assumptions that define the used scenarios; however, it is clear that the more the demand varies the more relevant is the economic advantage of using a variable number of work shifts throughout the planning horizon.

5. Conclusions

This paper discusses the implications of considering, in strategic capacity planning, the possibility of using a variable number of work shifts, which is hardly considered in the literature, and proposes a model that allows treating any type of relationship between costs and the intensity in the use of equipment, which is related to the types and number of active shifts during the planning horizon..

A computational experiment shows that the model can be solved in acceptable computing times for moderate values of the number of types of equipment and of work shifts. A second experiment illustrates the behaviour of the model in different scenarios and, comparing the optimal solutions to those obtained under the assumption that the work shifts cannot vary during the planning horizon, shows the significant differences between them, both in terms of their shape and the optimal value of the objective function.

Our future research on this problem will focus on expanding the size of the instances that can be solved in acceptable computing times for a decision making process in strategic capacity planning and in the development of a real industrial case to test and improve the utility of the proposed model.

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Symbol	Definition
T	Number of periods of the planning horizon.
E	Number of types of equipment.
W	Number of types of work shifts.
S	Set of system states.
U_t	Set of allowed state transitions in period t ($t = 1,\ldots, T$). $(i, j) \in U_t$ means that is allowed to find the system in state i at the beginning of period t and in state j at the end of period t, with $i, j \in S$.
V_{tk}	Set of state transitions where some or all workers of the work shift k are dismissed at the end of the period t ($k = 1,\ldots, W$; $t = 1,\ldots, T$).
H	Initial cash balance.
I_t, J_t	Respectively, interest rates applicable to positive and negative balances in the debit/credit bank account at the end of period t ($t = 1,\ldots,T$).
B_{tij}	Profit of transition from state i to state j in period t ($(i, j) \in U_t$, $t = 1,...,T$).
C_{tij}	Cash account balance between collections and payments when the system transitions from state i to state j in period t ($(i, j) \in U_t$; $t = 1,\ldots,T$).
A_{tki}	Increasing, generated in period t, of the cost of dismissing the workforce of the work shift k, if the system is in the state i ($t = 1,\ldots,T$; $k = 1,\ldots, W$; $i \in S$).
F_{kij}	Proportion of workers in work shift k that are dismissed if the system transitions from state i to state j ($(i, j) \in \bigcup_{t=1}^{T} V_{tk}$; $k = 1, \ldots, W$).
M_{tki}	Upper bound on the cost of dismissing the workforce of the work shift k at the end of the period t, if the system is in the state i at the beginning of the period ($k = 1,\ldots, W$; $i \in S$; $t = 1,\ldots,T$).
D	Maximum negative cash balance (expressed as an absolute value).
R	Corporate tax rate.
N	Number of compensation periods for tax liabilities (integer positive value).
P_t, L_t	Respectively, upper bounds on profit and loss before tax for period t ($t = 1,\ldots,T$).

Table 1 Data

Symbol	Definition
$x_{tij} \in \{0,1\}$	1 if the system is in states *i* and *j*, respectively, at the beginning and at the end of period *t* ($\forall (i,j) \in U_t$; *t* = 1,…,*T*); 0 otherwise.
$z_t \in \{0,1\}$	1 if profit before tax for period *t* is positive; 0 if it is positive (*t* = 1,…,*T*).
$m_{tk} \in \mathbb{R}^+$	Cost of dismissing all the workers of the work shift *k* if they were dismissed at the end of period *t* (*t* = 1,…,*T*; *k* = 1,…, *W*).
$f_{tk} \in \mathbb{R}^+$	Cost of fire workers of the work shift *k*, fired at the end of period *t* (*t* = 1,…,*T*; *k* = 1,…, *W*).
$i'_t, j'_t \in \mathbb{R}^+$	Absolute values of positive and negative bank balance, respectively, at the end of period *t*, before collecting the interest income or paying the interest charge generate in period *t*, (*t* = 1,…,*T*).
$i_t \in \mathbb{R}$	Bank balance at the end of period *t* (*t* = 1,…,*T*).
$u_{tn} \in \mathbb{R}^+$	Income tax liability for period *t* offset in year *t*+*n* (*t* = 1,…,*T; n* = 1,…, *N*).
$b_t \in \mathbb{R}^+$	Income tax base for year *t* (*t* = 1,…,*T*).
$p_t, l_t \in \mathbb{R}^+$	Absolute values of profit and loss before tax for period *t* (*t* = 1,…,*T*), respectively.

Table 2 Variables

Work shift	*Production equipment*						
	1	*2*	*3*	*4*	*5*	*6*	*7*
1	4,500	7,000	10,000	12,500	15,000	25,000	35,000
2	4,500	7,000	10,000	12,500	15,000	25,000	35,000
3	4,500	7,000	10,000	12,500	15,000	25,000	35,000
4	1,800	2,800	4,000	5,000	6,000	10,000	14,000
5	1,800	2,800	4,000	5,000	6,000	10,000	14,000

Table 3. Production capacity of the work shifts and types of equipment used in the computational experiment, in units of product per period. In the second computational experiment (section 4.2) were used the values corresponding to the shaded cells.

Case Number	*T*	*W*	*E*	*Number of variables*	*Number of constraints*	*Computing time (s.)*
1	10	2	3	2,143	3,030	0
2	10	2	4	5,143	6,030	1
3	10	2	5	8,143	9,030	3
4	10	2	6	11,143	12,030	6
5	10	2	7	14,143	15,030	12
6	10	3	3	17,013	25,056	30
7	10	3	4	33,253	49,056	160
8	10	3	5	49,493	73,056	442
9	10	3	6	65,733	97,056	987
10	10	3	7	81,973	121,056	7,574
11	10	4	3	89,840	169,784	4,421

Table 4. Number of variables and constraints, and computing time for the first computational experiment.

Case Number	*T*	*W*	*E*	*Number of variables*	*Number of constraints*	*Computing time (s.)*
1	10	2	3	1,359	1,724	0
2	10	2	4	2,699	2,694	0
3	10	2	5	4,039	3,664	1
4	10	2	6	5,379	4,634	1
5	10	2	7	6,719	5,604	2
6	10	3	3	4,109	4,392	1
7	10	3	4	7,109	7432	3
8	10	3	5	10,109	10,472	5
9	10	3	6	13,109	13,512	10
10	10	3	7	16,109	16,552	16
11	10	4	3	8,839	10,734	6
12	10	4	4	14,159	17784	15
13	10	4	5	19,479	24,834	34
14	10	4	6	30,119	38,934	61
15	10	4	7	30,119	38,934	104
16	10	5	3	15,549	22,670	27
17	10	5	4	23,849	36,310	74
18	10	5	5	32,149	49,950	159
19	10	5	6	40,449	63,590	285
20	10	5	7	48,749	77,230	486

Table 5. Number of variables and constraints, and computing time for the first computational experiment with hierarchised work shifts.

Labour costs	*Work shift*		
	1	*2*	*3*
Low	450	450	540
Medium	4,500	4,500	5,400
High	45,000	45,000	54,000

Table 6. Values corresponding to the three assumptions of labour costs for each work shift, operating the production equipment 1 in the first period. In monetary units (m.u.).

Economies of scale scenario	*Production equipment*		
	1	*2*	*3*
Low	450,000	682,000	946,000
Medium	450,000	633,000	802,000
High	450,000	545,000	577,000

Table 7. Cost of acquiring the equipment in the first period, for the three economies of scale assumptions. In monetary units (m.u.).

Scenario			*Period (work shift can change)*										*Period (work shift cannot change)*									
Demand	*Economies of scale*	*Labour costs*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Constant	L	L-M-H	1(3)					1(3)					1(3)					1(3)				
	M	L-M	1(3)					1(3)					1(3)					1(3)				
	M	H	3(1)										3(1)									
	H	L-M-H	3(1)										3(1)									
Increasing	L	L-M	2(1)		(2)					2(2)		(3)	2(3)							2(3)		
	L	H	2(1)		(2)					2(2)		(3)	2(3)								2(3)	
	M	L-M	2(1)		(2)					2(2)		(3)	2(3)							2(3)		
	M	H	2(1)		(2)					2(2)		(3)	3(2)									
	H	L-M-H	3(1)					(2)					3(2)									
Decreasing	L	L-M-H	2(3)	(2)		2(2)					(1)		2(3)			2(3)						
	M	L-M-H	2(3)	(2)		2(2)					(1)		2(3)			2(3)						
	H	L-M	3(2)					(1)					3(2)									
	H	H	3(2)					(1)					3(2)			3(2)						
Incr-Decr	L	L-M-H	2(1)	(2)			(3)	2(3)	(2)			(1)	2(3)					2(3)				
	M	L-M	2(1)	(2)			(3)	2(3)	(2)			(1)	2(3)					2(3)				
	M	H	2(1)	(2)			(3)	2(3)	(2)			(1)	3(2)									
	H	L-M-H	3(1)			(2)				(1)			3(2)									

Table 8. Optimal decisions corresponding to the 36 scenarios that result from combining the different assumptions of each one of three factors (evolution of demand, economies of scale and labour costs). Each row corresponds to a single scenario or to a set of scenarios. L, M and H in the cells of the second and third columns refer to, respectively, Low, Medium and High levels. The cells in the *Period* columns show the optimal decision at the beginning of each period when work shifts can be changed and when they cannot be changed. An 'n(m)' decision means that equipment of type n must be acquired/renewed and m work shifts must be active; An '(m)' decision means that equipment must not change and m work shifts must be active; a blank cell means that neither the equipment nor the number of active work shifts must change.

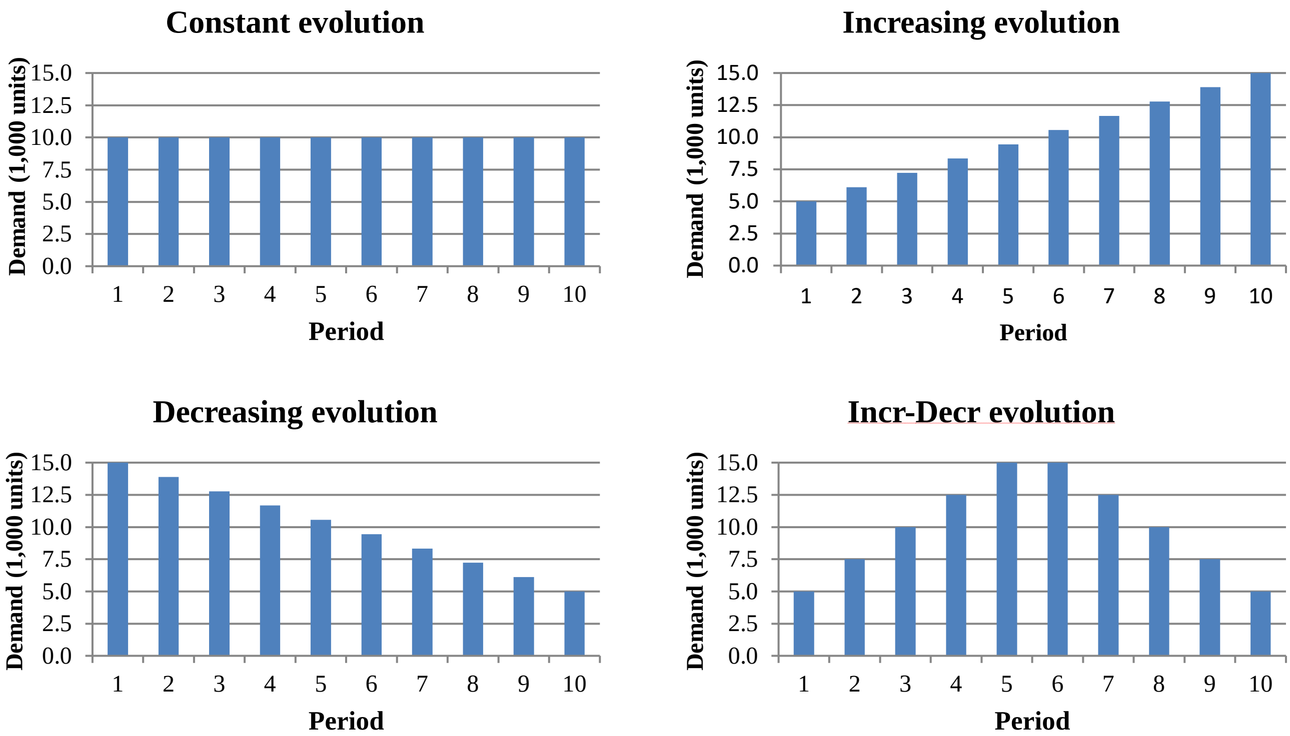

Fig. 1. The four assumptions on the evolution of the demand